14

Terrestrial Magnetism
and
Atmospheric Electricity

(Now entitled Journal of Geophysical Research)

An International Quarterly Journal

Volumen 40

1935

TERRESTRIAL MAGNETISM
AND
ATMOSPHERIC ELECTRICITY

X

TERRESTRIAL MAGNETISM

AND

ATMOSPHERIC ELECTRICITY

(Now entitled Journal of Geophysical Research)

AN INTERNATIONAL QUARTERLY JOURNAL

Volumen 40

1935

Reprinted with the permission of the original publishers

JOHNSON REPRINT CORPORATION

NEW YORK, NEW YORK

First reprinting, 1960, Johnson Reprint Corporation

Printed in Germany
Anton Hain K. G., Meisenheim/Glan

TERRESTRIAL MAGNETISM
AND
ATMOSPHERIC ELECTRICITY

AN INTERNATIONAL QUARTERLY JOURNAL

Founded by
LOUIS A. BAUER

Conducted by
J. A. FLEMING

Department Terrestrial Magnetism, Carnegie Institution
of Washington, D. C.

With the Assistance of

VOLUME 40
MARCH—DECEMBER, 1935

THE JOHNS HOPKINS PRESS
BALTIMORE, MARYLAND
1935

TABLE OF CONTENTS

MARCH, 1935

JUNE, 1935

SEPTEMBER, 1935

DECEMBER, 1935

VILHELM CARLHEIM-GYLLENSKÖLD
1859-1934

Terrestrial Magnetism
and
Atmospheric Electricity

VOLUME 40 MARCH, 1935 No. 1

RANDOM FLUCTUATIONS, PERSISTENCE, AND QUASI-PERSISTENCE IN GEOPHYSICAL AND COSMICAL PERIODICITIES

BY J. BARTELS

Abstract—The statistical aspects of the application of harmonic analysis, introduced by A. Schuster in his famous paper on the investigation of hidden periodicities, are discussed on the basis of recent developments in the theory of probability. Between the two extreme cases of random fluctuations and persistent waves, hitherto discussed exclusively, the intermediate case of quasi-persistence is introduced and recognized as a common phenomenon in the time-functions of meteorology, geophysics, and cosmical physics. Statistical methods, based on the conception of the harmonic dial, are given for dealing with quasi-persistence and its effect on tests for persistent waves, and they are generalized for the case of periodicities of other form than that of the sine-wave. Typical examples are given illustrating various forms of random fluctuations, quasi-persistence, and persistence, as well as questions related to harmonic analysis, such as the periodogram, non-cyclic change, curvature-effect, equivalent length of sequences, effective expectancy, random walk, interference, and the infective property of quasi-persistence on adjacent periods (see summary at end of paper).

(I) INTRODUCTION

1. *The problem*—Investigations on periodicities, cycles, recurrence-tendencies, and similar phenomena in geophysics proceed, in general, in three stages: (1) Analytical transformations of the observational data, for instance, harmonic analysis; (2) statistical studies on the results of these transformations, testing the degree of their significance; (3) physical explanations of the significant periodicities, for instance, by rotation-periods of the celestial bodies, by free or forced wave-motions or oscillations, etc.

These three stages are not in every case of equal importance, nor is their order invariable. Tidal theory, for instance, starts from the well-known movements of the celestial bodies and develops a specially adapted harmonic analysis, and there is hardly a need for the statistical viewpoint. The situation is, however, different with respect to the great number of geophysical periodicities in which the length of the period is given beforehand and only the form of the actual periodic variation in this interval is wanted, for instance, in the case of the solar and lunar diurnal variations and the annual variations, which occur in practically every geophysical phenomenon. Here statistical methods have been applied successfully, for instance, in the case of the solar diurnal magnetic variations, which show a marked day-to-day variability,[1,2] or in the case of the lunar diurnal variations of terrestrial magnetism or of atmospheric pressure, where small periodic changes are masked by much larger time-changes of different character, or in the case of the semi-

[1]S. Chapman and J. M. Stagg, London, Proc. R. Soc., A, 123, 27-53 (1929); 130, 668-697 (1931).
[2]J. Bartels, Terr. Mag., 37, 291-302 (1932).

annual variation of magnetic activity. There is a large and promising field for further use of such methods.

It seems, however, even more urgent to improve the procedure for testing the significance of such periodicities in which not even the lengths of the periods or recurrence-intervals are known or suspected from the outset. A large number of periods and cycles has been claimed in atmospheric temperature, rainfall,[3] solar radiation,[4] earthquakes, and even in business-activity,[5] while only very few of them have been generally recognized. This strange result has brought about a state of uncertainty and instinctive distrust which sometimes even affects the attitude towards perfectly sound periodicities. An attempt is made in this paper to discuss the elementary principles underlying research of periodicities. The main reasons for the contradictory results will be found in the lack of adequate combination of harmonic analysis with the theory of probability in its modern form.[6]

2. *Schuster's periodogram*—The "Investigation of hidden periodicities" published in 1898 by A. Schuster in this Journal[7] has become famous because it is the first successful attempt to "introduce a little more scientific precision into the treatment of problems which involve hidden periodicities" by applying the theory of probability. A. Schuster calculated his "periodogram" for 25 years of records of magnetic declination at Greenwich[8] and for sunspot-data,[9] modifying his original method according to the optical analogy between the periodogram and the spectrum of a luminous disturbance. A number of periodograms have been calculated since then; considerable progress in the practical application of the Schuster method, speeding up the heavy arithmetical work connected with it, has been made by K. Stumpff,[10] using instrumental methods, and L. W. Pollak,[11] who analyzed the international magnetic character-figure (designated C in this paper) for the years 1906 to 1926 using punched cards and Hollerith tabulating machines.

3. *A short review of literature*—Since Schuster's papers were written, a number of investigations in pure mathematics and theoretical physics have appeared bearing on subjects which are connected with periodogram-analysis—though this connection is not expressly mentioned and, sometimes, not even realized. Since these studies may be utilized for a revision and development of the periodogram-method, some of them may be enumerated here. On the analytical side, the theory of "almost periodic functions" created by H Bohr[12, 13] generalizes the ordinary Fourier series by considering sums of sine-waves with frequencies which

[3]See, for instance, the puzzling list of periods ranging from a few hours to 260 years in Sir Napier Shaw's Manual of Meteorology, vol. 2, pp. 312-327, Cambridge, 1928.

[4]C. G. Abbot, Smithson, Misc. Coll., **87**, No. 9 (1932); **87**, No. 18 (1933); **89**, No. 5 (1933).

[5]Edwin B. Wilson, Science, **80**, 193-199 (1933); Quart. J. Economics, 375-417 (May 1934).

[6]R. von Mises, Wahrscheinlichkeitsrechnung, Leipzig und Wien, 1931; E. Kamke, Einführung in die Wahrscheinlichkeitstheorie, Leipzig, 1932; A. Kolmogoroff, Grundbegriffe der Wahrscheinlichkeitsrechnung, Ergebn. Math., **2**, Nr. 3, Berlin (1933); A. Khintchine, Asymptotische Gesetze der Wahrscheinlichkeitsrechnung, Ergebn. Math., **2**, Nr. 4, Berlin (1933).

[7]Terr. Mag., **3**, 13-41 (1898).

[8]Cambridge, Phil. Trans., **18**, 107-135 (1899).

[9]London, Phil. Trans. R. Soc., A, **206**, 69-100 (1906).

[10]K. Stumpff, Analyse periodischer Vorgänge, Berlin, 1927.

[11]L. W. Pollak, Prager Geophysikalische Studien, Heft 3 (Čechoslovak. Statistik, Reihe 12, Heft 13), Prague, 1930.

[12]Harald Bohr, Fastperiodische Funktionen, Ergebn. Math., **1**, Nr. 5, Berlin (1932).

[13]A. S. Besicovitch, Almost periodic functions, Cambridge, 1932; N. Wiener, The Fourier integral and certain of its applications, Cambridge, 1933.

are *not* entire multiples of a fundamental frequency. On the statistical side, the fundamental problem variously named "random vibrations," "random flights," or "random walk" (Irrfahrt), has been treated by Lord Rayleigh,[14] on whose first paper A. Schuster based his periodogram, J. C. Kluyver,[15], K. Pearson,[16], G. Pólya,[17] and G. I. Taylor.[18] Some relations can also be found to papers on Brownian movement, or eddy-diffusion in the atmosphere.[19, 20, 21] A part of the optical analogy, the superposition of light-waves with random phases, has been treated by M. von Laue[22] and A. Einstein.[23] A. Basch's theory of "error-tensors"[24] developed for geodetical purposes must also be mentioned. A. Glogowski,[25] in a dissertation on hidden periodicities, does not sufficiently emphasize the statistical viewpoint and misconstrues Schuster's methods.

Of the many papers dealing with geophysical and cosmical periodicities, a few may be selected as containing theoretical discussions of the periodogram-method. G. U. Yule[26] discusses the effect of superposed fluctuations and disturbances on harmonic analysis. Sir Gilbert Walker[27] defines criteria for reality of periods. L. Weickmann's discovery of "symmetry-points" in the records of atmospheric pressure entailed a number of studies on periodicity in general.[28, 29] H. H. Turner[30] considered discontinuities in meteorological phenomena. Leo Keller[31] amplifies the mathematical system of periodography in a form suitable for geophysical applications.

4. *Plan of this paper*.—It is not proposed to give here a bibliographical account of the contributions of the various authors to the theory of periodogram-analysis. It seems to be more convenient to derive the new results directly by using elementary graphical illustrations of harmonic analysis.

It would have been possible to derive the results of this paper in a quite general way, discussing mathematical-statistical properties of "populations" formed by a number of vectors in two or more dimensions. However, it seemed more appropriate to show the need for these considerations by dealing with time-functions representing actual geophysical phenomena. After the introduction of the conception of persistence and quasi-persistence as contrasted with random fluctuations,

[14] Lord Rayleigh, Phil. Mag., **10**, 73-78 (1880); **36**, 429-449 (1918); **37**, 321-347, 498-515 (1919). Reprinted in Scient. Papers **1** and **6**, Cambridge, 1899 and 1920.

[15] J. C. Kluyver, Amsterdam, Proc. Akad. Wet., **8**, 341-350 (1906).

[16] K. Pearson, A mathematical theory of random migration (Math. contrib. to the theory of evolution, **15**), London, 1906.

[17] G. Pólya, Zürich, Mitt. Physik. Ges., **19**, 75-86 (1919).

[18] G. I. Taylor, London, Proc. Math. Soc., **20**, 196 ff. (1922).

[19] O. G. Sutton, London, Proc. R. Soc., A, **135**, 143-165 (1932).

[20] L. F. Richardson and J. A. Gaunt, London, Mem. R. Met. Soc., **3**, No. 30 (1930).

[21] O. F. T. Roberts, London, Mem. R. Met. Soc., **4**, No. 37 (1933).

[22] M. von Laue, Ann. Physik, **47**, 853-878 (1915); **48**, 668 ff. (1915).

[23] A. Einstein, Ann. Physik, **47**, 879-885 (1915).

[24] Wien, SitzBer. Akad. Wiss., Math.-Nat. Klasse, Abt. IIa, **137**, 583-598 (1928).

[25] A. Glogowski, Beiträge zur Auffindung verborgener Periodizitäten, Münster i. W., 1929.

[26] G. U. Yule, London, Phil. Trans., A, **226**, 267-298 (1927).

[27] Sir Gilbert Walker, London, Quart. J. R. Met. Soc., **51**, 337-346 (1925); London, Mem. R. Met. Soc., **1**, No. 9 (1927); **3**, No. 25 (1930); Mon. Weath. Rev., **59**, 277-278 (1931); London, Proc. R. Soc., A, **131**, 518-532 (1931). See also D. Brunt, Memoirs R. Met. Soc., **2**, No. 15 (1928), the discussion in London, J. R. Met. Soc., **54**, 299-303 (1928), and R. A. Fisher, London, Proc. R. Soc., A, **125**, 54-59 (1929).

[28] L. Weickmann, Beitr. Geophys., **34**, 244-251 (1931).

[29] K. Stumpff, Beitr. Geophys., **32**, 379-411 (1931); F. Dilger, Beitr. Geophys., **30**, 40-95 (1931).

[30] H. H. Turner, London, Quart. J. R. Met. Soc., **41**, 315-336 (1915); **42**, 163-173 (1916); **43**, 43-60 1917).

[31] L. Keller, Beitr. Physik frei. Atmos., **19**, 173-187 (1932).

if often appeared unnecessary to repeat the definitions in general abstract formulations. Furthermore, we regard throughout the paper all time-functions as given by values at equidistant intervals of time. This assumption clarifies the argument and holds in most geophysical applications. Continuous functions of time could have been treated in exactly the same way, replacing the sums by integrals, without introducing a fundamentally different conception; in fact, continuous recording is practically represented by values at very short intervals of time.

The standpoint taken in the present paper is the outcome of work on periods in meteorology and terrestrial magnetism, and has been discussed during several years in a number of talks at the Department of Terrestrial Magnetism of the Carnegie Institution of Washington and in courses of lectures at Berlin University. Exact proofs for some theorems involving theory of probability are omitted here and will be given in a later paper, to appear in the series "Ergebnisse der Mathematik" (Berlin, J. Springer).

(II) Harmonic Analysis as a Mathematical Representation of the Observations

5. *Principle of harmonic analysis*—Records of geophysical phenomena yield functions of time, f(t), which for further research are mostly transformed into a series of values for equal intervals of time, for instance, hourly, daily, monthly, annual, etc. The record may cover the time $t=0$ to $t=T$. For convenience, another time-variable, $x=t\times 2\pi/T$, is introduced so that the length of the record, as measured by x, is 2π. The number of values (or ordinates) given may be r; that is, the times (or abscissae) $x_1, x_2, \ldots x_r$ divide the time-interval 0 to 2π into r equal parts, and y_ρ may be the value of the variable for the time

(5.1) $$x_\rho=\rho(2\pi/r)$$

No attention is paid, at this stage, to the value y_0 at the time $x_0=0$ (see section 16).

Consider sine-functions and cosine-functions of frequency $\nu=0, 1, 2, \ldots k$, that is, completing ν cycles in the interval 0 to 2π [lengths of periods $p_\nu=T/\nu$], and their sum

(5.2) $$\phi_k(x)=a_0+(a_1\cos x+b_1\sin x)+(a_2\cos 2x+b_2\sin 2x)$$
$$+\ldots+(a_k\cos kx+b_k\sin kx)$$

Harmonic analysis consists in determining the coefficients $a_0, a_1, b_1, \ldots a_k, b_k$ so that $\phi_k(x_\rho)$ approximates the given ordinates y_ρ, in other words, that the *residuals* $[y_\rho-\phi_k(x_\rho)]$ are as small as possible. This problem is readily solved if it is put into the form that the average of the squared residuals

(5.3) $$s_k^2=\Sigma_\rho\,[y_\rho-\phi_k\,(x_\rho)]^2/r$$

shall be made a minimum.

Since ϕ_k contains $(2k+1)$ coefficients and shall represent r ordinates, we consider only values of k so that

(5.4) $$2k+1 \leqq r$$

Then it can be shown that the coefficients are given by the equations

(5.5) $$r\,a_0=\Sigma_\rho\,y_\rho,\ (r/2)\,a_\nu=\Sigma_\rho\,y_\rho\cos\nu x_\rho,\ \text{and}\ (r/2)\,b_\nu=\Sigma_\rho\,y_\rho\sin\nu x_\rho$$

where the sums are taken for $\rho=1, 2, \ldots r$, and ν runs from 1 to k. a_0 is the arithmetic mean of the y_ρ, and (a_ν, b_ν) are called the harmonic coefficients of the set y_ρ of ordinates. If r is an even number, the formula for $a_{r/2}$ is

(5.6) $$a_{(r/2)}=(-y_1+y_2-y_3+y_4-\ldots+y_r)/r$$

which differs from the formula for $\nu<r/2$ in so far as the right-hand sum is divided by r and not by $(r/2)$.

From the linear form of the equations (5.5), the theorem on *superposition* of different functions is easily verified, namely: A finite number of ordinates $y'_1, y'_2, \ldots y'_r; y''_1, y''_2, \ldots y''_r; \ldots$ may be given, and (a'_ν, b'_ν) may be the harmonic coefficients for the set $y'_1, y'_2, \ldots y'_r$, etc. Then a set of ordinates formed by the linear combination $A'y'_\rho+A''y''_\rho+\ldots$ $(\rho=1, 2, \ldots r)$, with constants $A', A'', \ldots$, has the harmonic coefficients $(A'a'_\nu+A''a''_\nu+\ldots, B'b'_\nu+B''b''_\nu+\ldots)$. This is known as the *additive property of the harmonic coefficients*, or principle of *superposition.*

The formulae (5.5) do not contain any reference to k, that is, the number of terms of the series $\phi_k(x)$. Each harmonic coefficient is therefore determined *independently*, regardless of the number of additional harmonic terms involved. This is a consequence of the so-called *orthogonality* of sine-waves and cosine-waves with periods which are submultiples of one and the same main period.

A proof of the formulae (5.5), and a discussion of some other points such as smoothing, non-cyclic variation, etc., is given in the *appendix.*

6. *The harmonic dial*—The sine- and cosine-functions of frequency ν can be combined into a *sine-wave* with (positive) amplitude c_ν and phase α_ν

(6.1) $a_\nu \cos \nu x+b_\nu \sin \nu x=c_\nu \sin (\nu x+\alpha_\nu)$, with

(6.2) $a_\nu=c_\nu \sin \alpha_\nu$, $b_\nu=c_\nu \cos \alpha_\nu$ and $c_\nu^2=a_\nu^2+b_\nu^2$, $\tan \alpha_\nu=a_\nu/b_\nu$

These relations can be illustrated in the *harmonic dial* for the frequency ν. In a plane coordinate system, in which a_ν is measured upward, and b_ν to the right (Fig. 1), the expression (6.1) is represented by a point P

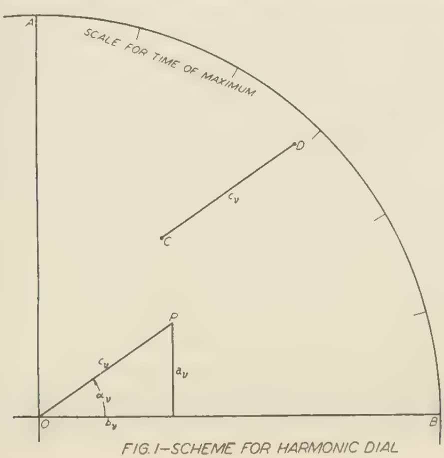


FIG. 1—SCHEME FOR HARMONIC DIAL

having the rectangular coordinates a_ν, b_ν, and, because of (6.2), the polar coordinates (c_ν, α_ν); or, also, by the vector OP having the projections a_ν, b_ν on the axes, the length c_ν and the azimuth α_ν. This vector will be called $\mathbf{c}_\nu$. The first of the ν maxima of the wave (6.1) occurs when $(\nu x+\alpha_\nu)=90°$, that is, at the time $x_{max}=[90°-\alpha_\nu]/\nu$. Therefore, $\alpha_\nu=90°$ corresponds to $x_{max}=0$, $\alpha_\nu=0°$ to $x_{max}=90°/\nu$, etc. It is therefore possible to indicate, on a circle around the origin, the times x_{max} (or t_{max}, expressed in the original time t) for the waves represented by vectors pointing in that direction. This gave the name to the diagram, because, in a semi-diurnal wave [time interval T from 0^h to 12^h], $t=1^h$ corresponds to $x=2\pi/12=30°$, and the scale for t_{max} becomes the ordinary dial of a clock.

The "blank" for a harmonic dial of a certain frequency contains the origin O, a linear scale for the amplitudes c_ν [or a number of circles around the origin designating certain values of c_ν] and a circular scale at the edge, marked with the occurrence of the maximum and, incidentally, giving the length of the period p_ν. Changes of units for c_ν or of time origin [for instance, from local to Greenwich time in dials for diurnal waves] are easily indicated by renumbering the respective scales. Each point P entered, as a dot, in this blank represents, by the vector OP, a sine-wave of the period p_ν. Since the blanks for harmonic dials of the period p_ν for the intervals $t=0$ to T, T to $2T$, $2T$ to $3T$, etc., are identical except for the numbering of the circular scales, which differ by multiples of T, they can all be combined into that for the interval $t=0$ to T, because the various intervals can be indicated by marking the dots P.

7. *Vector-addition in harmonic dials and the average vector*—It is sometimes convenient to ascribe to each vector CD on the harmonic dial the same meaning as to the parallel vector OP starting at the origin, so that all parallel vectors of equal length denote the same sine-wave. Then, the additive property of the harmonic coefficients [section 5] has its graphical analogy in the usual *vector-addition.*

A number (say, n) of sine-waves of equal frequency ν may be indicated as vectors $\mathbf{c}'_\nu$, $\mathbf{c}''_\nu$, . . . starting at the origin O, and plotted as dots denoting the ends of the vectors. If these sine-waves are added and divided by n, the average sine-wave has the harmonic coefficients $[(a'_\nu+a''_\nu+\ldots)/n]$, $[(b'_\nu+b''_\nu+\ldots)/n]$ and is therefore represented by the mass-center of the n dots, or the *average vector* $(\mathbf{c}'_\nu+\mathbf{c}''_\nu+\ldots)/n$.

This remark is often used as follows: Suppose the number r of ordinates is an entire multiple of the frequency ν, say, $r=\nu r_1$. Then the angles νx_ρ (5.1) are $\nu\rho\,(2\pi/\nu r_1)=\rho(2\pi/r_1)$, so that (apart from irrelevant multiples of 2π), $\nu x_1=\nu x_{r_1+1}=\nu x_{2r_1+1}=\ldots$, etc. The equation (5.5) for a_ν (and for b_ν) can therefore be rearranged as follows

$$\begin{aligned}(7.1)\qquad a_\nu=(2/r)\,\Sigma_{\rho=1}^{r}\,y_\rho\cos\nu x_\rho=1/\nu\,&[(2/r_1)\,\Sigma_{\lambda=1}^{r_1}\,y_\lambda\cos\lambda\,(2\pi/r_1)\\&+(2/r_1)\,\Sigma_{\lambda=1}^{r_1}\,y_{r_1+\lambda}\cos\lambda\,(2\pi/r_1)+\ldots\\&+(2/r_1)\,\Sigma_{\lambda=1}^{r_1}\,y_{(\nu-1)r_1+\lambda}\cos\lambda\,(2\pi/r_1)]\end{aligned}$$

Comparing the first term in the bracket with (5.5), we realize that it is the coefficient for frequency 1 of the ordinates y_1 to y_{r_1}, and the second term is the coefficient for frequency 1 of the ordinates y_{r_1+1} to y_{2r_1},

etc., and a_ν is the average of these ν coefficients. In other words, if a period p comprises an interval represented by r_1 ordinates, and ν such intervals are given, then the harmonic analysis of the total of νr_1 ordinates gives, for the period p, harmonic coefficients which are the arithmetic means of the ν harmonic coefficients computed from each single interval of r_1 ordinates.

In another arrangement, (7.1) becomes

$$(7.2)\qquad a_\nu=(2/r_1)\ \Sigma_{\lambda=1}^{r_1}\ (1/\nu)\ (y_\lambda+y_{r_1+\lambda}+y_{2r_1+\lambda}+\ldots+y_{(\nu-1)r_1+\lambda})\ \cos\lambda\ (2\pi/r_1)$$

This is the basis of many schemes for numerical harmonic analysis, starting by writing the ordinates in ν rows of r_1 each, and then analyzing the averages of the r_1 columns.

8. *International magnetic character-figure C and harmonic dial for 27-day period*—Examples demonstrating the use of the harmonic dial for research on solar and lunar diurnal variations and for annual variations have been given formerly.[32] For the purpose of this paper, the series of the daily international magnetic character-figures C has been selected, comprising the 10,206 days between January 11, 1906, and December 20, 1933. C indicates the degree of magnetic activity for each Greenwich day by one of the figures 0.0 (denoting very quiet conditions), 0.1, 0.2, etc., to 2.0 (denoting very great disturbances). The rotation-period of the Sun, of about 27 days, is reflected in C in the recurrence of quiet and disturbed times.[33] This recurrence is demonstrated in graphical day-by-day records published in this JOURNAL.[34] For these diagrams, the whole series has been divided into 27-day intervals. For convenience, we shall refer to these intervals as "rotations" numbered 1 (beginning January 11, 1906) to 378 (beginning November 24, 1933). In each rotation the days are numbered 1 to 27. The dates of the first days in each rotation can be taken from the diagram in Volume 39 of this JOURNAL or from the table on Figure 15 of this paper; the dates are repeated, with a shift of one or two days (after leap-years), every second year, since $2\times365=27\times27+1$.

The character-figures C for the years 1906 to 1926 have been used in Pollak's publication.[11] It may be remarked, however, that it is not intended here to demonstrate again the 27-day recurrence or to repeat Pollak's periodogram-analysis; the series of C is only taken as a suitable illustration of the general argument, which will gradually lead to other conclusions than those drawn by Pollak.

For each of the 378 rotations, the harmonic coefficients of the sine-wave of 27-day period were computed and the results are represented in the harmonic dial of Figure 2. The dots are distributed in a "cloud" around the origin without, apparently, preferring any direction; the average vector, that is, the mass-center of the cloud formed by all dots, indicated by a cross, falls close to the origin. The largest amplitude is 0.760 unit of C [or $0.760C$] for rotation No. 208, beginning May 1, 1921, and containing the heaviest magnetic disturbances [about May 12

[32] J. Bartels, Zs. Geophysik, **3**, 389-397 (1927); Handbuch d. Experimentalphysik, **25**, I. Teil, 167 ff., 631 ff. (Leipzig, 1928); Sci. Mon., **35**, 110-130 (1932); Terr. Mag., **37**, 22-27, 291-302 (1932).

[33] C. Chree and J. M. Stagg, London, Phil. Trans. R. Soc., A, **227**, 21-62 (1927).

[34] Terr. Mag., **37**, 42 (1932), for the years 1906 to 1930; **39**, 201-202 (1934), for the years 1923 to 1933 together with a similar diagram for sunspots.

to 21] of our series; the maximum of this wave falls near May 16. Amplitudes of less than $0.01C$ occur in rotations Nos. 61 and 372. The diagram will be referred to in later discussions.

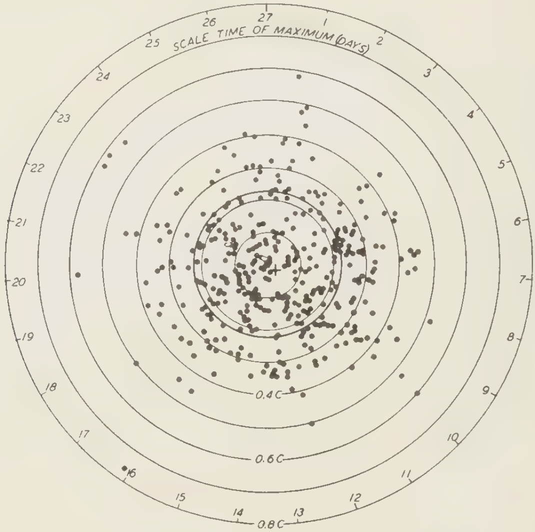


FIG. 2—HARMONIC DIAL, INTERNATIONAL MAGNETIC CHARACTER-FIGURE C, 1906-1933, FOR EACH OF 378 INTERVALS OF 27 DAYS BEGINNING JANUARY 11, 1906 (=DAY 1)—SINE-WAVES OF 27-DAY PERIOD

9. *Graphical interpretation of harmonic analysis*—While we shall not go into the much-discussed details of practical harmonic analysis,[35] that is, the actual evaluation of the equation (5.5) for the coefficients, a graphical interpretation of these equations, using the principle of superposition (section 5), will be helpful later.

The character-figures C for rotation No. 275, starting April 14, 1926, have been plotted in the top row of Figure 3A. This set of 27 ordinates can be conceived as a sum of 27 primitive sets, in each of which all ordinates are zero except one; the first three of these sets are plotted in Figure 3A. Generally speaking, the set of ordinates

$$y_1, y_2, y_3, \ldots, y_r \tag{9.1}$$

is equivalent to the sum of the primitive sets

$$\left\{\begin{array}{l} y_1, \ 0, \ 0, \ldots, 0 \\ 0, \ y_2, \ 0, \ldots, 0 \\ 0, \ 0, \ y_3, \ldots, 0 \\ \ldots\ldots\ldots\ldots\ldots \\ 0, \ 0, \ 0, \ldots, y_r \end{array}\right. \tag{9.2}$$

[35] For practical harmonic analysis see C. Runge and F. Konig, Numerisches Rechnen, pp. 208-231, Berlin, 1924; also E. T. Whittaker and G. Robinson, The calculus of observations, London, 1924. For some schemes used in geophysical applications see C. R. Duvall and C. C. Ennis, Terr. Mag., **32**, 151-162 (1927); J. Bartels, Beitr. Geophysik, **28**, 1-10 (1930); and the book of K. Stumpff already noted under footnote 10. A great help in numerical work is given by L. W. Pollak, Handweiser zur harmonischen Analyse (Prager Geophysikal. Studien Heft 2), which has appeared as Czechoslovakische Statistik, Reihe 12, Heft 10, Prague, 1928, while the same author's "Rechentafeln zur harmonischen Analyse," Leipzig, 1926, can in general be replaced by Crelle's Rechentafeln or by the slide-rule.

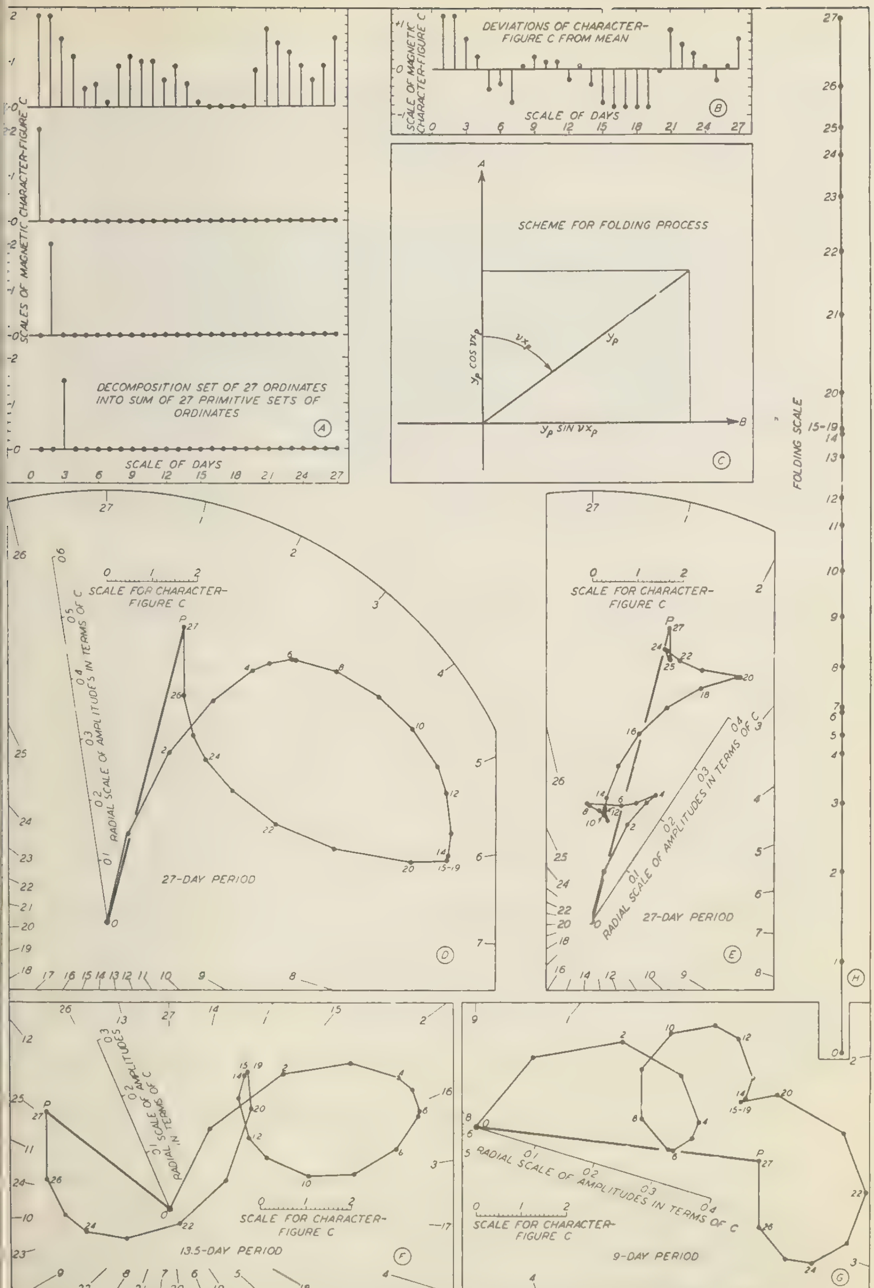


FIG. 3—GRAPHICAL HARMONIC ANALYSIS, OR FOLDING PROCESS, FOR INTERNATIONAL MAGNETIC CHARACTER-FIGURES FOR THE 27 DAYS, APRIL 14 TO MAY 10, 1926, FOR PERIODS OF 27, 13.5, AND 9 DAYS

According to (5.5), the harmonic coefficients, multiplied by $(r/2)$, for each primitive set are given, for the frequency ν, by $y_\rho \cos \nu x_\rho$, $y_\rho \sin \nu x_\rho$ (where $\rho = 1, 2, \ldots, r$): the representation in the harmonic dial (Fig. 3*C*) is a vector of length y_ρ forming the angle νx_ρ with the direction *OA*, because the projections of this vector are equal to the coefficients (times $r/2$). The sum of these r primitive vectors has, according to (5.5), the projections $(r/2)\, a_\nu$, $(r/2)\, b_\nu$ and represents therefore the sine-wave for the original set (9.1). In our example—the set of 27 ordinates in the top row of Figure 3*A*—$r = 27$, $x_1 = 2\pi/r = 13°.3$: for a sine-wave of 27-day period, $\nu = 1$, the angles νx_ρ for the successive primitive vectors are 13°.3, 26°.7, 40°.0, . . . , 360°.0, and the whole construction of summing the vectors consists in joining together the ordinates y_ρ, changing successively the direction clockwise by 13°.3 (Fig. 3*D*). The vector between *O* and the end-point, *P*, should be divided by $(r/2) = 27/2$ to obtain the amplitude c_1: instead, we can measure it in a scale enlarged (27/2) times (radial scale for *OP* indicated in Fig. 3*D*). Thus, we see from Figure 3*D*, comparing it with Figure 1, that the orthogonal coordinates of *P*, in units of *C*, are $a_1 = +0.48$, $b_1 = +0.13$, and its polar coordinates $c_1 = 0.50$, $\alpha_1 = 76°$; the sine-wave, therefore, is

$$+0.48 \cos x + 0.12 \sin x = 0.50 \sin (x + 76°) \tag{9.3}$$

Its maximum occurs about the time $x = 14°$, or $t = 14 \times (27/360) = 1.05$ days, or, since the time 1 day designates Greenwich noon of April 14, 1926, about 1 o'clock in the afternoon of that day.

If all the ordinates $y_1, y_2, \ldots y_r$ were equal, the construction in Figure 3*D* would lead to a regular polygon ending at the origin, that is, to vanishing coefficients, as could be expected. From the principle of superposition it follows, therefore, that a positive or negative constant can be added to all ordinates without changing the harmonic coefficients. For instance, the arithmetic mean a_0 can be subtracted, which amounts to measuring the ordinates in positive or negative *deviations* from the level a_0 (Fig. 3*B*); the construction of Figure 3*E*, plotting negative ordinates in the reverse direction, leads, of course, to the same point *P* as Figure 3*D*.

Figures 3*F* and 3*G* are analogous to Figure 3*D* and show the construction of the harmonic coefficients with frequencies $\nu = 2$ and 3, or periods of 13.5 and 9 days. The scales for the time of maximum are entered on scales around Figures 3*F* and 3*G*, while the scales for *OP* are the same in all diagrams 3*D*, 3*E*, 3*F*, and 3*G*. The sine-waves of frequencies 2 and 3, in units of *C*, are

(9.4) $+0.16 \cos 2x - 0.20 \sin 2x = 0.26 \sin (2x + 141°)$ with maxima on days 11.6 and 25.1

(9.5) $-0.06 \cos 3x + 0.46 \sin 3x = 0.46 \sin (3x + 353°)$ with maxima on days 2.4, 11.4, and 20.4

10. *The harmonic folding process*—The constructions in Figures 3*D*, 3*F*, and 3*G* can be interpreted as follows: Imagine a *folding scale* having links of the lengths of the ordinates y_ρ. If stretched out, as illustrated in Figure 3*H*, its entire length is equal to the sum of the ordinates, in the general case, $r\, a_0$. Suppose now each joint is turned by the angle $360°/27 = 13°.3$; we then obtain Figure 3*D*; by turning each

joint by $2\times 360°/27$ or $3\times 360°/27$, we obtain Figures $3F$ and $3G$. In general, this bending of the folding scale by the angles $\nu\times 2\pi/r$ furnishes $(r/2)\ a_\nu$ and $(r/2)\ b_\nu$, and the distance of the end-point from the origin is $(r/2)\ c_\nu$. This idea of the *harmonic folding process*, as it can be termed, will be helpful later.

Usually, only the result of the folding process, P, is retained in the harmonic dial. Nevertheless, and although for most actual computations numerical or mechanical harmonic analysis is preferable to graphical analysis, it is sometimes useful to recall the folding process as producing the vector OP, because it reveals the contribution of each single ordinate to the final vector. This contribution is particularly clear in the folding of the deviations from the arithmetic mean [the folding rule itself having then positive and negative links]; in Figure $3E$, the large positive ordinates 1 to 3 and the large negative ordinates 15 to 18 make the largest strides towards P. For illustration, Figure 4 shows, for an exact cosine-

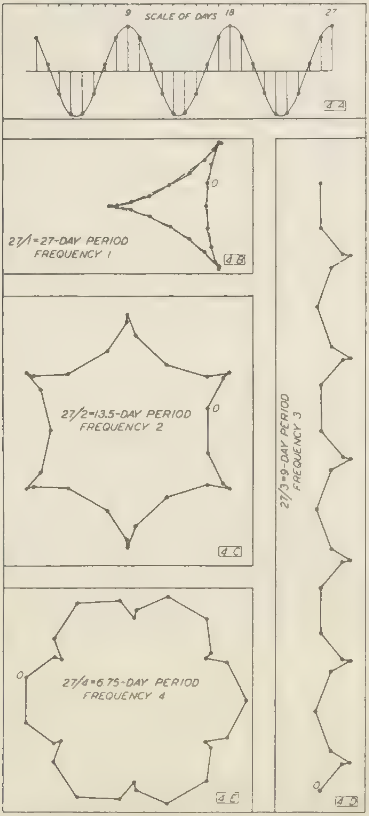


FIG. 4—A PERIOD OF FREQUENCY 3 IN 27 ORDINATES AND ITS GRAPHICAL HARMONIC ANALYSIS, OR FOLDING PROCESS, FOR PERIODS OF FREQUENCY 1, 2, 3, & 4

wave of 9-day periods, the folding process for the frequencies 1 to 4 (periods 27, 13.5, 9, and 6.75 days) of a set of 27 days.

The folding process is also a good illustration of the remark at the end of section 7.

11. *Standard deviations for sine-waves and residuals*—An *exact* representation of the set of r ordinates y_ρ is obtained by a series $\phi_k(x)$ of the form (5.2) if it is extended so that the number of coefficients $a_0, a_1, b_1, \ldots a_k, b_k$ is equal to that of the ordinates; if r is uneven, then $k=(r-1)/2$, while for even values of r the last term is $a_{(r/2)} \cos (rx/2)$, with $a_{(r/2)}$ given by (5.6). This procedure to represent a set of ordinates in the interval $t=0$ to T as a sum of sine-waves, as well as the approximation obtained for smaller values of k, is a *purely mathematical affair* and involves in no way the physical nature of the phenomenon described by these ordinates. Especially the fact that the sum $\phi_k(x)$ is periodic, repeating its values after intervals which are entire multiples of T, does not imply a similar property of the geophysical phenomenon outside the range of observation. The question of the physical meaning of the various sine-waves, and the possibility of "*forecasting*" *by means of periodicities* requires, therefore, additional tests, statistical in nature, which will be discussed later.

With less than r coefficients, the series $\phi_k(x)$ gives only an approximation, the degree of which can be estimated in the following way: The deviations of the given ordinates y_ρ from their respective arithmetic mean a_0 may be called

(11.1) $$z_\rho = y_\rho - a_0 \text{ (for values of } \rho = 0, 1, 2, \ldots r)$$

The standard deviation ζ may be defined as usual, that is, ζ^2 is the average of the z_ρ^2. It can be easily calculated from the y_ρ^2 and a_0: for $z_\rho^2 = y_\rho^2 - 2y_\rho a_0 + a_0^2$ and summing over $\rho=1$ to r gives $\Sigma z_\rho^2 = \Sigma y_\rho^2 - 2a_0 \, \Sigma y_\rho + r a_0^2$; replacing Σy_ρ by ra_0 and dividing by r, we obtain the well-known formula

(11.2) $$\zeta^2 = \Sigma y_\rho^2 / r - a_0^2$$

It can be shown (Appendix 1) that the average value of $\phi_k(x_\rho)$ is a_0, and its standard deviation η_k is given by $\eta_k^2 = (a_1^2 + b_1^2 + a_2^2 + b_2^2 + \ldots + a_k^2 + b_k^2)/2$ or, applying (6.2)

(11.3) $$\eta_k^2 = (c_1^2 + c_2^2 + \ldots + c_k^2)/2$$

except in the [geophysically irrelevant] case of the exact representation and r even, when the last term in the bracket is $2\, a_{(r/2)}^2$. Furthermore, the standard deviation s_k of the residuals, defined by (5.3), yields, on evaluation (see Appendix 1), the remarkably simple expression

(11.4) $$s_k^2 = \zeta^2 - \eta_k^2$$

or

(11.5) $$s_k^2 = \zeta^2 - (c_1^2 + c_2^2 + \ldots + c_k^2)/2 = (1/r) \; \Sigma \; y_\rho^2 - a_0^2 - (c_1^2 + c_2^2 + \ldots + c_k^2)/2$$

Each additional harmonic term reduces, therefore, the residuals by subtracting half of its squared amplitude from ζ^2, the squared standard deviation of the given ordinates. This applies also if only one or a few terms of ϕ_k are selected, for instance, the waves with frequencies 2 and 4.

In the case of exact representation all residuals and therefore s_k^2 are zero so that, from (11.5),

$$(11.6) \qquad (c_1^2+\ldots+c_{l-1}^2)/2+a_l^2=\zeta^2 \text{ (with } r \text{ even, } l=r/2)$$

$$\text{or} \qquad (c_1^2+\ldots c_l^2)/2=\zeta^2 \text{ (with } r \text{ uneven, } l=(r-1)/2)$$

For convenience, we shall put, for r even, $a_{(r/2)}\sqrt{2}=c_{(r/2)}$, so that the second equation always holds. This equation (for convenience, we shall only consider the case of r uneven) furnishes an estimate for the *upper limit of the remaining coefficients*, if a number of coefficients, up to the index k, have already been computed; because, from (11.6) and (11.5)

$$(11.7) \qquad c_{k+1}^2+c_{k+2}^2+\ldots+c_l^2=2\,\zeta^2-c_1^2-c_2^2-\ldots-c_k^2=2\,s_k^2$$

The square of the largest coefficient among the coefficients of the terms with higher frequency than k can therefore be not larger than the right-hand side, $2\,s_k^2$.

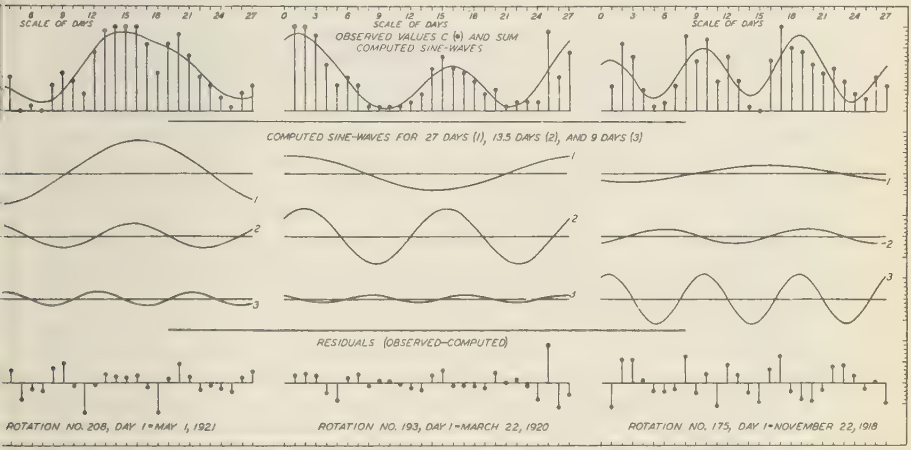


FIG. 5—HARMONIC ANALYSIS OF INTERNATIONAL MAGNETIC CHARACTER-FIGURE C FOR THREE INTERVALS OF 27 DAYS (ROTATIONS), SHOWING C, SUM OF COMPUTED SINE-WAVES OF 27-, 13.5-, AND 9-DAY PERIODS, SEPARATE SINE-WAVES, AND RESIDUALS

12. *Examples*—Figure 5 illustrates the harmonic analyses of the international magnetic character-figure for the three 27-day rotations No. 208 (day 1 = May 1, 1921), No. 193 (day 1 = March 22, 1920), and No. 175 (day 1 = November 22, 1918) which, in this order, have the greatest amplitude c_1, c_2, and c_3 for the waves of frequency 1, 2, and 3, or 27-, 13.5-, and 9-day period, found in any of the 378 rotations analyzed. Rotation 208 was mentioned at the end of section 8 as containing the heavy disturbances of May 12 to 21, 1921. Figure 5 gives, for each of the three rotations, in the first row the observed C, and the sum of three sine-waves, then the three sine-waves separately, and, finally, the residuals or differences between the observed C and the sum of the sine-waves. Some numerical values are given in Table 1; day 0 (or 27) is the origin of time, $\alpha=90°$ means that a maximum of the sine-wave occurs on day 27. The standard deviation ζ refers to the observed values of C, η_3 to the sum of the three sine-waves $[\eta_3^2=(c_1^2+c_2^2+c_3^2)/2]$, and s_3 to the residuals $(s_3^2=\zeta^2-\eta_3^2)$. The unit used is 0.01 unit of C.

TABLE 1—*Harmonic analysis of the three rotations with the largest amplitudes c_1, c_2, and c_3 (unit for a_0, c_1, c_2, c_3, ζ, η_3, and s_3 is 0.01C)*

Rotation number	Arith-metic mean	27-day period		13.5-day period		9-day period		Standard deviations		
	a_0	c_1	a_1	c_2	a_2	c_3	a_3	ζ	η_3	s_3
			°		°		°			
208	90	76	235	29	32	16	318	66	59	29
193	76	42	79	66	40	9	121	62	55	27
175	90	20	240	17	290	60	60	55	46	30

13. *Generalized harmonic dial*—The equations given in section 11 suggest the conception of a *generalized harmonic dial*[36] consisting of a rectangular coordinate-system in $2k$ dimensions, the axes assigned to $a_1, b_1, \ldots a_k, b_k$. Our set of r ordinates [or deviations z_ρ] is then represented by a single point P in this system, or the vector OP, and superposition is again represented by vector addition. The ordinary harmonic dials for the various frequencies are two-dimensional projections of the generalized dial. For the exact representation [if r is even, the last coordinate entered is not $a_{(r/2)}$, but $c_{(r/2)}=\sqrt{2}\times a_{(r/2)}$], the length of the vector OP is, according to (11.6), equal to $\zeta\sqrt{2}$. All sets of ordinates with the same standard deviation ζ are therefore exactly represented by a point on the sphere with radius $\zeta\sqrt{2}$. The formulae (11.4) and (11.5) for approximate representation can also be easily interpreted in this geometrical illustration. While, of course, actual drawings cannot be made, the conception of the generalized dial will be found useful in certain applications, especially for the transition from sine-waves to periodicities of other form (section 40).

14. *The ordinary periodogram*—The *periodogram* of a function f(t) in the interval $t=0$ to T is a diagram in which the amplitudes c_ν of the sine-waves are plotted against their frequencies ν or their periods T/ν. A. Schuster[8,9] himself favored later the use of c_ν^2 (instead of c_ν) as ordinate in order to simplify the statistical considerations based on the periodogram [or, as he called it, the periodograph], but for actual plotting c_ν is preferred as an illustration.

For the series of the international magnetic character-figure C for the years 1906-1926 used in Pollak's[11] paper, with a total of $r=7670$ days, an exact representation would be obtained by the same number of coefficients, namely, the average $a_0=0.62C$ [C is, as always, used to denote the unit of character-figures], 3834 amplitudes c_ν and as many phases a_ν, and, finally, the coefficient a_{3835} for a cosine-wave of two-day period. As in (11.6), we put again $c_{3835}=\sqrt{2}\ a_{3835}$. The periods p_ν of the successive waves of frequencies $\nu=1, 2, 3, \ldots$ would be, in days, $p_1=7670$, $p_2=3835$, $p_3=1917.5$, ..., $p_{20}=383.5$, $p_{21}=365.2$ (a year), ..., $p_{42}=182.6$ (6 months), ..., $p_{191}=40.16$, $p_{192}=39.95$, ..., $p_{255}=30.08$, $p_{256}=29.96$, ..., $p_{852}=9.002$, $p_{853}=8.992$, ..., $p_{958}=8.006$, ..., $p_{2556}=3.0008$, $p_{2557}=2.9996$, ..., $p_{3834}=2.0005$, $p_{3835}=2.0000$. These few values indicate that the difference in the length of the successive periods T/ν is very small for the high frequencies, because $p_\nu-p_{\nu+1}=$

[36]J. Bartels, Pub. Nat. Res. Council, Trans. Amer. Geophys. Union, 12th annual meeting, pp. 126-131 (1931).

$T/\nu(\nu+1)$ is approximately proportional to $1/\nu^2$. The labor involved in computing all coefficients would be very great. Pollak selected the periods 3 to 7 full days and 21 to 40 full days and their halves and thirds, because they are most easily calculated, the angles νx_p in (5.5) repeating themselves after each period.

The fact that most of these periods selected by Pollak are not entire submultiples of 7670 must not be overlooked; however, if we omit a few days at the end of the series, they become submultiples of the slightly reduced number of days [for instance, 17 goes in 7667], so that the amplitudes c can still be said to be derived from practically the whole series.

Around the periods 9.00 and 30.0 days, waves for the additional periods of 8.95, 8.96, . . . , 9.05 days and 29.7, 29.8, . . . , 30.2 days were inserted by Pollak, using Darwin's scheme of approximation [section 38]. I have computed, in addition, the amplitudes for four submultiples of a year (periods 3, 4, 6, and 12 months). Pollak's periodogram, with these additions, is reproduced in Figure 6; for clearness, the scale for abscissae is not uniform, spreading between 8.95 to 9.05 and 29.5 to 30.5 days, and changing as indicated.

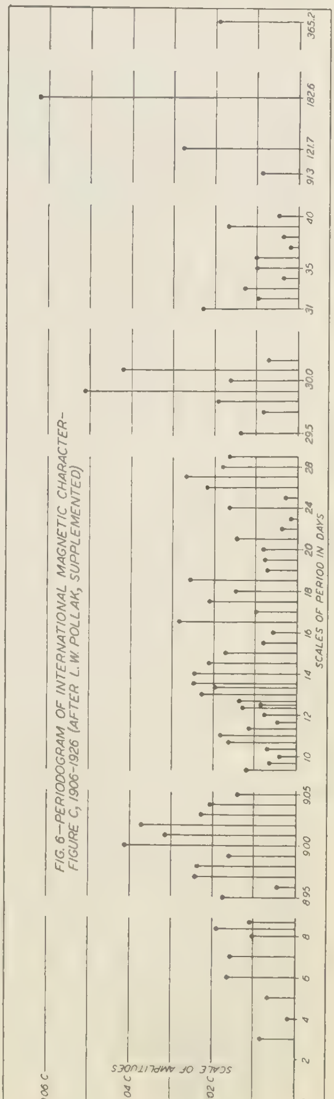


FIG. 6—PERIODOGRAM OF INTERNATIONAL MAGNETIC CHARACTER-FIGURE C, 1906-1926 (AFTER L. W. POLLAK, SUPPLEMENTED)

The periodogram gives only the amplitudes c_ν. The phases a_ν could be indicated by writing them down, or making the periodogram three-dimensional, combining the separate harmonic dials for each frequency by aligning them along their origins, like wheels on a common axis, which would correspond to the base-line in the ordinary periodogram. A mixed two-dimensional dial of all frequencies, indicating c_ν, a_ν by vectors, seems, however, to be confusing.

15. *Discussion of Pollak's periodogram*—We shall apply formulae (11.3) to the 73 amplitudes c_ν calculated by Pollak. The sum of the c_ν^2 is $0.02388C^2$, and therefore the standard deviation η_k of the sum of these 73 sine-waves is given by $\eta_k^2 = 0.01194C^2$, or $\eta_k = 0.1093C$. This

means that the approximative series ϕ_k, which is the sum of the arithmetic mean $a_0 = 0.621C$ and of these 73 sine-waves, has values which deviate from a_0 only by a few tenths of the unit C. The poor approximation of ϕ_k is even better illustrated by applying (11.4). The standard deviation of the 7670 daily values of C, for the years 1906 to 1926, is $\zeta = 0.461C$. The standard deviations of the residuals, s_k, is given by $s_k^2 = \zeta^2 - \eta_k^2 = 0.21266 - 0.01194 = 0.20072C^2$, or $s_k = 0.448C$. If, therefore, the sum ϕ_k of the 73 sine-waves is subtracted from the given values of C, the fluctuation of the residuals, measured by $s_k = 0.448C$, is practically the same as the fluctuation of the given values of C, measured by $\zeta = 0.461C$.

Are, then, the 73 sine-waves of the selected frequencies at least distinguished by large amplitudes, as compared with the rest of the total of 3835 amplitudes? The answer is suggested by (11.7). The sum of c_ν^2 for the remaining 3762 sine-waves is $2s_k^2 = 0.40144C^2$. If all amplitudes except one were zero, this one amplitude would be $0.63C$ a case obviously ruled out by a mere glance at the original series. If, on the other hand, all remaining amplitudes should have the same value c', this would be given by $(c')^2 = 2s_k^2/3762$, or $c' = 0.0103C$. The sum of the squares of 73 of the remaining amplitudes would then be $73(c')^2 = 0.0078C^2$. This is distinctively less than the sum of the squares of the 73 amplitudes for the actually selected waves, which above was given as $0.0239C^2$. The answer to our question is therefore affirmative.

We must remember, however, that the 73 selected frequencies are in no way equally distributed between all frequencies: from the list for the lengths of all periods given in section 14 it is seen that 958 periods are longer than 8 days, and the remaining 2877 shorter. Of the selected periods, 67 belong to the former and only six to the latter group: on the average, one out of 14 periods has been actually computed in the group of periods longer than eight days, but only one out of 480 periods in the group of shorter periods. This remark will be used later (section 32).

16. *Non-cyclic variation, selection-, or curvature-effect*—Harmonic analysis can be applied to all functions of time, f(t), occurring in geophysics, and will result in a satisfactory approximation of f(t) by a sum of sine-waves. It has already been said (section 11) that the significance of each sine-wave has to be tested, as will be described later. Apart from these tests, it will be easier to trace the real periodicities if such parts of f (t), which are obviously non-periodic, are separated before the harmonic coefficients are discussed.

A typical case of a non-periodic part is the *secular variation* in terrestrial magnetism, which, in the course of a day or a month, can be considered as a linear one-sided trend. In computing diurnal variations, its effect is seen in a systematic difference between the values for successive midnights, the midnight-difference, or *non-cyclic variation*. Another, and even more effective, cause for non-cyclic variations in terrestrial magnetism is the recovery after disturbances. f(t) can be freed from such effects by subtracting a suitable linear function of time, either by correcting the ordinates before the harmonic analysis, or by correcting the coefficients after the analysis. [The formulae deduced numerically by C. C. Ennis[47] can be derived in general terms (see ap-

[47]Terr. Mag., 32, 161-162 (1927).

pendix 4).] There has been some discussion on the feasibility of such *non-cyclic corrections*. They should be applied only if it is certain that the non-cyclic variation is due to an approximately linear function. This seems to be the case, for instance, in the average diurnal variations of magnetic horizontal intensity on quiet days, which show a systematic increase from midnight to midnight, and of those on disturbed days, showing a decrease.

More troublesome to eliminate is a systematic (mostly parabolic) *curvature*, which appears in *selecting* certain parts of a function of time. The classical case is the computation of the average diurnal variation of atmospheric pressure on clear and cloudy days in extra-tropical latitudes, which, in effect, amounts to selecting from the barogram and superposing, intervals of 24 hours between successive midnights, with high pressure (for clear days) and intervals with low pressure (for cloudy days). Now the general curvature of each single interval will be systematic so that, after non-cyclic correction, the average diurnal variation for clear days will show a pronounced maximum about noon, and that for cloudy days a pronounced minimum about noon. That these maxima and minima have nothing to do with an actual diurnal variation can be proved by selecting intervals of 24 hours between successive noons, which will show the maximum in the average clear-day variation about midnight. The possibility of such an effect, which was found in various phenomena by the author,[38] has often been overlooked, leading to curious misinterpretations. By suitable arrangement, this effect can be determined separately and corrected for.

(III) Statistical Principles—Random Walk

17. *The random walk with equal stretches*—The basis for all statistical considerations on periodicity is the problem of the "*random walk*," formulated, in its simplest case, by K. Pearson[39] as follows: "A man starts from a point O and walks a distance l in a straight line; he then turns through any angle whatever and walks a distance l in a second straight line. He repeats this process n times. I require the probability that after these n stretches he is at a distance between r and $(r+\mathrm{d}r)$ from his starting point, O." Figure 7[40] illustrates the case $n=27$; in addition, the random azimuths of the successive stretches are marked, in the upper left corner, by dots on a circle with radius l, in order to demonstrate (as in section 7) that the mass-center of these dots, as the average of the n-vectors of length l, is removed from the center of the circle by exactly $1/n$ of the distance between the starting and the end-point of the random walk.

The problem as well as its generalizations—for instance, to the case of stretches varying in length [section 18], or to more than two dimensions—has been amply discussed.[14-24] We need here only the following asymptotic expression for large values of n. Only the main theorems will be cited and discussed here; as to exact proofs, see the remark at the end of section 4.

[38] J. Bartels, Ann. Hydrogr., **51**, 153-160 (1923); Beitr. Physik frei. Atmos., **11**, 51-60 (1923); Terr. Mag., **37**, 18-20 (1932). See also S. Chapman and M. Austin, London, Quart. J. R. Met. Soc., **60**, 23-28 (1934).

[39] K. Pearson, Nature, **72**, 294 (1905).

[40] Constructed by taking the azimuth's from L. H. C. Tippett, Random sampling numbers (Tracts for computers, No. 15, Cambridge, 1927).

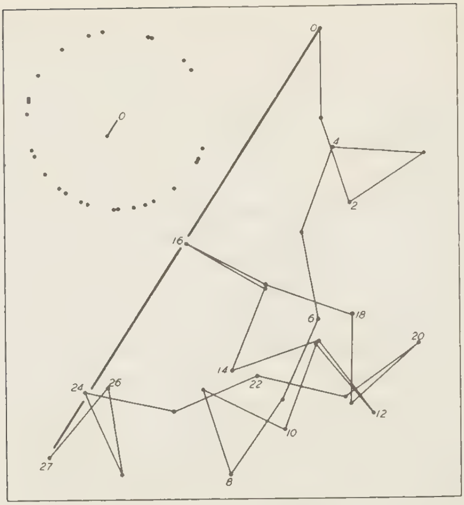


FIG. 7—RANDOM WALK WITH EQUAL STRETCHES

The random walk of n stretches may be repeated a great number (N) of times. The distance reached in each case may be called $L_1(n)$, $L_2(n)$, ..., $L_N(n)$. Then it can be shown that the average square-distance, defined by

$$M^2(n) = \lim_{N\to\infty} [(L_1^2(n) + \ldots + L_N^2(n))/N] \tag{17.1}$$

is simply nl^2. $M(n)$, called the *expectancy*,[41] is therefore given by

$$M(n) = l\sqrt{n} \tag{17.2}$$

and the probability $w(r)\mathrm{d}r$ that a distance between r and $(r+\mathrm{d}r)$ is reached is (with $e^x = \exp x$) given by

$$w(r) = (2/M^2)\, r \exp(-r^2/M^2) \tag{17.3}$$

This curve, for which examples are given later (Fig. 9), reaches a maximum for $r = M/\sqrt{2}$ and has an inflection-point at $r = M\sqrt{6}/2$. As always, "probability" means distribution of "relative frequency," that is, $w(r)\mathrm{d}r$ is the limit, for $N\to\infty$, of the ratio of the number of distances falling between r and $(r+\mathrm{d}r)$, to the number N of all distances.

$w(r)\mathrm{d}r$ is the probability for the end-point falling between r and $(r+\mathrm{d}r)$, that is, within an area $2\pi r\mathrm{d}r$; the probability for the end-point falling within an infinitesimal area $\mathrm{d}a$ is therefore $(1/\pi M^2) \exp(-r^2/M^2)\mathrm{d}a$. If we plot these probabilities as vertical ordinates on the plane of the random walk, we obtain a symmetrical bell-shaped surface, produced by the rotation of a curve which is $(1/M\sqrt{\pi})$ times an ordinary normal Gaussian frequency-curve for standard deviation $M/\sqrt{2}$.

[41]This definition of the expectancy, as the square-root of the average square-distance, makes the formulae simple. Some authors prefer to call $M\sqrt{\pi}/2$ the expectancy for a reason given at the end of section 17.

It is convenient to express the distance r as a multiple of the expectancy M

$$r = \kappa M \tag{17.4}$$

Then the probability that a distance between κM and $(\kappa + d\kappa)M$ is reached, is $w(\kappa)d\kappa$, with

$$w(\kappa) = 2\kappa \exp(-\kappa^2) \tag{17.5}$$

The total probability $W(\kappa)$ that a distance greater than κM is reached, is obtained by integrating $w(\kappa)$ from κ to ∞, giving

$$W(\kappa) = \exp(-\kappa^2) \tag{17.6}$$

TABLE 2—*Probability $W(\kappa) = \exp(-\kappa^2)$ that a random walk reaches a point beyond a circle with radius κM*

κ	$W(\kappa)$	κ	$W(\kappa)$	κ	$W(\kappa)$	κ	$W(\kappa)$
0.0000	1.0	0.8326	0.5	2.146	10^{-2}	4.015	10^{-7}
0.3246	0.9	0.9572	0.4	2.628	10^{-3}	4.292	10^{-8}
0.4724	0.8	1.097	0.3	3.035	10^{-4}	4.552	10^{-9}
0.5972	0.7	1.269	0.2	3.393	10^{-5}	4.799	10^{-10}
0.7147	0.6	1.517	0.1	3.717	10^{-6}	5.257	10^{-12}

The higher values in Table 2 apply, of course, only to large values of n, because, for instance, with $n=16$, $M=4l$, and the greatest possible distance, with all 16 stretches in line, is $16l=4M$, so that $W(4)=0$ in this case. For values of κ smaller than $\sqrt{n}$, however, the formula (17.6) is a very good approximation, and it is hardly ever necessary in geophysical applications to use the exact distribution-formulae worked out by K. Pearson,[16] and replacing (17.3) for small values of n; it is sufficient to note for later use that (17.2) remains valid for small values of n, including $n=2$.

In Table 2, the value $\kappa = \sqrt{\log \text{nat } 2} = 0.8326$, with $W(0.8326) = 0.5$ is of special interest, because a circle with the radius 0.8326 M (usually called the *probable radius*, though this expression is misleading) divides the plane into two areas in which the end-point of the random walk may fall with equal probability.

The *arithmetic mean* of the L_1, L_2, . . . , that is, limes $(L_1+L_2+\ldots+L_N)/N$, can be shown to be

$$M\sqrt{\pi}/2 = 0.8862M \tag{17.7}$$

18. *Random walk with unequal stretches*—The statistics of a random walk, for which the successive stretches are unequal, say, $l_1, l_2, \ldots l_n$, obey, under certain conditions, the same set of formulae as that in section 17. The conditions and the proof are fully given by A. Khintchine[6]; it is sufficient here to say that the N sets of n stretches, $l'_1, l'_2, \ldots l'_n$; $l_1'', l_2'', \ldots l_n''; \ldots; l_1^{(N)}, l_2^{(N)}, \ldots l_n^{(N)}$, used for each walk must be taken at random from a common "supply" of stretches (nN in number): the

frequency-distribution of the lengths of the single stretches in this supply is arbitrary in wide limits[42]; for instance, it can be itself of the form of the equation (17.3). We obtain then the solution for the problem of the random walk if we simply define the *expectancy l of the single vectors* by

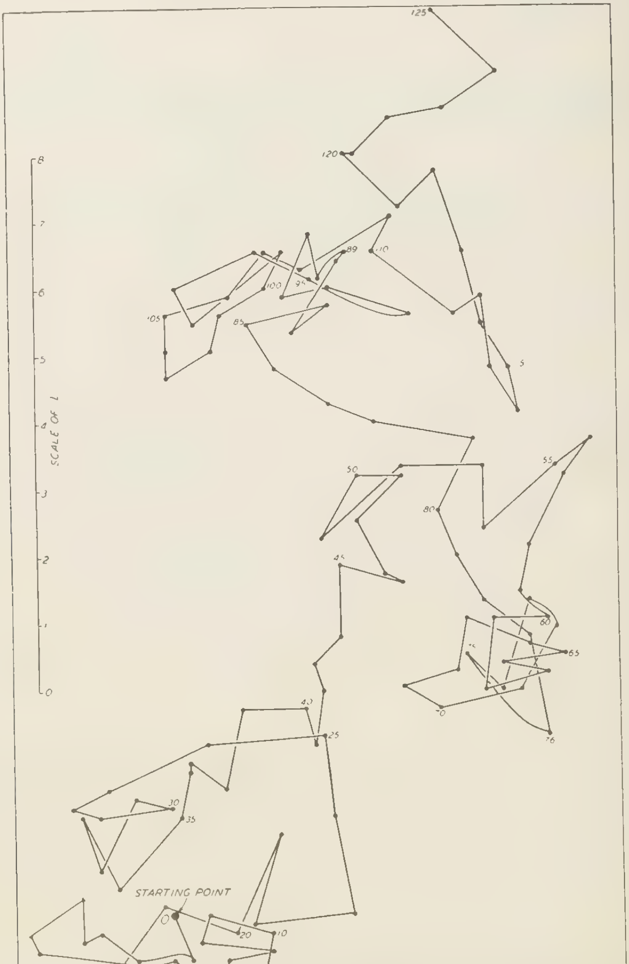


FIG. 8—RANDOM WALK WITH UNEQUAL STRETCHES

[42]This result was not known to K. Stumpff, who in his interesting paper on periodicities in sunspot-numbers (Prager Geophysikalische Studien, Heft 4—Čechoslovak. Statistik Reihe 12, Heft 14, Prague, 1930) discusses a special case of frequency-distribution, differing from the normal curve, and finds, of course, by rather intricate analysis, the general result of Khintchine for this special distribution.

(18.1) $$l^2=[(l_1')^2+\ldots+(l_n')^2+\ldots+(l_1^{(N)})^2+\ldots+(l_n^{(N)})^2]/nN$$

and apply the equations numbered (17.2) to (17.6); that is, the random walk leads, with the same probability, to distances between r and $(r+\mathrm{d}r)$ from the origin as if it had consisted of n equal stretches of length l, where l is given by (18.1).

An example of such a random walk of $n=125$ stretches—again constructed with the help of Tippett's random numbers[40]—is given in Figure 8; the amplitudes are distributed around their average value l according to a normal Gaussian law with standard deviation $0.39l$ (derived from the random numbers in Sir Gilbert Walker's paper of 1930[27]).

19. *The expectancy for an average vector, the $1/\sqrt{n}$ law*—Our formulae can be readily used for another geometrical problem, which is only a formal modification of the random walk. If we conceive each stretch of length $l_1, l_2, \ldots l_n$ as a vector $\mathbf{l}_1, \mathbf{l}_2, \ldots \mathbf{l}_n$, the line between O and the end-point of the random walk is the vectorial sum $\mathbf{l}_1+\mathbf{l}_2+\ldots+\mathbf{l}_n$, and $1/n$ of its length is the average vector. If each single vector is plotted with O as starting point, and its end-point indicated by a dot, then the mass-center of the dots represents, again, the average vector $(\mathbf{l}_1+\mathbf{l}_2+\ldots+\mathbf{l}_n)/n$, much as indicated in section 7. The distribution of the average vector for a large number (N) of random walks (of n stretches each) is therefore a reduction of the distribution for the vectorial sum in the ratio $1:n$. For the sum, (17.2) gives the expectancy $l\sqrt{n}$; therefore the probability for the average vector is governed by (17.3) and (18.1), with the expectancy m (defined by the average square m^2 of the average vector) given by

(19.1) $$m=l/\sqrt{n}$$

Since l, according to (18.1), is the expectancy of the single vectors, and m that of the average of n vectors, we can formulate as follows: Averages for n random vectors have an expectancy which is the original expectancy of the single vectors reduced in the ratio $1/\sqrt{n}$.

20. *Comparison of harmonic dial and random walk*—The main application of the theory of probability to geophysics consists in finding, for a given set of observed quantities, a suitable statistical analogue which can be accepted as representing the idealized case reached if the number of observations, under the same conditions, could be infinitely increased.

The random walk, in the modification of section 19, offers itself as the statistical analogue to such harmonic dials as Figure 2, showing 378 sine-waves of 27-day period (amplitudes c, phases α) in the character-figure C. We shall first ask whether the "cloud" of 378 points on the dial is distributed so that each point can be regarded as the end-point of a random walk made under the same conditions. This puts $N=378$ and leaves n arbitrary. As the parameter governing the distribution we compute the expectancy M, where M^2, analogous to (17.1), is defined as the average of the squares, c^2, of the amplitudes for the 378 waves, and find $M=0.262C$.

In order to find the frequency-distribution, we count out how many amplitudes c fall in classes between equidistant limits. These limits, chosen according to the conventional rule that about 20 classes should be occupied, are 0, $0.036C$, $0.072C$, etc. The numbers of amplitudes

in each class [the limits of which are marked by vertical lines] are entered as ordinates in Figure 9, and are compared with the theoretical frequency-distribution (probability), that is, with the curve computed, with the expectancy $M=0.262C$, from (17.6), the theoretical frequency between $\kappa_1 M$ and $\kappa_2 M$ being, of course, $N[W(\kappa_1)-W(\kappa_2)]$. The observed

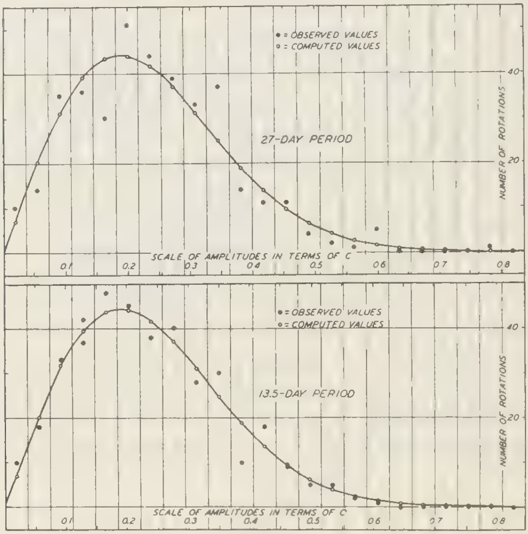


FIG. 9—NUMBER OF ROTATIONS (INTERVALS OF 27 DAYS) HAVING FOR INDICATED PERIODS IN INTERNATIONAL MAGNETIC CHARACTER-FIGURE C, 1906–1933, AMPLITUDES BETWEEN 0 AND 0.036 C, 0.036 C AND 0.072 C, ETC.

frequencies agree fairly well with the theoretical curve, the differences appearing to be of accidental nature. Only the isolated highest amplitude $c=0.760C$ might need some comment. However, if it is expressed as a multiple κM of M, we obtain $\kappa=2.90$, for which, after (17.6) $W(\kappa)$ is about 1/4500, meaning that, on the average, 1 out of 4500 amplitudes should be even greater than $0.760C$; it is therefore not strange that one occurs already among the first 378 amplitudes observed, that is, in 1/12 of the average number 4500. [The limitation of C to values between 0.0 and 2.0 excludes, of course, amplitudes c of sine-waves over a certain theoretical limit, which implies a restriction on the use of (17.6) for higher values of κ. The theoretical limit mentioned for c is, by the way, not $1.00C$, as one might guess, but about $4/\pi=1.27C$, furnished, for instance, by a succession of 14 days with $C=2.0$ followed by 13 days with $C=0.0$.]

From the observations described in section 8, for each "rotation" of 27-day length, the sine-wave of frequency 2, or 13.5-day period, was also computed. The 378 amplitudes obtained in this way, applying the same analysis as in the case of the sine-waves with 27-day period, give the expectancy $M=0.264C$, and the frequency-curve in Figure 9. The highest amplitude is $0.657C$, with $\kappa=2.49$, and $W(2.49)\sim 1/500$, even greater than above, and practically not much different from 1/378.

Another test consists in deriving, from the 378 amplitudes, the "probable radius" and the "arithmetic mean," for the cloud, which should (section 17) theoretically equal $0.833M$ and $0.886M$, respectively. The cloud for the 27-day period actually gives for these ratios 0.86 and 0.88, and that for the 13.5-day period gives 0.82 and 0.87. The agreement with the theoretical values is satisfactory, because the deviations of these "observed" ratios from the theoretical values may be expected to be of the order $1/\sqrt{N}$, or 0.05. The probable radius is also drawn in Figure 2.

So far as these tests go—and *only* so far—each of the $N=378$ vectors in the harmonic dial can therefore be conceived as the result of a random walk of n stretches of lengths l_1', l_2', ... l_n'; l_1'', l_2'', ... l_n''; ... $l_1^{(378)}$, ... $l_n^{(378)}$, where the stretches vary at random around a mean-square value l, formed as in (18.1). Only the parameter $M=l\sqrt{n}$, the expectancy, is prescribed by the observations, while n can be chosen arbitrarily, with $l=M/\sqrt{n}$ following. Of course, the equivalent interpretation of section 19 can also be applied, conceiving each vector in the dial as the average vector of n random vectors.

21. *Harmonic dial and average vector*—This interpretation of section 19 can also be applied to our dial in Figure 2 in another way. This time we put $n=378$, and consider the hypothesis that our dial in Figure 2 is just a sample of a great number, N, of dials, each representing 378 vectors, with the same (or only slightly different) expectancy $l=0.262C$ for the single vectors. In each of these hypothetical dials, we consider the average vector, just as in Figure 2, where its end-point is indicated by a cross. Then, according to (19.1), the expectancy of this average vector is $m=l/\sqrt{n}=0.262/\sqrt{378}=0.0135C$, and the frequency-distribution around the origin is governed by (17.3), with the expectancy m put for M. Now, in our dial Figure 2, the average vector is actually found to be $0.0336C$, or $2.49m$. According to (17.6), a value exceeding $2.49m$ should occur only once in about 500 cases. Here it seems doubtful whether it might be assumed as merely accidental that, in the one and only dial actually obtained, a large average vector should be obtained such as might be expected only once in about 500 trials: still, the probability 1/500 for chance is generally considered not so small as to warrant a definite claim that the observations considered (in our case, the vectors plotted in Fig. 2) do not correspond to the statistical analogue (random walk) with which they are compared. By the way, 1/500 is roughly the chance, that, in throwing a coin, a predetermined side appears nine times in succession.

If, in all 378 sets, the 27-day period were perfectly persistent, that is, would have the same amplitude and the same phase (or time of maximum) persisting throughout the 28 years, then the average vector would be exactly equal to the single vectors, that is, the average amplitude would be M, instead of $m=M/\sqrt{378}$. This would give $\kappa=\sqrt{378}-20$, and the possibility $W(\kappa)$ for chance would become practically zero. If the 27-day period should vary at random, we would obtain values of κ around 1. The value $\kappa=2.49$ actually obtained could be interpreted as meaning that there is a probability of 500:1 that the 27-day period contains at least a small persistent part. We shall see later why this

interpretation—which is commonly used in applications of periodogram-analysis—is not warranted (section 36).

In the case of the waves with 13.5-day period, we obtain the expectancy of the average vector $m = 0.0136C$, while the average vector actually computed is $0.0200C = 1.47m$, and the probability for chance is $W(1.47) = 1/9$, much higher than in the case of the 27-day period.

22. *Remarks on the probability of chance* (*κ-test*)—We have followed the generally adopted convention in calling $W(\kappa)$ the "probability of chance." This is, of course, only a short expression for the exact definition of $W(\kappa)$, which may be repeated for the case of average vectors. From the amplitudes of n single vectors for the same frequency, we calculate the expectancy of these single amplitudes analogous to (18.1), divide it by $\sqrt{n}$ and thus obtain, according to (19.1), the expectancy for the amplitude of the average vector of that frequency; this expectancy, our m of sections 19 and 21, will be called $\mathbf{c}$ from now on. It is based on the assumption of complete independence of the single vectors. By actual calculation (*vectorial* sum, division by n) of the average vector, we find its amplitude c, and calculate $\kappa = c/\mathbf{c}$. $W(\kappa)$ is exactly the probability that, under random-walk conditions, an amplitude greater than $c = \kappa\mathbf{c}$ should be found. In other words, if the random walk is repeated N times, about $NW(\kappa)$ times a distance greater than $\kappa\mathbf{c}$ should be reached, or about once if it is repeated $(1/W(\kappa))$ times. If $W(\kappa)$ is very small, that is, $[(1/W(\kappa)]$ is very large, it is reasonable to assume that the conditions of random walk, or pure chance, do not hold because in the one and only case considered we have obtained a result which *should* occur only very rarely, and the suspicion is justified that some systematic regularity is contained in the distribution of the single vectors—which will be seen later.

Ad. Schmidt and Sir Gilbert Walker[27] have called attention to the following point: If only *one* frequency is considered, the considerations regarding $W(\kappa)$ hold. But some authors, having calculated $\mathbf{c}$, c, and κ for each of a number (say 100) of independent frequencies, picked out that frequency with largest κ, say, κ_1. Then, of course, we must ask for the probability that once in 100 independent cases a value greater than κ_1 times its expectancy should occur, and that is 100 $W(\kappa_1)$.

Since the question whether $W(\kappa_1)$ is small enough in order to exclude chance is a matter of opinion anyway—one in a million is often considered as an upper limit—it is not necessary in most cases to consider the more accurate formulae introduced by Sir Gilbert Walker. He asks for the probability that, on random-walk conditions, the 100 independent values of κ should all be smaller than κ_1, and finds $\{1 - [1 - W(\kappa_1)]^{100}\}$; this, however, is, for small $W(\kappa_1)$, practically 100 $W(\kappa_1)$. If the observational material is large enough, some objections raised by Brunt[27] do not hold. Much more serious is a common mistake in the choice of the expectancy $\mathbf{c}$, which it is the object of this paper to indicate (section 32 and following).

23. *Elliptical distributions*—The discussion of the properties of N sine-waves of the same frequency has, by the harmonic dial, been transformed into the geometrical analysis of the equivalent N vectors plotted from the origin, or of the "cloud" formed by their N end-points. We have seen that, under random-walk conditions, this cloud approaches

circular symmetry around the origin for large values of N. In actual geophysical work, however, especially for diurnal and seasonal variations, the cloud can have quite different shapes. If, for instance, the phenomenon contains a regular sine-wave of the frequency considered, with constant phase and amplitude, which is superposed by random fluctuations (introduced, for instance, by errors of observation), the cloud of points will be circular, but centered around the point A representing the regular sine-wave instead of around the origin O. If the regular sine-wave has constant phase, but a varying amplitude, the cloud is stretched into elliptical shape, and this elliptical distribution will be recognized most easily if the superposed irregular fluctuations are comparatively small.

Such elliptical distributions, which, in the most general case, have been discussed from the standpoint of the theory of probability by A. Khintchine,[6] have been found in the diurnal variations of terrestrial magnetism; statistical methods for computing the ellipses and further discussion of their physical meaning are given in a former paper in this JOURNAL.[2]

If the center A of the cloud is well outside the origin O, it is sometimes desirable to consider each single vector $\mathbf{c}=OP$ to consist of the regular vector $OA=\mathbf{r}$ (with constant amplitude r) and an irregular part $AP=\mathbf{i}$

$$\mathbf{i}=\mathbf{c}-\mathbf{r} \tag{23.1}$$

The expectancies of $\mathbf{c}$ and $\mathbf{i}$—computed in the usual way by summing the squares of the amplitudes, dividing by the number of vectors, and taking square root—may be c and i. The following relation is convenient for changing from c to i, or vice versa, namely

$$i^2=c^2-r^2 \tag{23.2}$$

This formula is a two-dimensional generalization of (11.2), because (23.1) corresponds to (11.1). [The proof is simple: The coefficients a and b of the vectors $\mathbf{i}$ and $\mathbf{c}$ follow (11.1) and (11.2) separately, and the squares of the vector-amplitudes are a^2+b^2.] If, therefore, we have calculated, for a cloud of points, the expectancy for distances of these points from any origin O, we obtain the expectancy for the distances of the points from their mass-center A by subtracting r^2, where r is the distance OA.

24. *The expectancy of sine-wave amplitudes calculated from rare events occurring at random*—In the preceding paragraphs, we have applied the conception of the random walk to vectors representing sine-waves in the harmonic dial. It can, however, also be applied to the actual calculation of the harmonic coefficients, as represented in the folding process (section 10), and this will establish a statistical relation between a set of random ordinates and its harmonic coefficients, which was the starting point of A. Schuster.[7]

The original conception of the random walk, with stretches of equal length, is the geometrical expression for the harmonic analysis of a function of the following type: Consider a long time-interval T, (20 years, say), divided into a large number r of equal intervals (about ten million minutes of time). The ordinates are put equal to 1 for minutes characterized by a (comparatively rare) event, for instance, the beginning

of a magnetic storm with sudden commencement at a given observatory, and zero for all other minutes. The total number r_1 of events, or ordinates 1 will then be small (in our example not more than, say, 500) compared with the number r_0 of ordinates O ($r_1 << r_0$) and they will be scattered over the whole interval T considered. Take, then, a sine-wave of high frequency, say, $\kappa = 240$, with a period of one month; in the folding process, this means that the directions of the links describe a full swing of 360° per month. If, now, the events are scattered at random (like the atomic disintegrations in radioactive material), the folding process will lead to a diagram equivalent to a random walk with $n = r_1$ stretches.

In order to introduce the theory of probability, we must again hypothesize that our interval of observation T is a sample of a large number N of such time-intervals of length T with the same statistical properties, the average number of events in each interval being r_1. For a given period, the random walk of the folding process will lead to distances $L_1(r_1), \ldots L_N(r_1)$, and their relative frequency, or the distribution of the end-points, will be governed by the formulae (17.2) and (17.3), with $M = \sqrt{r_1}$. Now, according to sections 9 and 10, the amplitudes c_ν of the sine-waves are obtained by dividing the distance L by half of the number $(r_0 + r_1)$ of ordinates. If, therefore, we define, analogous to (17.1), the *expectancy* $\mathbf{c}_\nu$ of the amplitude by

$$\mathbf{c}_\nu^2 = (c_{\nu 1}^2 + c_{\nu 2}^2 + \ldots + c_{\nu N}^2)/N \tag{24.1}$$

we obtain

$$\mathbf{c}_\nu = 2\sqrt{r_1}/(r_0 + r_1) \tag{24.2}$$

The remarkable feature of this result is that the expectancy $\mathbf{c}_\nu$ does not depend on the length of the period, or the *frequency* ν.

25. *Random walk and folding process, equipartition of the variance*—Some caution is necessary in applying the idea of the random walk to the ordinary case of equidistant ordinates, in which the directions in the folding process are limited to a few submultiples of 360°, such as in Figures 3*D*-3*G*. The theorem can be formulated most clearly if we use in the folding process, not the ordinates y_ρ themselves, but their deviations z_ρ from the mean a_0, $z_\rho = y_\rho - a_0$ ($\rho = 1, 2, \ldots r$), as illustrated in Figure 3*E*.

A great number N of sets of r ordinates may be given. The average of all Nr ordinates shall be zero, and the sum of their squares may be $Nr\zeta^2$, so that ζ is their standard deviation; nothing else will be assumed except that the ordinates are "random numbers," quite independent of each other.

Such sets can, for instance, be obtained by drawing ordinates at random from a great supply of ordinates having normal (Gaussian) frequency-distribution with standard deviation ζ and combining them to sets of r each. In each individual set, the arithmetic mean a_0^* of the r ordinates will not be zero, and the standard deviation ζ^* will not be exactly ζ, but, on the average for all N sets, according to well-known statistical laws,

$$(a_0^*)^2 = \zeta^2/r \text{ and } (\zeta^*)^2 = \zeta^2(r-1)/r \tag{25.1}$$

This example is, however, by no means the most general case for which the following theorem holds, because the frequency-distribution

of the supply of ordinates may deviate from the normal law in wide limits.[43]

Each set of r ordinates is subjected to harmonic analysis yielding, for r uneven, $(r-1)/2$ amplitudes c_ν and phases α_ν; for r even we obtain $(r-2)/2$ amplitudes c_ν and phases α_ν and $a_{(r/2)}$. We form, for each frequency ν separately, the average square amplitude, or expectancy, $\boldsymbol{c}_\nu$ for all N sets, defined by (24.1) and, in the same way, $\boldsymbol{a}_{r/2}$. Then it can be shown that

$$(25.2) \qquad \boldsymbol{c}_\nu = 2\zeta/\sqrt{r};\ \boldsymbol{a}_{r/2} = \zeta/\sqrt{r}$$

The independence, for random ordinates, of $\boldsymbol{c}_\nu$ of frequency ν can be termed the law of the *equipartition of the variance* (where variance is the expression introduced by R. E. Fisher[44] for the square of the standard deviation). Because, taking the case of r uneven, we know from (11.3) that each amplitude c_ν contributes $c_\nu^2/2$ to the variance η_k^2 of the sum ϕ_k of sine-waves. If, therefore, we write down (11.3) for each set, sum up, and divide by N, we obtain

$$(25.3) \qquad (\zeta^*)^2 = (\boldsymbol{c}_1^2 + \boldsymbol{c}_2^2 + \ldots + \boldsymbol{c}_{(r-1)/2}^2)/2$$

If we assume[45] equipartition, or $\boldsymbol{c}_1^2 = \boldsymbol{c}_2^2 = \ldots = \boldsymbol{c}_{(r-1)/2}^2 = \boldsymbol{c}^2$, we obtain $(\zeta^*)^2 = \boldsymbol{c}^2(r-1)/4$. Remembering that, because of (25.1), $(\zeta^*)^2 = \zeta^2 (r-1)/r$, we obtain $\boldsymbol{c}^2 = \zeta^2\, 4/r$, that is, (25.2).[46]

The former formula (24.2) appears now as a special case of (25.2), because, in the example considered in section 24, $\zeta = \sqrt{r_1/r}$ and $r = r_0 + r_1$.

In a single set of r ordinates, c_ν can have any (positive) value, but the frequency-distribution of c_ν in a large number N of sets is governed by (17.3) to (17.6), with $M = \boldsymbol{c}$; the total probability that a single c_ν exceeds $M\boldsymbol{c}$, is again $W(\kappa) = \exp(-\kappa^2)$.

26. *Periodogram for random fluctuations*—The periodogram, as defined in section 14, can be plotted for each of the N sets of r ordinates considered in section 25; each periodogram shows, against the abscissae ν, the individual amplitudes c_ν as ordinates which, if it is desired, can be connected by a more or less arbitrary line. The *mean periodogram*, representing the average of all sets, shows *the expectancy* $\boldsymbol{c}_\nu$, defined by (24.1), as a function of ν; according to the law of equipartition (25.2) the mean periodogram for random ordinates would show a straight line at the distance $2\zeta/\sqrt{r}$ above the horizontal axis (only declining, for r even, to $\zeta/\sqrt{r}$ for the highest frequency $\nu = r/2$).

It may be noted that the mean periodogram does not only depend on the standard deviation ζ, but also on r. If, for instance, we divide 100,000 random ordinates into $N=1000$ sets of $r=100$ ordinates, the mean periodogram is only half as large as if we divide the material into $N=4000$ sets of $r=25$ ordinates.

The discussion of this paragraph applies at once to the case that the ordinates of any given function have accidental and independent ob-

[43] See section 18.

[44] R. E. Fisher, Statistical methods for research workers, 3rd ed., Edinburgh, 1930.

[45] The remarks given above are only illustrations, not a proof of the law of equipartition. For a simple proof, insert (5.5) into $c_\nu^2 = a_\nu^2 + b_\nu^2$ and add for all N sets.

[46] In the case considered, it has been necessary to distinguish between r and $(r-1)$, because r, the number of ordinates in a single set, may be as small as 3. But in all cases where the total number of observations appears in the equations, we shall generally not question scrupulously whether $(N-1)$ should stand for N, because we take N so large that this difference should not matter. In other words, observational material in which the addition or omission of one or a few observations should alter the conclusions seriously, is not considered sufficient for a statistical treatment.

servational errors, with standard error ζ. Then the periodogram of the true ordinates is superposed by the periodogram $2\zeta/\sqrt{r}$ of the errors, the superposition, for each separate frequency, following (23.2). The influence of observational errors on the harmonic coefficients is, however, mostly negligible in geophysical applications; it has been often mistaken for the influence of the actual irregular fluctuations of the observed quantity, which are fundamentally different in nature and will be shown to have, in each case, a peculiar type of mean periodogram (section 30).

(IV) PERSISTENT PERIODICITIES

27. *The expectancy as a function of the length of period*—The definition (24.1) of the expectancy $\boldsymbol{c}_\nu$ can, of course, at once be extended to the case that the Nr given ordinates represent a real geophysical phenomenon. The discussion in section 20 can therefore, simply by putting $\boldsymbol{c}$ for M, be expressed in the following way: With $N=378$ and $r=27$, that is, from 378 sets of 27 character-figures C for consecutive days (rotations), we obtain the expectancy $\boldsymbol{c}=0.262C$ for sine-waves of 27-day period computed from single rotations.

Since the expectancy $\boldsymbol{c}_\nu$ is the basis for all further discussion, it is necessary to consider the reliability of $\boldsymbol{c}_\nu$ if it is derived from N sets. It is clear that a single set, that is, a single amplitude c_ν, is a bad approximation for $\boldsymbol{c}_\nu$, because the single values of c_ν vary as expressed by (17.3) with $\boldsymbol{c}_\nu$ for M (see the probability-curves in Fig. 9). We imagine a very large supply of amplitudes c_ν. If we take, at random, N amplitudes c_ν from this supply, we shall compute an approximate expectancy $\boldsymbol{c}_\nu^{(N)}$ which differs from $\boldsymbol{c}_\nu$. If we repeat the computation for another set of N amplitudes, and another, etc., the $\boldsymbol{c}_\nu^{(N)}$ will be distributed around $\boldsymbol{c}_\nu$. This scattering, for large values of N over, say, at least 25, can be expressed by the standard deviation of the $\boldsymbol{c}_\nu^{(N)}$, which is approximately

$$\boldsymbol{c}_\nu/\sqrt{2N} \tag{27.1}$$

If N is large, this distribution around $\boldsymbol{c}_\nu$ approaches the normal law of errors; for smaller values of N, the distribution has been calculated by A. Schuster.[8]

We now turn to a fundamental consideration. The character-figures C for consecutive days are certainly not independent, since a magnetically quiet or disturbed time generally extends over a few days in succession. A number of statistical considerations are available for testing the degree of this dependence of consecutive values of C; for instance, by adding the figures C for two, three, and more consecutive days. If the standard deviation of C is $\zeta(1)$, and the standard deviations of the sums for 2, 3, etc., days, each divided by $\sqrt{2}$, $\sqrt{3}$, etc., are $\zeta(2)$, $\zeta(3)$, etc., respectively, then, on complete independence, we should expect $\zeta(1)=\zeta(2)=\zeta(3)=\ldots$, so that the ratios $\zeta(2)/\zeta(1)$, $\zeta(3)/\zeta(1)$, ... can be taken as measures of dependence. This test is mentioned here because its two-dimensional analogue will be used later for testing quasi-persistent periods.

Although, of course, no harmonic analysis is needed, and, in fact, would be clumsy for testing independence, we are, on the other hand, interested in the effect of dependence on the harmonic coefficients,

especially, on the expectancy $\mathbf{c}_\nu$. It should, of course, make the expectancies for periods of a few days smaller than those for longer periods, instead of equipartition as expressed in (25.2). We shall test this assumption for the 27-day period in the series of international character-figures C. In this case, with $\zeta = 0.467C$ (this value of ζ for the interval 1906 to 1933 is only slightly higher than that, $0.461C$, for the interval 1906 to 1926 used in section 15) and $r = 27$, equipartition (obtained, for instance, by mixing up the daily figures at random) would give, from (25.2) the expectancy $2\times 0.467/\sqrt{27} = 0.180C$. The actual expectancy $\mathbf{c}_\nu$ has been obtained for the 27-day period in section 20 (where it was called M), namely, $\mathbf{c}_\nu = 0.262C$, and its standard deviation, according to (27.1), is $0.262/\sqrt{2\times 378} = 0.0095C$. The actual expectancy $0.262C$ exceeds therefore the equipartition value $0.180C$ by nearly nine times its standard deviation: the difference between the two values is therefore significant, not "accidental," and proves that the *expectancy* as derived by (24.1) from the actual amplitudes obtained by harmonic analysis from single rotations *depends definitely on the length of the period.*

28. *Persistent periodicities*—A sine-wave of period p is called *persistent* if it is repeated with the same amplitude and the same phase in all intervals of length p. Is it possible to trace such a persistent sine-wave if it is superposed on other fluctuations? The answer is affirmative, provided the number N of periods p contained in the interval of observations is sufficiently large. The procedure is suggested by the preceding discussion: Each single interval of length p is subjected to harmonic analysis and yields a sine-wave of period p, which, if represented in the harmonic dial of period p, is the vector sum of two sine-waves, namely, the persistent sine-wave and another "accidental" sine-wave, for which the average square amplitude, calculated according to (24.1) from the N accidental sine-waves for the single intervals, may be $\mathbf{c}$. Then the average sine-wave of period p computed from all Np observations will be the vector-sum of the persistent wave (of amplitude c) and an average "accidental" sine-wave, the amplitude of which, according to section 19, is of the order $\mathbf{c}/\sqrt{N}$. Therefore, so small as c may be as compared with $\mathbf{c}$, in the average taken over a sufficient number N of periods the persistent wave will finally overwhelm the "accidental" waves produced by the non-persistent fluctuations which mask the hidden periodicity in the original data.

This process of reducing the average of the accidental wave is best visualized in the harmonic dial for the period p: The dial showing the sine-waves obtained from single intervals of length p will be a cloud of points widely scattered; but the cloud on the dial of the average sine-waves of period p obtained from a number of intervals of length Np will be reduced with respect to the end-point A of the persistent sine-wave vector, in the ratio $1/\sqrt{N}$, till, with N increasing infinitely, the whole cloud contracts into A.

The determination of the atmospheric tides of lunar origin has been so far the greatest "triumph" of this $1/\sqrt{N}$ law,[47] because, at extra-tropical

[47] S. Chapman, London, Mon. Not. R. Astron. Soc., **78**, 635-638 (1918); see also foot-note 50 and the second and third references in foot-note 32, and J. Bartels, Quart. J. R. Met. Soc., **51**, 173-176 (1926). Since then, the determination of the lunar semidiurnal variation of atmospheric temperature at Batavia, 1866-1928, with an amplitude of 0°.009 Centigrade, has added an even better example: S. Chapman, London, Proc. R. Soc., A, **137**, 1-24 (1932).

stations, the expectancy c is about 0.30 mm mercury, for semidiurnal waves computed from 24 hourly values of atmospheric pressure, while c is only 0.01 mm, so that 900 days are needed to bring the accidental waves down to the level of the persistent wave and 100 years to reduce the accidental part to about $c/6$.

In a wider sense, also such periods can be called persistent (and traced in the same way), which have a constant period p, a phase fluctuating a few degrees around an average value, and a variable amplitude. Most diurnal and annual waves in meteorological or terrestrial-magnetic phenomena are of this nature. The reduction of the elliptical distributions discussed in section 23 follows the same $1/\sqrt{N}$ law, unless the averages are taken for systematically selected single intervals (section 16).

29. *Example: The semiannual persistent wave in terrestrial-magnetic activity; the summation-dial*—In our 28-year series of international magnetic character-figure C, only the period of six months can be definitely considered as persistent. In Figure 10, the harmonic dials have been plotted for sine-waves of six-month period, at the left computed from

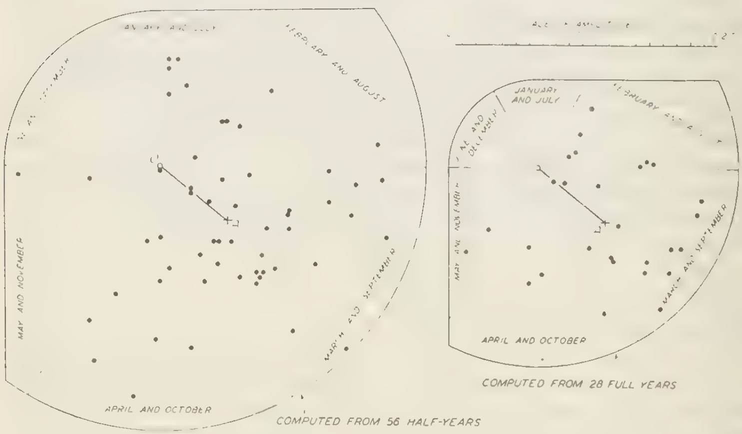


FIG. 10—HARMONIC DIALS FOR 6-MONTHLY SINE-WAVES IN THE INTERNATIONAL MAGNETIC CHARACTER-FIGURE C

the 56 half-years, at the right computed from the 28 calendar years; of course (section 7), each dot in the right-hand diagram is the mass-center of two dots on the left. The average wave for all 28 years has the amplitude $c = 0.0675C$, and its phase is given by maxima which occur about the dates March 22 and September 20, very near the equinoxes; it is represented by the average vector OA which, of course, is the same in both diagrams. The expectancy for single waves (vectors reckoned from origin O and combined according to (24.1)), is $c = 0.111C$ at the left, $c = 0.096C$ at the right. For the average of 56 or 28 accidental waves we should expect therefore $0.111/\sqrt{56} = 0.0148C$, and $0.096/\sqrt{28} = 0.0181C$. The actual average vector, $0.0675C$, is $\kappa = 4.6$ and 3.6 times as large. With these values of κ, Table 2 for $W(\kappa)$ gives only a proba-

bility of about 10^{-9} or 10^{-6} that the waves are accidental. That the analysis based on the half-years gives even better results than that based on full years is easily understood, because the expectancy $0.096C$ obtained from full years is relatively more increased by the presence of the persistent wave than the expectancy, $0.111C$, obtained from half-years.

For the "accidental" or "irregular" vectors, reckoned from A, (23.2) gives the expectancy for single vectors $0.088C$ or $0.068C$, and for averages of 56 or 28 vectors, $0.0118C$ or $0.0129C$. This makes $\kappa = 5.7$ or 5.2, and $W(\kappa)$ smaller than 10^{-12}.

The basis of this discussion has been the comparison between a random walk and the gradual vectorial addition of the single vectors in the harmonic dial represented in Figure 10. This summation has been represented in Figure 11 (in a diagram which may be called *summation-dial*); the decisive preponderance of the directions indicating maxima near the equinoxes excludes all similarity with a "random walk" and illustrates the "reality" of the 6-month wave, which has just been quantitatively proven by the κ-test.

A more detailed analysis of this persistent semiannual wave and a discussion of its physical nature may be found in a former paper.[48]

30. *Mean periodogram for geophysical phenomena*—Our procedure of testing the reality of a periodicity consists in deriving a value for the expectancy c_ν which is based exclusively on harmonic analysis for single waves *of the same period*. This value represents therefore, in exactly the right manner, the combined effect of the standard deviation ζ of the given ordinates *and* the dependence of successive ordinates (section 27). The latter is present in most geophysical cases in so far as high values and low values of the ordinates occur in groups.[49] If, therefore, we cut the series of ordinates into sets of r successive ordinates, the arithmetic mean in each single set will, in general, differ *more* from the arithmetic mean of all ordinates than in the case of independence; in other words, the stand-

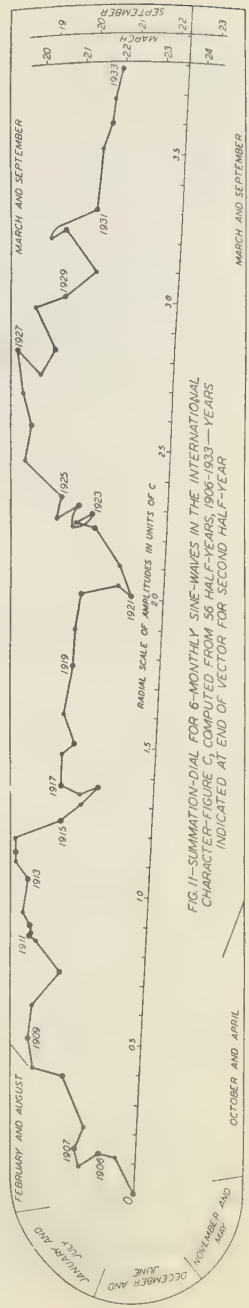


FIG. 11—SUMMATION-DIAL FOR 6-MONTHLY SINE-WAVES IN THE INTERNATIONAL CHARACTER-FIGURE C, COMPUTED FROM 56 HALF-YEARS, 1906-1933—YEARS INDICATED AT END OF VECTOR FOR SECOND HALF-YEAR

[48] Terr. Mag., **37**, 22-27 (1932).

[49] The analogy to the Lexis theory of dispersion is obvious; this theory is described in every textbook on the theory of probability (for instance, that of Kamke[6] or R. E. Fisher[44]), and has been applied by F. Baur to meteorological phenomena [Met. Zs., **47**, 381-389 (1930)]. In relation to periodicities, the Lexis theory must be modified, or specialized, as will be seen later (section 40).

ard deviation of the arithmetic means for sets of r successive ordinates will be *greater* than the random value $\zeta/\sqrt{r}$. [Example: International magnetic character-figure C, 1906 to 1933, has standard deviation for single daily values $\zeta=0.467C$; if we form arithmetic means for each of the 378 rotations (intervals of $r=27$ days), their standard deviation is found to be $0.148C$; if the values C for successive days were independent, this value should be only $0.467/\sqrt{27}=0.090C(\pm 0.003)$.] On the other hand, if, in each single set, the deviations of each of the r ordinates from the arithmetic mean for that set are formed, their standard deviation ζ_r will be *smaller* than the standard deviation ζ of all ordinates, the ratio ζ_r/ζ increasing to unity with increasing r (in the case considered, $\zeta_{27}=0.444C$). From (11.6), it follows therefore that the expectancy for smaller periods (computed from sets of a few ordinates) will be, in general, smaller than that for longer periods.

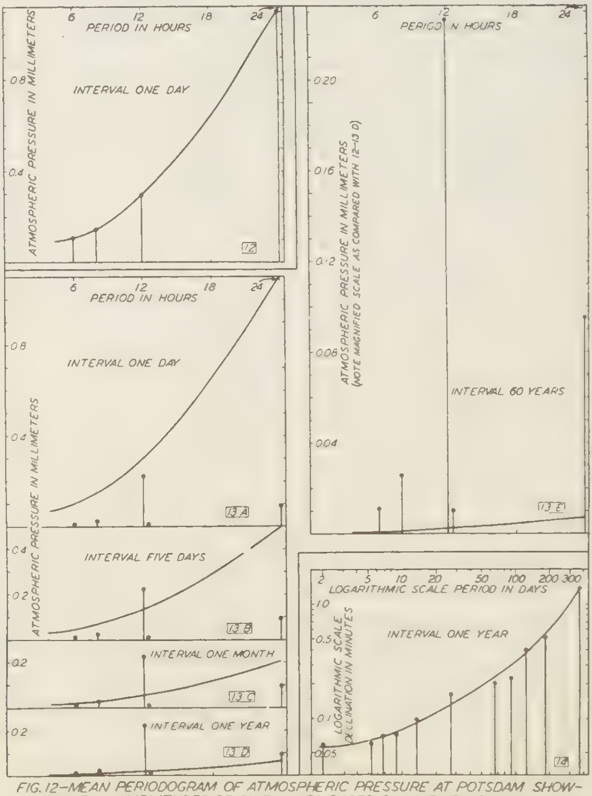


FIG. 12—MEAN PERIODOGRAM OF ATMOSPHERIC PRESSURE AT POTSDAM SHOWING MEAN AMPLITUDES OF SINE-WAVES BASED ON HARMONIC ANALYSIS FROM SINGLE-DAY INTERVALS FOR PERIODS 6, 8, 12, AND 24 SOLAR HOURS

FIG. 13—AMPLITUDES OF PERSISTENT SINE-WAVES FOR PERIODS 6, 8, 12, AND 24 SOLAR HOURS AND FOR 12 LUNAR (=12.4 SOLAR) HOURS CONTRASTED WITH MEAN PERIODOGRAMS FOR INTERVALS AS INDICATED

FIG. 14—MEAN PERIODOGRAM OF MAGNETIC DECLINATION AT GREENWICH, SHOWING MEAN AMPLITUDES OF SINE-WAVES BASED ON HARMONIC ANALYSIS FROM SINGLE-YEAR INTERVALS (AFTER SCHUSTER)

As an example, consider the hourly values of atmospheric pressure observed at Potsdam, Germany.[50] The expectancies for the sine-waves with 6-, 8-, 12-, and 24-hour periods, computed from single sets of $r=24$ hourly values, are, in mm of mercury, 0.11, 0.14, 0.30, and 1.11. They are entered in the mean periodogram of Figure 12. The free-hand curve drawn through the four ordinates can reasonably be expected to represent the actual mean periodogram, that is, the expectancy c as a function of the period p.

We now make the assumption—to be tested by its consequences—that the harmonic coefficients of period p, computed from different sets of r observations, are independent and do not contain a persistent part. Then the "random-walk" theory (19.1) is applicable, and we can at once obtain the mean periodogram for the amplitudes computed from the average harmonic coefficients won by harmonic analysis of sets of $2r$, $3r$, . . . , in general, of qr ordinates, because (7.1) the vectors for the average sine-waves derived by the harmonic analysis of qr ordinates are the averages of the q sine-waves computed from r ordinates each. From (19.1) it follows, therefore, that we obtain the mean periodogram for sets of qr ordinates simply by reducing the mean periodogram for sets of r ordinates in the ratio $1/\sqrt{q}$. This applies, of course, only to such periods p which are submultiples of the interval represented by r ordinates. The reduction can, by increasing q, in actual computation, be continued till the whole set, $q=Nr$, of available ordinates is subjected to harmonic analysis. Persistent waves of amplitudes c greater than $c/\sqrt{N}$, where c is the expectancy for that particular period, will then be discovered, and the ratio $\kappa=c/(c/\sqrt{N})$ will indicate the degree of reliability.

Figure 13 shows, in the curves, the mean periodogram for waves from 6- to 24-hour periods in atmospheric pressure at Potsdam, calculated from single days ($r=24$), and the mean periodogram for waves computed from $q=5$, 30, 365 days and 22,000 days (60 years), obtained by reducing the curve for single days in the ratio $1/\sqrt{q}$; for clearness, Figure 13*E* has a scale magnified ten times. The persistent waves of 6, 8, 12, and 24 solar hours, of amplitudes 0.011, 0.026, 0.226, and 0.095 mm have been indicated by vertical lines in each periodogram, and, in addition, the lunar tidal wave of period 12 hours 25 minutes, with amplitude 0.011 mm. It is striking how the persistent waves, with increasing number q of days, gradually pierce the mean periodogram, which represents the veil of the non-periodic fluctuations hiding the persistent waves. One year of observations (Fig. 13*D*) is sufficient to extract the solar 12- and 24-hourly waves, while 60 years of observations (Fig. 13*E*) are necessary to press the level of the mean periodogram in the neighborhood of 12-hour period down to one-fifth of the amplitude of the actual lunar tidal wave.

31. *A. Schuster's example of a mean periodogram*—The idea of the mean periodogram as drawn in Figure 12 is the outstanding contribution of A. Schuster to the study of periodicities. In fact, on page 122 of his second paper,[8] he gives an actual mean periodogram showing the same

[50] J. Bartels, Ueber die atmosphärischen Gezeiten, Berlin, Veröff. Preuss. Met. Inst., No. 346 (1927). The values given above are the *probable* radii, as they were actually calculated; the expectancies should be about 20 per cent higher, but that does not matter for our purpose here.

characteristic feature as our Figure 12. In order to show the details more clearly, logarithmic scales for both periods and amplitudes have been used in Figure 14, which represents Schuster's calculations based on the daily means of magnetic declination at Greenwich, 1871-1895, corrected for the non-cyclic variation due to secular variation (section 16), and shows the expectancies for sine-waves from 2- to 365-day periods supposed to be calculated from single years of observations, that is, the average amplitudes, computed according to (24.1) from the amplitudes obtained by harmonic analysis of $N=25$ sets of $r=365$ ordinates each. Figure 14 corresponds to Figure 12; the vertical lines give the expectancies as calculated, and the smooth line has been drawn to fit approximately.

A. Schuster used his values for the expectancy to test the presence of a persistent wave with period between 25.5 and 27.5 days. Since the whole interval of observation is 9160 days, these periods would represent the frequencies $9160/25.5=358$ and $9160/27.5=332$, so that about 26 independent sine-waves lie between these limits. The greatest among them, calculated from all $N=25$ years, has an amplitude of $c=0'.0785$, while the chance value, with $c=0'.163$ for a single year, is $c/\sqrt{N}=0.163/5=0'.033$. Therefore $\kappa=0.0785/0.033=2.4$. This value, according to (17.6), should, if pure chance were working, be exceeded once in about 300 cases, and it cannot be claimed to be unusual if an event, occurring, on the average, once in 300 cases, occurs already in the 26 cases actually considered. Schuster considers also periods which are not entire submultiples of 9160 (see section 38), which increase the number of "independent" periods between 25.5 and 27.5 days to 4 times our number 26, or about 100; this is even more unfavorable to a claim that the greatest period found indicates a persistent wave, because the probability for chance becomes as high as $100/300=1/3$.

32. *Erroneous applications of the periodogram*—Unfortunately, neither the original and powerful method of Schuster, just described, nor its equivalent in the harmonic dial, as developed since 1922 by the present author, have been applied in any of the later papers dealing with the periodogram. This seems to be due partly to an exaggerated conception regarding the amount of labor needed to compute a large number of harmonic coefficients, partly to the fact that Schuster himself, in his paper on sunspots,[9] does not use his own method.

Most of the recent papers on periodograms (for instance, those of Pollak[11]) and Stumpff[10]) use the following substitute for the exact methods: The harmonic coefficients for a number of selected periods [in Pollak's case (section 14), 73 periods ranging between 2 and 40 days] are computed from the whole observational material, without effective subdivisions (that means, r is taken as the number of all observations). The amplitudes for these "trial periods," or, in some cases, their squares, are summed and divided by the number of the trial periods (in Pollak's case, 73); with this "expectancy," which is the substitute for our $c/\sqrt{N}$ of section 30, the amplitudes of the trial periods are compared, and the ratio κ of each amplitude to the "expectancy" is used to decide, by means of (17.6), on the reality of the large amplitudes.

A. Schuster,[9] in his paper on sunspots, recommends the following procedure for deducing the expectancy: From the whole of the observa-

tional material, without subdivisions, he computes the harmonic coefficients for a number of periods with lengths between 55 days and 24 years and enters their amplitudes in a periodogram. Then he goes on to say:

"It has been stated that in the absence of definite periods the expectancy of the intensity of the periodogram must be obtained from the periodogram itself in all cases where the events to be analyzed are not, as regards their succession, independent of each other. *The expectancy not depending on the period* we may select for the purpose any portion of the curve in which we have no reason to suspect any periodicities. The portion most suitable for this purpose in our case is that lying between 54 days and 1.5 years. Shorter periods must be avoided . . . owing to the fact that sunspots as a rule last several days. . . . Spots persist during more than one solar rotation. This effect will, however, disappear when the period is well above that of the solar rotation. When the periods come near to 1.5 years, the sub-periods of well-ascertained periodicities make their presence felt. Hence the limits chosen for calculating the natural intensity of the periodogram must be confined to about 35 days on the one hand and 1.5 years on the other."

It seems strange that Schuster, in the phrase printed here in italics, renounces his own discovery made in the second paper, and represented here in Figure 14. In fact, it is quite clear that in the case of sunspots the *expectancy must depend very largely on the period*, because of the general reasons discussed in section 30. This is confirmed by an independent calculation[48] of the expectancies for 6-monthly and 12-monthly periods in relative sunspot-numbers, 1872-1930; the amplitudes, calculated from single years, have, in the units of the relative sunspot-numbers, the expectancies 8.3 and 10.9. This distinct increase of the expectancy by about one-third of its value if the period lengthens from 6 to 12 months is likely to continue for longer periods.[51]

Now it seems extremely desirable to "clean the slate" of all uncertain periodicities and regard persistent periods as established only after the severest test. From this standpoint, the danger lies, of course, not so much in cases where the *assumed* expectancy for a certain period is greater than its proper value—though this might occasionally prevent the detection of an actual persistent wave—but in cases where it is *smaller*, because that makes the actually calculated amplitude appear more significant and entails higher values of κ. If, for instance, the proper value of κ is 2.15, indicating a probability for chance $W(\kappa) = 1/100$, an underestimated expectancy assumed at half the proper value would yield $\kappa = 4.3$, with $W(\kappa) = 10^{-8}$, which would erroneously appear to justify a claim for a persistent wave. Table 2 (section 17) illustrates the serious mistakes possible if the expectancy is assumed too low, and, consequently, κ too high, even by as little as one-fourth of the proper value. And such an underestimate of the expectancy is almost certain if, as sometimes suggested, the largest amplitudes of the periodogram are omitted in calculating (in the manner indicated) the expectancy on the ground that they might indicate persistent waves and raise the expectancy unduly.

The periodogram has been discussed here because it has been used so often in previous work. The author prefers the illustration of persistent waves in the harmonic dial for their period, with the cloud of points contracting, with increasing number N of periods combined, into the end-point of the persistent vector (section 28). The harmonic dial

[51] K. Stumpff in his paper on periodicities in sunspots (see foot-note 42) distinguishes at least between short and long periods, the division being taken at about 3 years' length. He follows, however, Schuster in adopting a common expectancy for the longer periods.

confines the attention to the period for which the persistence is to be tested, and avoids the confusion produced by mixing amplitudes for periods of different length, and, therefore, of different expectancy.

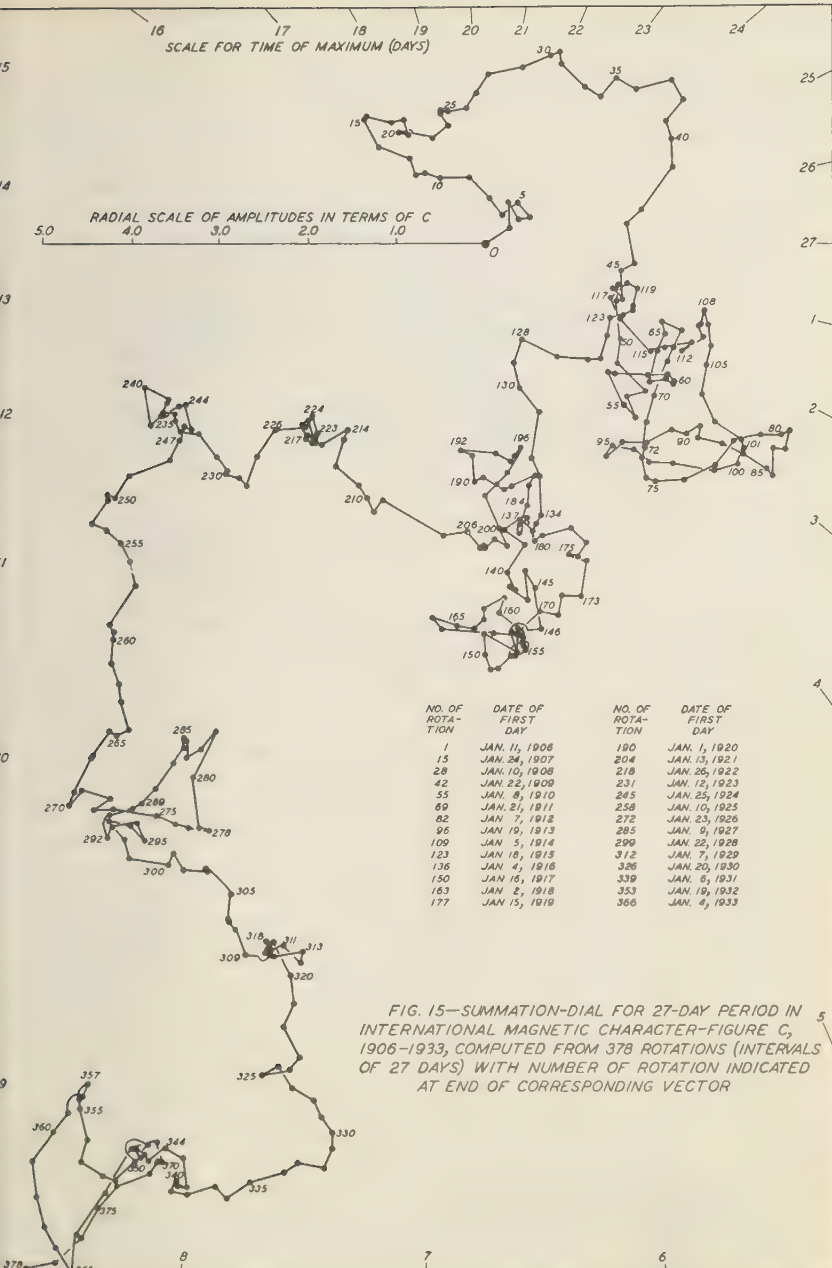
(V) Quasi-Persistence—Effective Expectancy

33. *Quasi-persistent waves*.—We call quasi-persistent such periodicities which are repeated with approximately the same phase and amplitude for a certain number of periods, forming what may be termed a *sequence*, each sequence ending more or less abruptly without any relation to other sequences. This conception is not restricted to sine-waves; in fact, the most striking example is offered by the diagrams[44] for the 27-day recurrences in terrestrial-magnetic activity as described by the international magnetic character-figure C. This recurrence-phenomenon is expressed in quasi-persistence of the various sine-waves with periods that are submultiples of 27 days. Here we shall consider those with periods of 27, 13.5, and 9 days; later (section 40) we shall formulate our results without reference to harmonic analysis or sine-waves.

Quasi-persistence is best studied in connection with the summation-dial, introduced in section 29; summation-dials for the periods of 27 and 13.5 days are reproduced in Figures 15 and 16.[52] (In order to get a better reproduction, the dial in Figure 15 has been turned by 90° from that in Figure 2, vectors with maxima on day 27 pointing to the right.) These diagrams illustrate the vector-addition, step by step, of the single vectors in the harmonic dial for successive rotations; for instance (Fig. 15 for 27-day periods), the vector from the origin O to the point marked 130 in the summation-dial is the sum of all vectors for the single rotations 1 to 130, inclusive; a reduction to 1/130 would give the average vector for the interval of time covered by these 130 rotations. Of course, the summation-dial can also be used to form other averages than those starting at the origin. For instance, the vector connecting the points marked 130 and 378, divided by $(378-130)=248$ would be the average vector for rotations 131 (because the vector connecting points marked 130 and 131 refers to rotation 131) to 378, inclusive. Where the track returns to approximately the same point, the average vector for the intervening rotations is small; for instance, on the summation-dial for the 13.5-day period, the points 140 and 344 fall so close together that they are less than $0.05C$ apart, according to the scale for the single vectors. This means that the average vector (or amplitude for the 13.5-day period) for all the 204 rotations 141 to 344 comprising the whole interval between May 18, 1916, and June 16, 1931, is smaller than $0.05C/204$, or $0.00025C$.

On the other hand, we can select, on the same diagram, Figure 16, long distances traversed in a few rotations. For instance, the points marked 324 and 349 are $6.38C$ apart, indicating an average vector, for the 25 rotations Nos. 325 to 349, of $6.38/25=0.255C$. Applying considerations analogous to section 21 we find, for these 25 rotations, the expectancy for the single vector equal to $0.332C$, and the expectancy for the average of 25 vectors therefore $0.332/\sqrt{25}=0.0664C$, this gives $\kappa=0.255/0.0664=3.84$, $W(\kappa)=4\times10^{-7}$. The probability $W(\kappa)$ for

[52]Figure 15 represents the summation of the single vectors in Figure 2, while Figure 16 represents the same process for the 13.5-day period, for which the analogue to Figure 2 is not reproduced here.



NO. OF ROTA-TION	DATE OF FIRST DAY	NO. OF ROTA-TION	DATE OF FIRST DAY
1	JAN. 11, 1906	190	JAN. 1, 1920
15	JAN. 24, 1907	204	JAN. 13, 1921
28	JAN. 10, 1908	218	JAN. 26, 1922
42	JAN. 22, 1909	231	JAN. 12, 1923
55	JAN. 8, 1910	245	JAN. 25, 1924
69	JAN. 21, 1911	258	JAN. 10, 1925
82	JAN 7, 1912	272	JAN. 23, 1926
96	JAN 19, 1913	285	JAN. 9, 1927
109	JAN 5, 1914	299	JAN. 22, 1928
123	JAN 18, 1915	312	JAN. 7, 1929
136	JAN 4, 1916	326	JAN. 20, 1930
150	JAN 16, 1917	339	JAN. 6, 1931
163	JAN 2, 1918	353	JAN. 19, 1932
177	JAN 15, 1919	366	JAN. 4, 1933

FIG. 15—SUMMATION-DIAL FOR 27-DAY PERIOD IN INTERNATIONAL MAGNETIC CHARACTER-FIGURE C, 1906–1933, COMPUTED FROM 378 ROTATIONS (INTERVALS OF 27 DAYS) WITH NUMBER OF ROTATION INDICATED AT END OF CORRESPONDING VECTOR

FIG. 16—SUMMATION-DIAL FOR 13.5-DAY PERIOD IN INTERNATIONAL MAGNETIC CHARACTER-FIGURE C, COMPUTED FROM 378 ROTATIONS, SIMILAR TO FIGURE 15

chance is so low in this case that even a multiplication by a few powers of ten (because of "selecting" this particular stretch from Figure 16, as mentioned in section 22) might not destroy the strong indication of a persistent wave of 13.5-day period within that interval of 25 rotations—vanishing, however, outside that interval.

Quasi-persistence is indicated in the summation-dials by sequences of vectors of approximately equal directions, for instance, the long sequences in the 13.5-day diagram between the points marked 216 to 231 (December 1921 to January 1923), or 324 to 338 (December 1929 to January 1931), which both correspond, of course, to distinct sequences in the former diagrams[34] for the 27-day recurrences. On the other hand, no such long sequences can be detected in certain parts of the summation-dials Figures 15 and 16, for instance, in the year 1926, rotations 272 to 284. These parts resemble closely the random walk pictured in Figure 8. And if we detect, in the random walk, Figure 8, the apparent "sequence" between the points 76 and 85, we are forced to give up the idea of distinguishing between random walk and quasi-persistence by a mere inspection of the summation-dial or haphazard considerations. In fact, the problem is to find a numerical measure for the geometrical property of the summation-dial which will give a clear distinction between random-walk conditions (Fig. 8), quasi-persistence (Figs. 15 and 16), and persistence (Fig. 11).

34. *Quasi-persistence measured by equivalent length σ of sequences*—In order to find such a measure, we consider a random walk, with the expectancy c of the single vectors. If we form the vectorial sum of every two successive vectors and divide it by 2, that is, if we form averages of every two successive vectors, we obtain a new set of vectors which has the expectancy $c/\sqrt{2}$; in general, if we average h successive vectors, these averages will have the expectancy $c/\sqrt{h}$ according to (19.1). If, however, we have a perfectly persistent wave without any superposed fluctuations, that is, if we have vectors of equal direction, the expectancy for the average of h successive vectors would be, of course, obtained as c.

We can express these conditions in another way. Suppose we compute, from N successive vectors given, the expectancies for the single vectors, for the averages of two vectors, etc., in general, for the averages of h successive vectors. [In order to be able to obtain a satisfactory approximation to the expectancy (section 27), the number of independent averages, roughly N/h, must not be too small; because of formula (27.1), h must not be much greater than about $N/50$, if we want the expectancy correct within 10 per cent.] We multiply the expectancy for the averages of h vectors by $\sqrt{h}$ [or, what amounts to the same, divide the expectancy for the sums of h vectors by $\sqrt{h}$], and obtain a value which we shall call $c(h)$. In the case of the random walk, $c(h)$ is always the same value $c=c(1)$, the expectancy for single vectors; but for persistent vectors, we obtain the ever-increasing value $c(h)=c\sqrt{h}$. In the case of quasi-persistence, $c(2)$ will be greater than $c(1)$, and $c(3)$ will be greater than $c(2)$, etc., but this increase will not continue proportionally to $\sqrt{h}$, as in the case of persistence, but will, in general, asymptotically approach an upper limit, limes $c(h)$, for $h=\infty$, say, $c(\infty)$.

If we now put $c(\infty)/c(1)=\sqrt{\sigma}$, we may call σ (which need not be an entire number), the *equivalent length of the sequences* of the quasi-persistent wave.

This designation of σ is justified as follows: In order to compute the expectancy for our quasi-persistent wave for large values of h, we can proceed as if the average of h single vectors, showing quasi-persistence, is the same as the average of (h/σ) random vectors of expectancy $c(1)$; in other words, as if, of the h vectors, every σ successive vectors are equal, and only h/σ vectors are independent. In fact, the average of (h/σ) random vectors has the expectancy $c(1)/\sqrt{h/\sigma}$, and multiplication with $\sqrt{h}$ gives $c(1)\sqrt{\sigma}$, that is, the same value $c(h)$ as actually computed.

Perhaps σ is the exact expression for what H. H. Turner[30] designated as a "chapter."

35. *Quasi-persistence in terrestrial-magnetic activity*—The actual computation for testing quasi-persistence was based on the summation-dials for the periods of 27, 13.5 and 9 days, the first two of which are reproduced in Figures 15 and 16. h was chosen equal to 4, 9, 16, and 25. Because of (17.7) the arithmetic mean of the amplitudes of a number of vectors, $(c'+c''+\ldots)/n$, distributed according to (17.3), is a constant fraction (0.8862) of the expectancy, defined as the square root of $[(c')^2+(c'')^2+\ldots]/n$. Now, as the law (17.3) can safely be taken as governing the distribution of the single vectors (Fig. 9) as well as their sums, we can be sure to make no systematic error in considering the ratios of the corresponding arithmetic means (which are somewhat easier to calculate) as sufficient approximations for the ratios of the expectancies $c(h)/c(1)$.

For instance, $c(4)/c(1)$ for the 13.5-day period was calculated as follows: The amplitudes of sums of four consecutive single vectors, namely, distances of the points marked O and 4, 4 and 8, . . . 372 and 376, as well as of the points 2 and 6, 6 and 10, . . . , 374 and 378, were measured on Figure 16. The sum of these 188 distances is 127.06 units of C, and the average distance therefore $127.06/188=0.6759C$. In order to deal with averages, and not sums, for four successive vectors, this value must be divided by 4, and then, in order to obtain the equivalent of $c(4)$ multiplied by $\sqrt{4}=2$. This gives $0.3380C$. This should, under random-walk conditions, be equal to the arithmetic mean of the lengths of the single vectors, that is, equal to the arithmetic mean of the $188\times4=752$ distances 0 to 1, 1 to 2, 2 to 3, . . . , 375 to 376 and 2 to 3, 3 to 4, . . . , 377 to 378 (in this arithmetic mean, all single vectors appear twice except those for the rotations 1, 2, 377, and 378). On actual calculation,

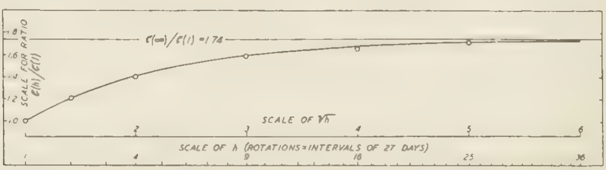


FIG. 17—QUASI-PERSISTENCE, INDICATED BY RATIO OF $c(h)$ (EXPECTANCY OF SUM OF h VECTORS DIVIDED BY $\sqrt{h}$) TO $c(1)$, FOR AVERAGE OF SINE-WAVES WITH PERIODS OF 27, 13.5, AND 9 DAYS IN THE INTERNATIONAL MAGNETIC CHARACTER-FIGURE C, 1906-1933; THE ASYMPTOTIC VALUE $c(\infty)/c(1)=1.74$ INDICATES EQUIVALENT LENGTH OF SEQUENCES $1.74^2=3.0$ ROTATIONS

this mean of these 752 distances is found to be only $0.2298C$. Consequently, our ratio $c(4)/c(1)$ is $0.3380/0.2298=1.470$. In the case of a perfectly persistent wave, we should have obtained for this ratio $\sqrt{4}=2$, and in the other extreme, the random case, ratio 1.

Table 3 shows the results of the calculations. The average ratios have been entered against $\sqrt{h}$ as abscissae, in Figure 17, and fit well into an exponential curve which approaches asymptotically $c(\infty)/c(1)=1.74$. Therefore, the equivalent length of sequences is $\sigma=1.74^2=3.0$ rotations.

TABLE 3—*Quasi-persistence in the international magnetic character-figure C, 1906 to 1933*

Period	$c(1)$	Ratio $c(h)/c(1)$ for $h=$				
		1	4	9	16	25
days 27	$0.262C$	1.000	1.410	1.561	1.632	1.766
13.5	$0.264C$	1.000	1.470	1.729	1.713	1.738
9	$0.236C$	1.000	1.322	1.478	1.611	1.608
Average		1.000	1.401	1.589	1.652	1.704

In addition, for the periods of 27 and 13.5 days, the ratios have been computed for $h=2$, for individual years. Only for the two years 1916 and 1917 (rotations 136 to 162), in the 27-day period, the ratio $c(2)/c(1)$ is smaller than 1, namely, 0.89 and 0.95; this is expressed in the summation-dial Figure 15, where the trace between the points marked 135 and 162 appears very irregular. For the 13.5-day period, the ratios for the same years are 1.20 and 1.12. Particularly high values of $c(2)/c(1)$, approaching the theoretical maximum of $\sqrt{2}=1.41$ for persistence, are found, in the 27-day period, for the years 1911 (1.38), 1913 (1.39), and 1933 (1.38); for the 13.5-day period, for the years 1922 (1.37) and 1930 (1.40). The average ratio for all rotations (years 1906 to 1933) are $c(2)/c(1)=1.196$ for the 27-day period, and 1.226 for the 13.5-day period; the average value 1.211 has been entered in Figure 17.

For $h=4$, the ratios $c(4)/c(1)$ were computed for 14 pairs of years 1906 and 1907, etc. All ratios are greater than 1, the lowest being, of course, 1.04, in the 27-day period for the two years 1916 and 1917, which already gave the lowest ratios $c(2)/c(1)$. Particularly high values of $c(4)/c(1)$, approaching the value $\sqrt{4}=2$ for persistency, are found, in the 27-day period, 1912 to 1913 (1.64), and in the years 1924 to 1925 (1.68), and, in the 13.5-day period, in the years 1930 to 1931 (1.80) and 1932 to 1933 (1.71).

In order to test the strong quasi-persistence noticeable in the former 27-day recurrence-diagram for the years 1928 to 1933,[31] the ratios $c(h)/c(1)$ were also computed for these years alone; they are, for $h=2$, 4, and 9

For the 27-day period: $h=2$, 1.252; $h=4$, 1.436; $h=9$, 1.661
For the 13.5 day period: $h=2$, 1.308; $h=4$, 1.698; $h=9$, 2.151

The highest value, 2.151, would already correspond to an equivalent length of the sequences of more than $\sigma = 2.151^2 = 4.6$ rotations.

A diagram equivalent to Figure 17 may be called the *characteristic* for the period p in the observational material.

36. *Influence of quasi-persistence on tests for persistence: Effective expectancy*—The vectorial average of N vectors which have the expectancy $\boldsymbol{c}(1)$, and quasi-persistence characterized by the equivalent length σ of the sequences (N great compared with σ), has an expectancy which, multiplied by $\sqrt{N}$, we have called, in section 34, $\boldsymbol{c}(N)$; it is therefore $\boldsymbol{c}(N)/\sqrt{N}$, and if N is great enough so that $\boldsymbol{c}(N)/\boldsymbol{c}(1)$ has the limiting value $\sqrt{\sigma}$, we can write $\boldsymbol{c}(1)\sqrt{\sigma}$ for $\boldsymbol{c}(N)$ and obtain for the expectancy of the average of N vectors

$$\boldsymbol{c}(1)\sqrt{\sigma}/\sqrt{N} \tag{36.1}$$

This value differs from the random value $\boldsymbol{c}(1)/\sqrt{N}$, obtained by assuming successive vectors **c** independent, *by the factor* $\sqrt{\sigma}$. If, in quasi-persistent waves, we search for persistent waves as described in sections 28-32, *this* value (36.1) must be taken as the expectancy with which the amplitudes actually found by vector-addition must be compared. *The crucial ratio κ of the amplitudes actually found to their expectancy is therefore reduced in the ratio $1/\sqrt{\sigma}$ against the ratio calculated on the assumption of random-walk conditions, or of independence of successive single vectors, without regard to quasi-persistence.* The consequences for the considerations on the probability for chance $W(\kappa)$ are sometimes decisive, because even a small decrease in κ may mean a large increase of $W(\kappa)$, according to Table 2 (section 16). $\boldsymbol{c}(1)\sqrt{\sigma}$ may appropriately be called *effective expectancy*, as contrasted to the *ordinary expectancy* $\boldsymbol{c}(1)$.

The decisive influence (36.1) of quasi-persistence on tests for persistence as well as on the uncertainty of average sine-waves derived from a large material can also be expressed in another way: Against random conditions, the effective number (N/σ) of the available observations is reduced to $1/\sigma$ of its apparent number N.

We can now adjust our considerations in section 21. There, assuming random-walk conditions and starting from the ordinary expectancy $\boldsymbol{c}(1)$, we obtained, for the average vectors of the 27-day and 13.5-day periods for the 378 rotations 1906 to 1933, the values $\kappa = 2.49$ and 1.47, with $W(2.49) = 1/500$ and $W(1.47) = 1/9$. Taking $\sqrt{\sigma} = 1.74$ (which is certainly not too high, judged from Table 3), the consideration of quasi-persistence, as expressed in the effective expectancy, reduces κ to 1.43 and 0.84, raising $W(\kappa)$ to $W(1.43) = 1/8$ and $W(0.84) = 1/2$. These "probabilities for chance" are so high that there can be absolutely no doubt about the absence of a noticeable persistence in these periods of 27 and 13.5 days; or, expressed more accurately, if persistent waves of these periods existed, the material at hand is not sufficient to trace them.

In the same way, we can dispose of the persistent wave of 9.00-day period which Pollak[11] believed to have traced in the international magnetic character-figure C for the years 1906 to 1926. He obtained for this wave an average amplitude of $0.0412C$, and, with his expectancy, disregarding quasi-persistence, a value $W(2.76) = 1/2000$, which he considered sufficiently low to exclude pure chance. Our own calculation

for the 9-day period gives, for the 284 rotations in the years 1906 to 1926, $c(1)=0.232C$ (only slightly different from the value $0.236C$ given, for all years 1906 to 1933, in Table 3); for $\sqrt{\sigma}$, we make, from Table 3, the conservative estimate 1.62 giving the effective expectancy $0.376C$. The expectancy for an average of 284 rotations is, therefore, $0.376C/\sqrt{284}=0.0224C$, and $\kappa=0.0412/0.0224=1.84$, with $W(1.84)=1/30$, which is not at all suspiciously low. The full series 1906 to 1933 gives, by the way, about the same indication.

37. *Infection of adjacent periods by quasi-persistence*—We have seen that the 27-day sine-wave period in the international magnetic character-figure C shows quasi-persistence with $\sigma=3.0$ rotations. It is easy to see that, for instance, the 28-day period must be affected by this quasi-persistence. Suppose, namely, the series of character-figures C to be divided into intervals of 28 days, beginning January 11, 1906. These single intervals would, on harmonic analysis, give amplitudes of 28-day sine-waves which are only little different from those of the 27-day sine-waves computed from the first 27 days in each 28-day interval; this is easily recognized by considering the folding process (section 10, and Fig. 3), in which the turning angles for the 27-day and 28-day periods, respectively, are $(360°/27)=13°.33$ and $(360°/28)=12°.86$, so that the successive links in the 28-day folding-process are only 0°.47, 0°.94, etc., less inclined against the vertical than the same links in the 27-day folding-process. Therefore, the ordinary expectancy $c(1)$ for the 28-day sine-waves will not differ greatly from that for the 27-day sine-waves. However, successive 28-day intervals begin always one day later than the successive 27-day rotation; if, in the summation-dial for sine-waves of 27-day period, the vectors for successive rotations have nearly the same phase, because of quasi-persistence, then, in the summation-dial for the 28-day sine-waves, because of the relative shift of 27-day and 28-day intervals, the vectors should have phases increasing about $(360°/27)=13°$ from one 28-day interval to the next [the maxima appearing to occur one day earlier in successive 28-day intervals]. Therefore, the general aspect of the summation-dial for 28-day sine-waves would be about the same as that for the 27-day sine-waves (Figure 15), except that successive vectors were turned by about 13° anti-clockwise. This would not greatly affect the values of $c(2)/c(1)$ and even $c(3)/c(1)$ (sections 34, 35); only for higher values of h, $c(h)/c(1)$ for the 28-day period may not increase to the same values as given in Table 3, so that the equivalent length σ of sequences for the 28-day period may be smaller than 3.0. In other words, the 28-day period will show quasi-persistence because it is "*infected*" by the quasi-persistence of the 27-day period. This makes it, apart from other reasons,[63] difficult to determine the exact length of the 27-day recurrence-interval, which might differ from 27 days by a few tenths of a day, and could be recognized as yielding the highest value of σ.

This kind of infection will diminish the greater the difference of the periods. For periods of 30 days, for instance, the infection by the 27-day period will be small. Actual calculation of 27-day periods and 30-day periods for the character-figure C for the two years 1924 and 1925 gave the following results—the values for 27-day period and 30-day period

[63] J. M. Stagg, Meteorological Office, Geophysical Mem. No. 40, London (1927).

being, in each case, printed after each other: Number of full intervals considered 27, 24; expectancy for single interval $c(1)=0.258C$, $0.248C$; $c(2)/c(1)=1.26$, 1.22; $c(4)/c(1)=1.68$, 1.21. This sample calculation allows one to assume, for the 30-day period for all years 1906 to 1926 used by Pollak, $\sqrt{\sigma}$ about 1.2, and the ordinary expectancy $c(1)=0.252C$, namely, $0.010C$ less than $c(1)=0.262C$ for the 27-day period in the same years 1906 to 1926, so that the effective expectancy becomes $c(1)\sqrt{\sigma}=0.302C$.

The expectancy for the average wave of Pollak's 255 intervals of 30 days is therefore $0.302C/\sqrt{255}=0.0189C$, and the same value will hold, with high approximation, for a wave of 29.9 days. The actually calculated sine-wave of Pollak for 29.9 days from his material has an amplitude of $0.0511C$. Therefore, $\kappa=(0.0511/0.0189)=2.70$, with $W(2.70)=1/1460$. This value might look suspiciously small, though not so small as the value of 1/110,000 which Pollak himself derives using a too low expectancy. But, since the 29.9-day period is picked, *because* of its high amplitude, out of 73 amplitudes actually calculated, the "probability for chance," according to section 22, is $(73/1460)=1/20$, and this is not small enough to warrant the definite assumption of persistence. There remains the possibility of long-range quasi-persistence, corresponding to Ad. Schmidt's idea of deep-seated long-lived foci in the Sun's surface-layers, with a rotation of about 30-day period.

38. *Interference*—The infectiousness of quasi-persistence, as described in section 37, is related to the general interference-phenomenon leading to the "*spurious periodicities*" which A. Schuster[8] discovered as complete analogues to the secondary maxima obtained in analyzing homogeneous light by a spectroscope of finite resolving power. His exact formula will be illustrated here by a straightforward application of the summation-dial which will yield an approximation sufficient for practical use.

Consider the persistent sine-wave of half-year period (section 29) in the international magnetic character-figure C, of amplitude $0.0675C$, and constant phase. The summation-dial of this period for the 56 half-years 1906 to 1933, freed from the irregular fluctuations exhibited in

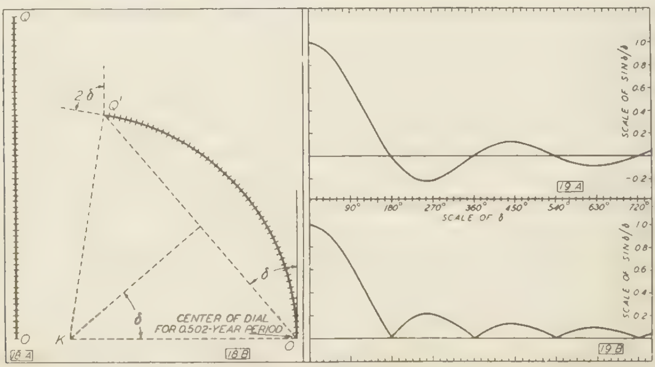


FIG. 18—INTERFERENCE, DEMONSTRATED IN SUMMATION-DIALS FOR 56 PERIODS OF 0.500 YEAR (18 A), AND 56 PERIODS OF 0.502 YEAR (18 B)
FIG. 19—FUNCTION SIN δ/δ

Figure 11, would be a succession of 56 perfectly aligned single vectors leading from the center O of the dial to the point Q (Fig. 18A).

What should we obtain if we would analyze the train of these 56 sine-waves of exactly $p=0.500$-year period for a slightly different period of length, say, $p'=0.502$ year? Fifty-six complete waves of period p' would cover an interval of 56.112 half-years. The harmonic dial for the 56 single waves of period p' in that interval would show amplitudes practically equal to that of the actual half-year period, but the maxima of each wave of period p' would shift gradually and occur earlier. Since $250\ p'=251\ p$, the phases of two periods would agree again after 250 intervals of length p'; consequently, the phase of the period p' shifts from one interval of length p' to the next by $(360°/250)=1°.44$. After 56 intervals, the phase-shift would be about 81°. We will call this angle 2δ. The summation-dial for the period $p'=0.502$ year is therefore approximately part of a circle (Fig. 18B), the length of the arc $O'Q'$ being equal to OQ, and the tangent of the circle drawn in Q' forming an angle of $2\delta=81°$ with the tangent drawn in O'.

Now the vectorial sum of the 56 sine-waves with period p' is represented by the straight line $O'Q'$, while the vectorial sum of the 56 sine-waves with period p is represented by OQ, which, as we have seen, is equal to the length of the arc $O'Q'$. The amplitudes of the average vectors of periods p and p' are obtained by dividing the vectorial sums by 56. Therefore, the ratio of the amplitudes of the average sine-waves with periods p' and p is equal to the ratio of the lengths of the chord and the arc $O'Q'$, or $\sin\delta/\delta$, as the auxiliary construction in Figure 18B indicates.

In general, consider a train of waves with a persistent period of length p, and suppose it to be analyzed for a period of slightly different length, $p'=p+\triangle p$, where $\triangle p/p$ is small. Putting $m=p/\triangle p$, we find $mp'=(m+1)p$, and since m is large, this will hold also if, instead of the exact value $m=p/\triangle p$, we take the nearest integer for m. Then our equation means that the interval covered by m periods p' is covered by $(m+1)$ periods p. This means that, in the summation-dial for period p', the shift of phase between a certain vector to the vector for an interval occurring mp' later is 2π, and therefore the phase-shift for successive vectors is $2\pi/m$. If the interval considered contains only n waves of period p, the angle 2δ of Figure 18B becomes $2\delta=2\pi n/m=2\pi n\ \triangle p/p$, and the ratio of the average amplitude of the "spurious" period p' to the average amplitude of the persistent wave with period p becomes

$$(38.1) \qquad \sin\delta/\delta \text{ with } \delta=\pi n\ \triangle p/p$$

where p is the length of the exact period, $(p+\triangle p)$ the length of the spurious period, and np the length of the whole interval analyzed. Because of the slight idealization assumed at the end of the summation-dial, this formula gives the correct value within the limit $1/n$, which is practically sufficient since n must be large enough anyway (see footnote 46).

The function $\sin\delta/\delta$ has been plotted in Figure 19A. The "spurious" amplitude vanishes for $\delta=\pi,\ 2\pi$, etc., that is, $\triangle p=p/n,\ 2p/n$, etc. This is only another expression for the independence of the harmonic coefficients in the series (5.2), because the length of the whole interval is np,

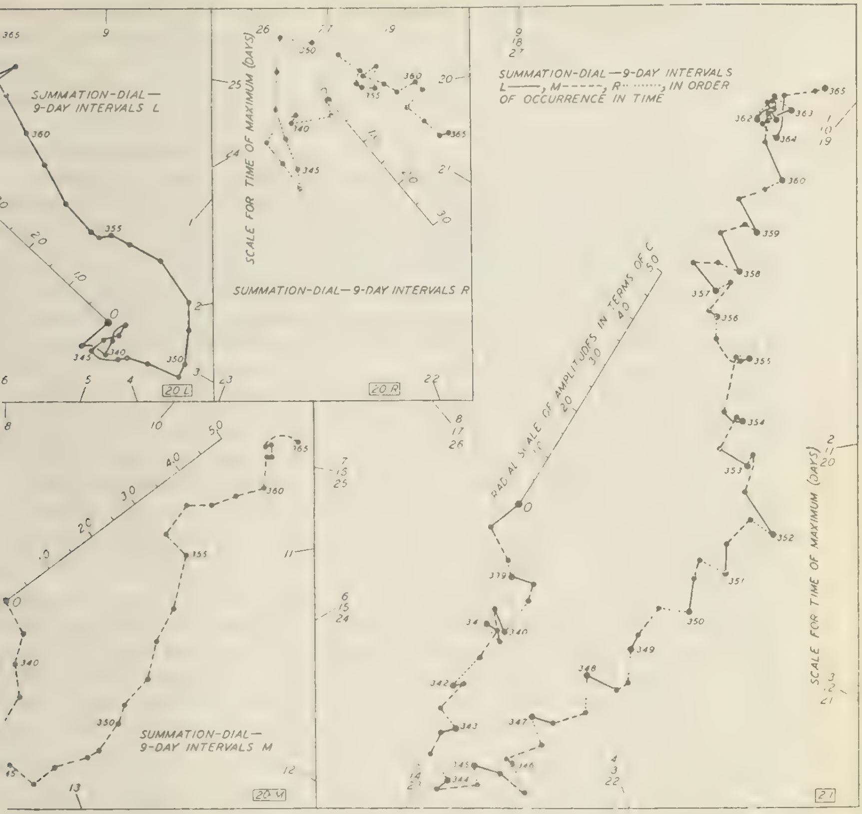
so that the period p has the frequency n (supposed to be high), and the frequencies $(n-1)$, $(n-2)$, ... have the periods $[np/(n-1)]$, $[np/(n-2)]$, ..., or, nearly, $[p+(p/n)]$, $[p+(2p/n)]$, etc. Of course, our discussion applies equally to periods $(p-\triangle p)$, etc., so that Figure 19 may be extended symmetrically to $\delta=0$. The negative sign of $\sin \delta/\delta$ between $\delta=\pi$ and 2π, 3π and 4π, etc., is, in our case, not significant and can be disregarded, as in Figure 19*B*.

If the observational material contains a persistent wave, the period p of which is no entire submultiple of the total interval T of observation, ordinary harmonic analysis would, in the series (5.2), not indicate the full amplitude c of this wave, but only a part of it in spurious periodicities. For instance, if $(n+0.5)p=T$, where n is an integer, we have $\triangle p/p=0.5/n$, and $\delta=\pi/2$, so that the waves with frequencies n and $(n+1)$ would show amplitudes with $0.637c$. It is therefore necessary to search the neighborhood of periods with suspiciously high amplitudes for the exact period of a possibly persistent period. This is done by Darwin's method of approximation, first used for the calculation of tides, and described by Stumpff and Pollak (foot-notes 10 and 11). The summation-dial, Figures 18*A* and 18*B*, is a reliable guide in applying this method, which approximates the arc $O'Q'$ in Figure 18*B* by a number of smaller chords; in other words, partial average vectors for the period p are calculated for a small number of groups—for instance, 10 groups of 4 half-years in our case—and these partial averages are combined in different ways, namely, *without* phase-shift to obtain the average vector for period p, and *with* appropriate phase shifts to obtain the average vector for adjacent periods $(p+\triangle p)$.

For another aspect of interference, see appendix section *A8*.

39. *Special kinds of quasi-persistence*—The interference-phenomenon described in section 38 can be conceived as some regular kind of quasi-persistence caused, by a persistent wave of period p, in waves with adjacent periods $(p+\triangle p)$. The diagram of Figure 17 for such an adjacent period would begin with interference "beats" similar to those of Figure 19*B*, but these oscillations would gradually be damped and end in the value $c(h)/c(1)$ for the actual quasi-persistence. A lunar wave derived from material with larger solar waves, for instance, in atmospheric pressure, or in the terrestrial-magnetic force, is a typical example.

A more general kind of quasi-persistence would be given by a case where, in the summation-dial, not the successive intervals, but perhaps the first, fourth, seventh, etc., exhibit a tendency to have the same phase. We obtain exactly this case, if we break up our 378 rotations of 27 days in the international magnetic character-figure C into $(3\times378)=1134$ intervals of 9 days. We shall designate each of these 9-day intervals by the rotation-number and distinguish the three thirds of each rotation by the letters L, M, R (left, middle, right). Figures 20 and 21 show, in different arrangement, the summation-dials for these sine-waves with 9-day periods for the years 1931 and 1932, comprising the 27 rotations Nos. 339 to 365, or 81 intervals of 9 days. Figure 21 shows the 81 vectors for the 9-day intervals as they follow each other in time, that is, for interval 339L, 339M, 339R, etc., up to 365R. The vectors for intervals L, M, R have been distinguished by drawing them in full, dashed, and dotted, respectively. In Figure 21 the number of each rotation is entered against the end (marked more boldly) of the last



0—SUMMATION-DIALS FOR 9-DAY PERIOD SINE-WAVES IN INTERNATIONAL MAGNETIC CHARACTER-FIGURE C, AND 1932, AS COMPUTED FOR 9-DAY INTERVALS L (DAYS 1–9), M (DAYS 10–18), R (DAYS 19–27), OF 27-DAY INTERVALS, ROTATIONS 339–365

1—SUMMATION-DIAL L-M-R FOR 9-DAY INTERVALS OF FIGURE 20

vector R for each rotation. (It is to be remarked that should we retain only these points marked in Figure 21 and omit the points for the ends of vectors L and M, we should obtain the summation-dial, on a three-fold magnified scale, for the 9 day periods calculated from *whole* rotations of 27 days, analogous to Figures 15 and 16, and discussed in section 35.) The time of maximum is indicated by the scales around the borders of the dials, by days 1 to 9 for L, 10 to 18 for M, 19 to 27 for R, according to the numbering of the days in the rotations.

Calculations similar to those in section 35, based on Figure 21, lead to the expectancy, for single vectors, $c(1) = 0.540C$, and for the ratios $c(h)/c(1)$, we obtain 1.109 for $h = 3$, 1.369 for $h = 6$, and 1.669 for $h = 9$.

On inspection of Figure 21, we find the feature indicated above, namely, practical independence for vectors immediately following each

other, but every third vector—for instance, those for M-intervals indicated by dashed lines—has a tendency to keep its phase. This is brought out more clearly in Figures 20L, 20M, and 20R, where the vectors for the L-, M-, and R-intervals are added separately and show considerable quasi-persistence. If we calculate $\mathfrak{c}(\mathrm{h})/\mathfrak{c}(1)$ for these three summation-dials of Figure 20, we obtain, on the average, 1.241 for $h=2$ and 1.395 for $h=3$. The contrast between the values of $\mathfrak{c}(h)/\mathfrak{c}(1)$ for $h=3$, namely, 1.109 for Figure 21, 1.395 for Figures 20L, 20M, and 20R, is the numerical expression for this particular kind of quasi-persistence, which could be called *intermittent*: The three thirds L, M, R of each rotation give nearly independent 9-day sine-waves, but corresponding thirds, for instance, the L-intervals alone, show strong quasi-persistence. This proves incidentally that the 9-day period has no self-existence, but is only a sub-period of the 27-day period, which is the actual periodicity.

The value $\mathfrak{c}(3)/\mathfrak{c}(1)=1.109$ for Figure 21, small as it is, is nevertheless greater than unity and reveals a weak degree of quasi-persistence, which, however, seems to be a general phenomenon in many cases in which dependent ordinates are divided into sets, because, for instance, a group of high ordinates, divided up by a limit between two sets, adds likewise, in both sets, to the cosine-coefficient a_ν of the successive sets.

The most general definition of quasi-persistence as distinguished from random conditions leads, of course, to the same fundamental difficulties encountered in a satisfactory definition of the term "accidental" in the theory of probability. In this respect, we refer to the books of Mises and Kamke[6], especially to the definition of the "fields of probabilities" discussed by Kamke.

From our much-used example of the 378 rotations in C, we can easily construct an illustration of these remarks. Imagine our 378 rotations divided into seven groups comprising rotations Nos. 1 to 54, 55 to 108, 109 to 162, . . . , and 325 to 378. In each group, mix the numbers of the rotations at random. Then draw the summation-dial for the 27-day period which, in the points 54, 108, . . . , 378 would be identical with that in Figure 15, and test for quasi-persistence. Obviously we should obtain a different curve from Figure 17, namely, $\mathfrak{c}(h)/\mathfrak{c}(1)$ would remain near unity for low values of h, because the mixing has produced random conditions for these, but with h approaching 54, $\mathfrak{c}(h)/\mathfrak{c}(1)$ will rise to the limiting value indicated in Figure 17.

If a persistent wave of amplitude c is present, $\mathfrak{c}(h)$ in Figure 17, with abscissa $\sqrt{h}$, would finally approach the line $c\sqrt{h}$, or $c\sqrt{N}$, if we write N for large values of h. The κ-test for persistent waves (section 36) can easily be applied to this characteristic, because $c\sqrt{N}/\mathfrak{c}(1)\sqrt{\sigma}=c/(\mathfrak{c}(1)\sqrt{\sigma}/\sqrt{N})$, and this is κ because of (36.1). We can therefore enter a uniform scale of κ, where $\kappa=1$ corresponds to the effective expectancy $\mathfrak{c}(1)\sqrt{\sigma}$ (see section 41).

40. *Periodicities of other form than sine-waves.*—The application of harmonic analysis to research on geophysical periodicities is sometimes criticized because the form of the periodicity—for instance, an average diurnal variation of magnetic declination—is said to be in no way connected with sine-waves, which are forced upon it by the purely mathematical process of harmonic analysis. In fact, a *physical* reason for expecting periodicities having the form of sine-waves is given only in a

few cases, for example, if the phenomenon is due to resonance in an oscillating system (semidiurnal wave of atmospheric pressure), or if it is caused by forces changing like sine-waves (tides), or wherever a differential equation of the type $y'' = -ky$ may hold. But a *mathematical* reason for applying harmonic analysis is always given, because it furnishes an adequate approximation, replacing the given ordinates by a few harmonic coefficients.

In order to meet the criticism mentioned, the 27-day recurrence-phenomenon in magnetic activity has been treated here as an example *just because* the harmonic analysis is, in this case, a mere mathematical affair, with no simple physical meaning ascribable to each of the separate sine-waves of 27-, 13.5-, and 9-day periods. Yet we have been able to develop the ideas of persistence and quasi-persistence in this material. There can be therefore no doubt that the same methods can be successfully applied in dealing with other geophysical and meteorological phenomena.

However, the following outline of a test for persistence, quasi-persistence, or random fluctuations will show how our methods can be generalized so that they do no more imply an explicit reference to harmonic analysis. Consider the international magnetic character-figure C, 1906 to 1933, arranged in 378 rotations, that is, written in 27 columns with 378 rows. Form, for each rotation, the standard deviation, take its square, sum up for all rows, divide by 378, take square root: the value obtained is called $\zeta(1)$. Add each two successive rows of C and divide by two, thus obtaining average 27-day variations for two rotations each. For these new average rows, form standard deviation, take its square, sum up, divide by number of average rows, take square root, multiply with $\sqrt{2}$; the value obtained is called $\zeta(2)$. In general, form average rows of h successive rotations, compute standard deviation for each row, square, sum up, divide by number of average rows, take square root, multiply by $\sqrt{h}$, so obtaining values called $\zeta(h)$. With random fluctuations, $\zeta(h) = \zeta(1)$; with persistent periodicities $\zeta(h) = \zeta(1)\sqrt{h}$; with quasi-persistent periodicities of 27 days, $\zeta(h) \rightarrow \zeta(1)\sqrt{\sigma}$, where σ is the equivalent length of sequences.[54]

Remembering formula (11.6), and the conception of the generalized harmonic dial (section 13), in which the vector is proportional to ζ, it is easily verified that this method corresponds exactly to the generalization of the two-dimensional summation-dial to the generalized harmonic dial. Instead of Figures 15 and 16, we should consider a track of vectors, a summation-dial, in 26 dimensions. In the case of the character-figure C, the value for σ obtained will be not far from that of $\sigma = 3.0$ rotations obtained from the three sine-waves of 27-, 13.5-, and 9-day periods, since the amplitudes of these waves contribute most of $\zeta(1)$.

In general, rows of r ordinates would be written down, and averages of h such rows formed. In particular, for $r = 1$, we should obtain some form of Lexis test of independence of successive ordinates, suitable for geophysical applications.

41. *General statistical test for periodicity in geophysical phenomena*—

[54] As to persistent waves, the method given above is materially the same as that given in 1924 by E. T. Whittaker and G. Robinson (see foot-note 35). It had been independently found and applied in the author's doctor thesis, Göttingen, 1922.

In short, the full test of a period p for quasi-persistence and persistence is obtained as follows: Divide, in a suitable way as shown in section 35, the whole interval T of observations into intervals of equal length hp. Compute the amplitudes of the sine-wave of period p for each interval and from these amplitudes compute their expectancy according to (24.1). Multiply this expectancy by $\sqrt{h}$ and obtain $\mathfrak{c}(h)$. Derive $\mathfrak{c}(h)$ for various values of h, beginning with $h=1$, and ending with a value of h so that hp is still only about 1/20 of T, so that the function $\mathfrak{c}(h)$, represented as ordinate against the abscissa $\sqrt{h}$, is properly determined. From this characteristic, the nature of the fluctuations can be judged. Instead of the amplitudes of sine-waves, standard deviations can be used as indicated in section 40.

In Figure 22, five typical cases are shown. They will be enumerated below, and we add tentatively a few more examples for each type of the characteristic:

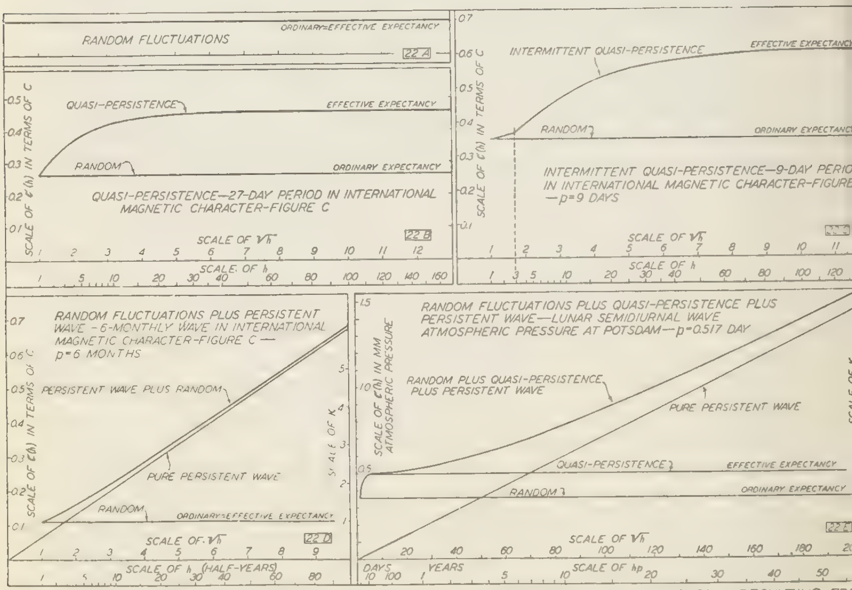


FIG. 22—$\mathfrak{c}(h)=\sqrt{h}$ TIMES ORDINARY EXPECTANCY FOR AVERAGE SINE-WAVES OF PERIOD p RESULTING FRO[M] HARMONIC ANALYSIS OF INTERVALS OF LENGTH hp AS FUNCTIONS OF $\sqrt{h}$ —FIVE TYPICAL CASES

(*A*) *Random fluctuations—$\mathfrak{c}(h)$ equal to the ordinary expectancy $\mathfrak{c}(1)$*—Disintegrations of radioactive substances; artificial examples obtained by random sampling (summation-dials resembling Figs. 7 and 8).

(*B*) *Random fluctuations plus quasi-persistence*—$\mathfrak{c}(1)$ increases asymptotically from the ordinary expectancy $\mathfrak{c}(1)$ to a limiting value, the effective expectancy $\mathfrak{c}(1)\sqrt{\sigma}$ (summation-dials resembling Figs. 15 and 16). 27-day period in terrestrial-magnetic activity (section 35), aurora, and in solar phenomena, due to solar rotation; many meteorological phenomena, for instance, the periods of from 20 to 40 days in at-

mospheric pressure (Weickmann), the waves of periods of a few days in rainfall (Defant), the period of 3.5 years in atmospheric pressure in the Indian Ocean, the cycles of Brückner, A. E. Douglass (tree-rings), and many others.

(*C*) *Intermittent quasi-persistence*—$\mathbf{c}(h)$ increases slowly for $\mathbf{c}(1)$ up to $c(h_0)$, from there follows example *B* (summation-dials resembling Fig. 21). Period p is submultiple of actual period of length $h_0 p$. Nine-day sine-waves in international magnetic character-figure *C* (section 39), and all cases of subperiods.

(*D*) *Random fluctuations plus persistence*—$\mathbf{c}(h)$ increases from $\mathbf{c}(1)$, approaching asymptotically the straight line $c\sqrt{h}$, where c is the amplitude of the persistent wave. Probability of chance for persistent wave judged by $W(\kappa)$, (17.6), with κ=ratio of $\mathbf{c}(h)$ to ordinary expectancy $\mathbf{c}(1)$ (summation-dials resembling Fig. 11). Six-monthly wave in terrestrial-magnetic activity (section 29); the period of about 11 years in sunspots (?) and its effects in geophysical phenomena; most annual variations in meteorology; cyclic variations in the radiation of variable stars.

(*E*) *Random fluctuations plus quasi-persistence and persistence*—Combination of *B* or *C* with *D*. Probability of chance for persistent wave judged by $W(\kappa)$, (17.6), with κ=ratio of $\mathbf{c}(h)$ to effective expectancy $\mathbf{c}(1)\sqrt{\sigma}$. All waves of 24 solar- and lunar-hour period, and their subperiods, in terrestrial magnetism, atmospheric electricity, meteorology, etc. Periods in sunspots other than 11 years. Biological and economical cycles. Quasi-persistence exhibited in the vectorial differences between the waves for single intervals and the persistent wave.

The illustration of these five cases in the *summation-dial* may finally be indicated: *A*—Random-walk; *B*—Modified random walk, so that each successive direction has a preference for the direction of the last vector; *C*—Like *B*, but the preferred direction is, for instance, that of the third vector before; *D*—Modified random walk preferring a *fixed* direction; *E*—Combination of *B*, or *C*, with *D*.

42. *Acknowledgments*—The numerical and graphical examples given in this paper were worked out with assistance given by C. C. Ennis and W. C. Hendrix at Washington, D. C., and by W. Zick at Eberswalde, Germany.

Appendix

A1. Harmonic analysis of equidistant ordinates: *Theorem I*—The interval $x=0$ to 2π is divided into r equal intervals, of length $2\pi/r$, by the abscissae $0, x_1, x_2, \ldots x_r$, where $x_\rho=\rho\, 2\pi/r$. A function $f(x)$ is given by the ordinates $y_\rho=f(x_\rho)$ for $\rho=1, 2, \ldots r$. The arithmetic mean of the y_ρ may be $f_0=\Sigma y_\rho/r$; their standard deviation may be ζ, defined by $\zeta^2=\Sigma(y_\rho-f_0)^2/r$. Consider a sum of sine- and cosine-functions of frequency $\nu=0, 1, \ldots, k$, with $k<r/2$

$$(A1.1) \qquad \phi_k(x)=a_0+\sum_{\nu=1}^{k}(a_\nu \cos \nu x+b_\nu \sin \nu x)$$

The coefficients a_0, a_ν, b_ν ($\nu=1, 2, \ldots k$) of $\phi_k(x)$ must be determined so that $\phi_k(x)$ approximates the given ordinates y_ρ of $f(x)$ in the meaning of

least-square adjustment, that is, so that the mean square s_k^2 of the residuals $y_\rho - \phi_k(x_\rho)$, that is

$$s_k^2 = \Sigma_\rho \, [y_\rho - \phi_k(x_\rho)]^2/r \tag{A1.2}$$

has the minimum value possible. The solution is

$$\begin{aligned} a_0 &= \sum_{\rho=1}^{r} y_\rho/r = f_0, \\ a_\nu &= \sum_{\rho=1}^{r} y_\rho \cos \nu x_\rho/(r/2), \\ b_\nu &= \sum_{\rho=1}^{r} y_\rho \sin \nu x_\rho/(r/2) \end{aligned} \tag{A1.3}$$

a_ν, b_ν are called harmonic coefficients. If r is an even number, a final term $a_{(r/2)} \cos (r/2)x$ can be added to $\phi_k(x)$, for which the minimum condition (*A1.2*) gives

$$a_{(r/2)} = (-y_1 + y_2 - y_3 + y_4 - \ldots + y_r)/r \tag{A1.3a}$$

Furthermore, the ordinates $\phi_k(x_\rho)$ of the approximating function have the average value

$$a_0 = \sum_{\rho=1}^{r} \phi_k(x_\rho)/r \tag{A1.4}$$

and their standard deviation η_k is given by

$$\eta_k^2 = \sum_{\nu=1}^{k} (a_\nu^2 + b_\nu^2)/2 \tag{A1.5}$$

or, for r even and $k = r/2$

$$\eta_{(r/2)}^2 = \sum_{\nu=1}^{(r/2)-1} (a_\nu^2 + b_\nu^2)/2 + a_{(r/2)}^2 \tag{A1.5a}$$

Finally for $k < r/2$

$$s_k^2 = \zeta^2 - \eta_k^2 = \zeta^2 - \sum_{\nu=1}^{k} (a_\nu^2 + b_\nu^2)/2 \tag{A1.6}$$

If the number of coefficients a_0, a_ν, b_ν in ϕ_k equals the number r of ordinates [for r even, $k = r/2$, for r uneven, $k = (r-1)/2$], the ordinates y_ρ are represented exactly by the ordinates of ϕ_k.

Proof—The proof is based on the fact that the system of functions 1, $\cos \nu x$, $\sin \nu x$ ($\nu = 1$ to k) are orthogonal[55] in the interval $x = 0$ to 2π. This fundamental property is expressed in the following formulae, in which the sums are extended from $\rho = 1$ to r and the indices ν and μ range between 1 and $(r-1)/2$, for r uneven, and between 1 and $r/2$ for r even, unless the index $r/2$, for r even, is expressly indicated by a separate formula.

$$\Sigma \cos \nu x_\rho = 0 \; (\nu < r), \; \Sigma \sin \nu x_\rho = 0 \; (\nu \leqq r) \tag{A1.7}$$

$$\Sigma \cos^2 \nu x_\rho = r/2, \; \Sigma \sin^2 \nu x_\rho = r/2 \; (\nu < r/2), \; \Sigma \cos^2 (r/2)\, x_\rho = r \tag{A1.8}$$

[55] Developments of arbitrary functions into series of orthogonal functions, such as sine-waves, spherical harmonics, etc., are discussed, for instance, in R. Courant and D. Hilbert, Methoden der mathematischen Physik, 2nd ed, Berlin, 1931.

$$(A1.9) \qquad \Sigma \cos \nu x_\rho \cos \mu x_\rho = 0,\ \Sigma \sin \nu x_\rho \sin \mu x_\rho = 0 \quad (\nu \neq \mu)$$

$$\Sigma \cos \nu x_\rho \sin \mu x_\rho = 0 \quad (\nu = \mu \text{ or } \nu \neq \mu)$$

We first prove ($A1.7$) by Moivre's theorem, namely, (writing exp z for e^z) $\cos \nu x_\rho + i \sin \nu x_\rho = \exp i\, \nu x_\rho = \exp i\, \nu\rho 2\pi/r = (\exp i\, \nu\, 2\pi/r)^\rho = q^\rho$, putting $q = \exp i\, \nu\, 2\pi/r$. Summing these equations from $\rho = 1$ to r, we obtain on the left hand $\Sigma \cos \nu x_\rho + i \sin \nu x_\rho$, and on the right hand the geometrical series

$$(A1.10) \qquad q + q^2 + q^3 + \ldots + q^r = q(q^r - 1)/(q-1)$$

But this is zero, for the denominator $(q-1) \neq 0$, since $1 \leqq \nu < r$ and $q^r = \exp i\nu 2\pi = 1$. The real and the imaginary part of the left-hand side must therefore also vanish, proving ($A1.7$).

The well-known formulae for $\cos (\nu+\mu)x$, $\cos (\nu-\mu)x$, etc., give at once for all values of ν and μ

$$(A1.11) \qquad \begin{cases} 2 \cos \nu x_\rho \cos \mu x_\rho = \cos (\nu+\mu)x_\rho + \cos (\nu-\mu)x_\rho \\ 2 \sin \nu x_\rho \sin \mu x_\rho = \cos (\nu-\mu)x_\rho - \cos (\nu+\mu)x_\rho \\ 2 \cos \nu x_\rho \sin \mu x_\rho = \sin (\nu+\mu)x_\rho - \sin (\nu-\mu)x_\rho \end{cases}$$

Summing from $\rho = 1$ to r, the right-hand sums vanish because of ($A1.7$), except $\Sigma \cos (\nu-\mu)x_\rho = r$ for $\nu = \mu$, because $\cos 0 = 1$, and $\Sigma \cos (\nu+\mu)x_\rho = r$ for $\nu = \mu = r/2$, r even, because $\cos r\, x_\rho = \cos 2\pi = 1$. That proves ($A1.8$) and ($A1.9$)[16].

We can now prove our theorem. First, ($A1.4$) follows from ($A1.7$), and if we form $(\phi_k(x_\rho) - a_0)^2 = (a_1 \cos x_\rho + b_1 \sin x_\rho + \ldots + a_k \cos kx_\rho + b_k \sin kx_\rho)^2$ and sum from $\rho = 1$ to r, ($A1.5$) follows from ($A1.8$) and ($A1.9$). We consider now a function $\phi_k^*(x)$ of the same form as $\phi_k(x)$, but with arbitrary coefficients a_0^*, a_ν^*, b_ν^*, while for $\phi_k(x)$ we take the coefficients defined by ($A1.3$). We consider the sum $\phi_k(x) + \phi_k^*(x)$ and shall find that this approximation to y_ρ is worse than that given by $\phi_k(x)$ alone, unless all coefficients of $\phi_k^*(x)$ disappear. We form the square of the residual (omitting the index k in $\phi_k(x_\rho)$ and $\phi_k^*(x_\rho)$ where it is not necessary)

$$[y_\rho - (\phi(x_\rho) + \phi^*(x_\rho))]^2 = y_\rho^2 + \phi(x_\rho)^2 + \phi^*(x_\rho)^2 - 2\, y_\rho\phi(x_\rho) - 2\, y_\rho\phi^*(x_\rho) + 2\, \phi(x_\rho)\phi^*(x_\rho)$$

Inserting the series ($A1.1$) for ϕ and its equivalent for ϕ^*, and adding

[16] All these formulae can easily be transformed into simple geometrical problems by means of the harmonic dial or our folding process (section 10). ($A1.7$) is, for instance, only the expression for the closing of a regular polygon, if star-shaped polygons are admitted. ($A1.8$) and ($A1.9$) refer to some kind of epicyclic motion, described by a point on the circumference of a circle revolving with frequency $(\nu+\mu)$, while its center revolves with frequency $(\nu-\mu)$ on another circle of the same radius and fixed center. This explains the regularity of Figure 4. With k circles with radii c_ν, each center moving with frequency ν on the circumference of the preceding circle, with beginning of the movement given by the phases a_ν, the movement of a point on the circumference of the outermost circle, projected on a vertical line, reproduces the function $\phi_k(x)$; this is the principle of tidal computing machines.

Of course, the harmonic dial is equivalent to the ordinary geometrical representation of complex numbers, because $c_\nu \sin (\nu x + a_\nu)$ is the imaginary part of $c_\nu \exp i\, (\nu x + a_\nu)$; our vector in the harmonic dial represents the "complex amplitude" $c_\nu \exp ia_\nu$, the factor $\exp i\nu x$ being common to all waves of frequency ν. This is the connection to the electrotechnical diagrams used for describing alternating currents (see, for instance, **17**, part 1, of the Handbuch der Experimentalphysik, Leipzig, 1934). For geophysical purposes, however, the special form of diagram described as harmonic dial is clearer.

up for $\rho=1$ to r, the terms on the right hand yield successively (the formulae applied being cited in brackets in each row)

$\sum_{\rho=1}^{r} y_\rho^2 = r\zeta^2 + ra_0^2$ After (11.2)

$\sum_{\rho=1}^{r} \phi(x_\rho)^2 = ra_0^2 + (r/2)\sum_{\nu=1}^{k}(a_\nu^2+b_\nu^2)$ After (*A1*.7) to (*A1*.9)

$\sum_{\rho=1}^{r} \phi^*(x_\rho)^2 = r(a_0^*)^2 + (r/2)\sum_{\nu=1}^{k}[(a_\nu^*)^2+(b_\nu^*)^2]$ After (*A1*.7) to (*A1*.9)

$-2\,\Sigma\, y_\rho\phi(x_\rho) = -2\,ra_0^2 - r\sum_{\nu=1}^{k}(a_\nu^2+b_\nu^2)$ After (*A1*.3)

$-2\,\Sigma\, y_\rho\phi^*(x_\rho) = -2\,ra_0a_0^* - r\sum_{\nu=1}^{k}(a_\nu a_\nu^*+b_\nu b_\nu^*)$ After (*A1*.3)

$2\,\Sigma\, \phi(x_\rho)\phi^*(x_\rho) = 2\,ra_0a_0^* + r\sum_{\nu=1}^{k}(a_\nu a_\nu^*+b_\nu b_\nu^*)$ After (*A1*.7) to (*A1*.9)

Therefore, the average square residual is, if we use (*A1*.5)

$$(A1.12)\qquad \sum_{\rho=1}^{r}[y_\rho - (\phi(x_\rho)+\phi^*(x_\rho))]^2/r = \zeta^2 - \eta_k^2 + (a_0^*)^2 + \sum_{\nu=1}^{k}[(a_\nu^*)^2+(b_\nu^*)^2]/2$$

The minimum value of the right-hand side is (*A1*.6), if $a_0^*=a_1^*=b_1^*=\ldots=a_k^*=b_k^*=0$. The case of $a_{r/2}$, for r even, is adequately covered by the proof.

Incidentally, (*A1*.12) proves a *corollary* to our main theorem. Thus if we require to approximate $f(x)$ by a sum of sine- and cosine-functions of frequency $\nu<r/2$, in which some of the frequencies are omitted (for example, if $r=12$, and we require approximation by $a_0+a_3\cos 3x+b_3\sin 3x+b_5\sin 5x$), the formulae (*A1*.3) remain valid, and (*A1*.6) also, if only the coecffiients actually used are inserted (in our example, $s^2=\zeta^2-(a_3^2+b_3^2+b_5^2)/2$). This may be proved by putting the coefficients of ϕ^* equal to the negative coefficients (*A1*.3) of the terms omitted, and applying (*A1*.12).

The proof given here does not make use of differential calculus, at the same time furnishing the corollary mentioned.

A2. Fourier series for continuous function, and harmonic coefficients for equidistant ordinates: Theorem II—A continuous function $\mathrm{f}(x)$ between $x=0$ and 2π may be developed into an infinite Fourier series

$$(A2.1)\qquad \mathrm{f}(x) = A_0 + \sum_{\nu=1}^{\infty}(A_\nu\cos\nu x + B_\nu\sin\nu x)$$

implying that $\mathrm{f}(x)$ complies with the conditions necessary for this development. Furthermore, the r equidistant ordinates $\mathrm{f}(x_\rho)=y_\rho$ may, by (*A1*.3), be represented by the finite series $\phi(x)$ (*A1*.1), with harmonic coefficients a_ν, b_ν ($\nu<r/2$). Then

$$(A2.2)\qquad a_0 = A_0 + A_r + A_{2r} + A_{3r} + \ldots$$

$$(A2.3)\qquad \begin{cases} a_\nu = A_\nu + A_{r-\nu} + A_{r+\nu} + A_{2r-\nu} + A_{2r+\nu} + \ldots \\ b_\nu = B_\nu - B_{r-\nu} + B_{r+\nu} - B_{2r-\nu} + B_{2r+\nu} - \ldots \end{cases}$$

and, for r even,

$$(A2.4)\qquad a_{r/2} = A_{r/2} + A_{3r/2} + A_{5r/2} + \ldots$$

Proof—In order to avoid excessive use of indices, the general proof may be abstracted from the following example: Put $r=12$, $\nu=5$: then $x_1=30°$, $x_\rho=\rho 30°$, $\cos\ (r-\nu)x_\rho = \cos\ (12-5)\rho 30° = \cos\ (\rho 360° - \rho 150°) = \cos\ \rho 150° = \cos\ 5x_\rho = \cos\ \nu x_\rho$. Similarly, $\sin\ (r-\nu)x_\rho = -\sin\ \nu x_\rho$, $\sin\ (r+\nu)x_\rho = \sin\ \nu x_\rho$. Therefore, the finite series $\phi(x)$, with coefficients given by ($A2.2$) to ($A2.4$), has, for $x=x_\rho$, ordinates equal to those of $f(x)$.

Take as an example $r=3$ and $\nu=1$. Then $a_1=A_1+A_2+A_4+A_5+\ldots$ In the analysis of annual values, a wave A_2 of frequency 2 in 3 years, that is, of period 1.5 years, can be mistaken for a wave A_1 of frequency 1 in 3 years, that is, of period 3 years. The reason is obvious since $\cos x$ and $\cos 2x$ have, for $x=0°$, 120°, and 240°, the same numerical values, namely, 1, -0.5, and -0.5.

A3. Smoothing—From a continuous function $f(x)$ with the period 2π, that is, $f(x)=f(x+2\pi)$, a smoothed function $g(x)$ may be derived by ascribing to each abscissa x the average of $f(x)$ for the interval $(x-\beta)$ to $(x+\beta)$, that is, $g(x) = \int_{-\beta}^{+\beta} f(x+\xi)d\xi/2\beta$. Then the Fourier series of $g(x)$ is

$$(A3.1)\qquad g(x) = A_0 + \sum_{\nu=1}^{\infty} (A_\nu \cos \nu x + B_\nu \sin \nu x)(\sin \nu\beta/\nu\beta)$$

If we plot a harmonic dial (section 6) for the vectors of sine-waves of period $2\pi/\nu$, that is for frequency ν, this equation means that the vector for $g(x)$ has the same direction, or the same phase, as that for $f(x)$, but the amplitude in $g(x)$ is reduced in the ratio $\sin \nu\beta/\nu\beta$. This function has been plotted (with $\delta=\nu\beta$) in Figure 19*A*. Negative sign of $\sin \nu\beta/\nu\beta$ means here reversal of phase, for instance: Average of $f(x)= \sin x$ for $\beta=3\pi/2$, that is, when smoothed over intervals of length 3π, $g(x) = -(2/3\pi) \sin x$.

Proof—Integrate each term of $f(x)$ in ($A2.1$): for instance, the integral of $\cos \nu(x+\xi)$ over $\xi=-\beta$ to $+\beta$ is $(1/\nu)\ (\sin\ \nu\ (x+\beta) - \sin\ \nu\ (x-\beta)) = (1/\nu)\ 2 \sin \nu\beta \cos \nu x$, and division by 2β gives the average $(\sin \nu\beta/\nu\beta) \cos \nu x$.

Application—Hourly means in terrestrial magnetism (day $=360°$, $\beta=7.5°$), monthly means (year $=360°$, $\beta=15°$), etc. In practice, for instance, the hourly means are submitted to harmonic analysis as if they were equidistant values observed at the half-hours, and then the harmonic amplitudes are corrected to "instantaneous values" by multiplication with $\nu\beta/\sin \nu\beta$. This procedure neglects the possibility of higher frequencies in $f(x)$ than $r/2$, discussed in section *A2*: this is, however, generally not serious because, if r is not too small, the waves with frequencies above $r/2$ are very much reduced by smoothing.

A4. Non-cyclic correction—The values of the ordinates for $x=0$

and 2π may be y_0 and y_r. For r ordinates, our form of harmonic analysis (section 5 and section *A1*) considers only the ordinates y_1 to y_r, giving of course, $\phi(0)=y_r$ instead of y_0. If

$$(A4.1) \qquad y_r - y_0 = d \text{ (non-cyclic change)}$$

we can apply a non-cyclic correction by adding, before harmonic analysis, an appropriate linear function, namely, adding to y_ρ $(\rho=0, 1, \ldots, r)$ the value

$$(A4.2) \qquad (d/2)-(d\rho/r)$$

If we submit these corrections to harmonic analysis, entering them for y_ρ in (*A1*.3), we obtain harmonic coefficients which we may call $\triangle a_\nu$ and $\triangle b_\nu$. Actual calculation gives the value of $\triangle a_0 = -d/2r$. In order to obtain $\triangle a_\nu$ and $\triangle b_\nu$, we can, because of (*A1*.7), omit the constant part $(d/2)$ and consider only $(-d\rho/r)$. Inserting this value in (*A1*.3), we obtain for $\triangle b_\nu$, putting $\epsilon = 2\pi\nu/r$

$$\triangle b_\nu = (2/r) \sum_{\rho=1}^{r} (-d/r)\ \rho \sin \rho\epsilon$$

$$\triangle b_\nu\ (-r^2/d) \sin (\epsilon/2) = \sum_{\rho=1}^{r} 2\rho \sin \rho\epsilon \sin (\epsilon/2) =$$

$$\sum_{\rho=1}^{r} \{\rho \cos\ (\rho-1/2)\epsilon - \rho \cos (\rho+1/2)\epsilon\}$$

$$= \sum_{\rho=1}^{r} \{(\rho-1/2) \cos (\rho-1/2)\epsilon-(\rho+1/2) \cos (\rho+1/2)\epsilon+ [\cos (\rho-1/2)\epsilon]/2+[\cos (\rho+1/2)\epsilon]/2\}$$

$$= \sum_{\rho=1}^{r} \{(\rho-1/2) \cos (\rho-1/2)\ \epsilon-(\rho+1/2) \cos (\rho+1/2)\epsilon\} + \cos (\epsilon/2) \sum_{\rho=1}^{r} \cos \rho\epsilon$$

By this rearrangement (known as "partial summation"), we can find the sum. The last sum vanishes because of (*A1*.7), and if we write out the successive terms for $\rho=1, 2, \ldots, r$ in the first sum we see that the positive and negative parts cancel, and only two remain from the terms with $\rho=1$ and $\rho=r$, namely, $\cos (\epsilon/2)/2-(r+1/2) \cos (r+1/2)\epsilon$, or $-r \cos (\epsilon/2)$ [since $r\epsilon=2\pi\nu$, and therefore $\cos (r+1/2)\epsilon = \cos (\epsilon/2)$]. Therefore, $\triangle b_\nu = (d/r) \cot (\epsilon/2)$. $\triangle a_\nu$ can in the same way, by multiplying by $\sin (\epsilon/2)$, be found as $-d/r$.

The non-cyclic correction can therefore be applied in the following simple way: Compute a_0, a_ν, b_ν from the given ordinates y_1 to y_r according to (*A1*.3), find the non-cyclic change $d=y_r-y_0$, add to a_0, a_ν, b_ν the corrections

$$(A4.3) \qquad \triangle a_0 = -d/2r, \quad \triangle a_\nu = -d/r, \quad \triangle b_\nu = +(d/r) \cot (\pi\nu/r)$$

Then $(a_0+\triangle a_0)$ is the arithmetic mean $(y_0/2+y_1+y_2+\ldots+y_{r-1}+y_r\ 2)\ r$, and $(a_\nu+\triangle a_\nu)$ and $(b_\nu+\triangle b_\nu)$ are the harmonic coefficients of corrected ordinates obtained by adding to the given ordinates a linear function which makes the ordinates for $x=0$ and $x=2\pi$ equal.

The general formulae ($A4.3$) give, for $r=24$ and $r=12$, the corrections computed numerically by C. C. Ennis[37] (whose $C=y_0-y_r=-d$).

If the number r of the ordinates becomes infinite, the formulae ($A4.3$) become $\triangle a_0=\triangle a_\nu=0$, and, because $x/\sin x$ becomes 1 for $x=\pi\nu/r$ decreasing to 0, $\triangle b_\nu=d/\pi\nu$. This is of course nothing but the coefficient of the Fourier series for the continuous linear function $(d/2\pi)\ (\pi-x)$, into which the non-cyclic correction ($A4.2$) is transformed by $r=\infty$, namely

$$(A4.4) \qquad (d/2\pi)\ (\pi-x)=(d/\pi)\ \{\sin x/1+\sin 2x/2+\sin 3x/3+\ldots +\sin \nu x/\nu+\ldots\}$$

This function is, by the way, discontinuous at $x=0$ and 2π, changing suddenly by the amount d. Finite partial sums of ($A4.4$) up to frequency ν exhibit therefore, near the discontinuity, the systematic lack of approximation known as *Gibbs' phenomenon*.[55] This is of little importance in geophysical applications, except as a warning that abrupt changes in the given function $f(x)$ can only be represented by including sine-waves of high frequency in the approximating series $\phi(x)$.

A5. Harmonic analysis and correlation—The correlation-coefficients between the given ordinates y_ρ (or their deviations $z_\rho=(y_\rho-a_0)$ from their arithmetic mean a_0) and the ordinates of the cosine-wave $\cos \nu x_\rho$ or the sine-wave $\sin \nu x_\rho$ are, respectively, $a_\nu/(\zeta\sqrt{2})$ and $b_\nu/(\zeta\sqrt{2})$, where ζ is the standard deviation of the y_ρ or z_ρ. [Indeed, the numerator of the correlation-coefficient is $\Sigma z_\rho \cos \nu x_\rho$, or because of ($A1.7$) and ($A1.3$), $\Sigma y_\rho \cos \nu x_\rho=(r/2)a_\nu$, and the denominator is the square root of the product $\Sigma z_\rho^2\ (=r\zeta^2)$ times $\Sigma \cos^2 \nu x_\rho\ (=r/2$, because of ($A1.8$)).] Harmonic analysis can therefore be conceived as computation of correlation-coefficients.

Another relation to correlation can be seen in the formulae used in deriving ($A1.12$), because they can be interpreted for the calculation of the correlation-coefficient of two sets of ordinates $\phi(x_\rho)$ and $\phi^*(x_\rho)$ from the respective harmonic coefficients.

A6. The method of exhaustion—From ($A3.1$), it follows that the smoothed function $g(x)$ does not contain any periods p for which $\sin \nu\beta=0$, that is, $\nu\beta$ is an entire multiple $m\pi$ of π, or, since the length of the period $p=2\pi/\nu$, no periods of lengths $p=2\beta/m$, for which the smoothing interval 2β is an entire multiple mp; adjacent periods are weakened. If, therefore, we form the difference $d(x)=f(x)-g(x)$, it contains the sine-waves of these periods in full intensity, and adjacent periods in nearly full intensity. That is, in $d(x)$, the periods with longer periods than 2β are suppressed in favor of the shorter periods. This process and several similar processes like differentiation or integration, have been recommended therefore in order to help finding periodicities. However, though they may be useful for reconnaissance work and illustrative purposes,[57] they do not lend themselves readily to the application of the statistical tests for persistence, etc.

$d(x)$ may again be smoothed for a longer interval β_1, and this successive smoothing and difference-formation—a process which could be

[57] See the papers and diagrams given by M. et Mme. H. Labrouste, Ann. Inst. Phys. du Globe, Paris, 7, 190 ff. (1929); 9, 99-101 (1931); 11, 93-101 (1933), Soixante-sixième Congr. d. Sociétés Savantes, 468-471 (1933); C.-R. Assemblée Lisbonne 1933, Union Géod. Géophys. Internat., Ass. Mag. Electr. Terr. Bull. No. 9, 292-295 (1934).

called method of *exhaustion*—has often been used as a substitute for harmonic analysis, not only because of the apparent saving of computing-labor but also because it has been thought to be independent of sine-waves (section 40). The method leads, however, in a roundabout way, to practically the same results as harmonic analysis, only obscuring its statistical aspect. The criticism[58] directed by H. H. Turner against an analysis of the sunspot-numbers, made by H. Kimura using the exhaustion-method, applies to a number of other papers.

A7. Refined computation of a persistent wave—As soon as the length of the period of a persistent wave is definitely known, its amplitude and phase can be obtained with an accuracy determined by the effective expectancy (section 36) and the number N of periods contained in the interval T of observation. Since, in general, N cannot be enlarged at will, the only possibility of increasing the accuracy is to lower the effective expectancy, for instance, by selecting, out of the whole interval T, suitable partial intervals with relatively smaller unperiodic variations. This has been done successfully in the computation of the lunar semi-diurnal waves in atmospheric pressure[47, 50]; fortunately, the error-estimates in the former paper[50] are based on monthly averages of the diurnal variation and need therefore not be revised after the effect of quasi-persistence has been detected. Of course, the selection of "quiet intervals" opens pitfalls which must be recognized, for instance, the curvature-effect (section 16). Particularly erroneous would be an attempt to compute an average vector from the single vectors with smaller amplitudes alone, for instance, from those in Figure 2 falling within the probable-error circle, because that would certainly lead to a systematic underestimate of the amplitude. But it would probably be admissible to compute the lunar 12-hour wave in pressure only from those days which have a small 24-hour wave. S. Chapman has proposed a scheme for a systematic reduction of the expectancy,[59] aiming at a corresponding increase of the accuracy with which the average sine-wave is obtained.

A8. Persistent waves with periodically changing amplitudes—The following formula is easily proved by applying (*A1.*11)

$$(A8.1) \qquad (c+2k\cos\mu x)\sin\nu x = c\sin\nu x + k\sin(\nu+\mu)x + k\sin(\nu-\mu)x$$

A wave with periodically changing amplitude is therefore equivalent to the sum of three persistent waves of different frequencies. This formula is much used in tidal theory, and can easily be demonstrated in the harmonic dial for frequency ν (two vectors of amplitude k, revolving with frequency μ in opposite directions around the end-point of the amplitude c).

Example—Terrestrial-magnetic activity, u_1-measure, 6-monthly wave,[48] amplitude varying in 11-year sunspot-cycle. Time t, origin at the beginning of a sunspot-year, increasing by 2π during one year. Then the 6-monthly wave is expressed by

$$[6.5+2.6\cos(t/11)]\sin(2t+261°) = 6.5\sin(2t+261°)$$
$$+1.3\sin(2t+(t/11)+261°)+1.3\sin(2t-(t/11)+261°)$$

[58]London, Mon. Not. R. Astr. Soc., 73, 543-552 (1913).
[59]Zs. Geophysik, 6, 396-420 (1930).

The frequencies (per year) of these terms are 2, 23/11, and 21/11, with periods 6.00, 5.74, and 6.29 months, respectively. Ordinary periodogram-analysis of a series of many years would therefore yield, besides the main 6-monthly wave, two waves of about one-fifth of its amplitude, and periods of 5.74 and 6.29 months; but this result differs only in form, not in physical content, from the statement of a 6-monthly wave of constant phase but variable amplitude.

Other examples are given by the case of solar diurnal waves with seasonally changing amplitudes (for instance, atmospheric temperature); the frequency of the solar diurnal wave is 365 per year and the frequency of the change of amplitude is 1 per year, so that the additional terms in (*A8*.1) have the frequencies 366 and 364 per year. The former has the period of a sidereal day, and this purely formal result has often been mistaken as a proof for influences of stars, etc.

Summary

(*a*) Every discussion of the physical causes of periodicities in geophysical and cosmical phenomena must be preceded by statistical studies testing the significance and reliability of these periodicities. This statistical viewpoint in the application of harmonic analysis was introduced by A. Schuster. The present paper gives, on the basis of the theory of probability, a new aspect and an improvement of these methods, generally called periodogram-analysis, or investigation of hidden periodicities. The scope of these results is not restricted to sine-waves.

(*b*) Following an introductory review of literature, harmonic analysis is discussed as a mathematical representation of time-functions, using vector-representation of sine-waves in the harmonic dial and the folding process as a graphical illustration of harmonic analysis. The degree of approximation between the given function and the sum of sine-waves is determined by the standard deviation of the residuals. The ordinary periodogram is introduced, and Pollak's periodogram for the international magnetic character-figure C is discussed.

(*c*) The generalized harmonic dial is introduced in order to prepare the transition from sine-waves to periodicities of other form. The nature of the non-cyclic variation and the selection- or curvature-effect, which is often misinterpreted, are discussed.

(*d*) The statistical laws for the random walk are described and applied, in various forms, to the harmonic dial and the folding process, using the conception of the summation-dial, expectancy c, and probability of chance (κ test, $1/\sqrt{n}$ law). For random fluctuations, the expectancy does not depend on the length of the period (equipartition of the variance).

(*e*) For geophysical phenomena, the expectancy depends definitely on the length of the period. This fact, often overlooked, is of decisive influence on tests for the reality of persistent periodicities, as demonstrated in several cases.

(*f*) Quasi-persistence, exhibited in limited sequences of successive waves, is described as a common phenomenon in geophysics and is measured by an index σ, the equivalent length of sequences. It affects the κ-test for persistent waves in so far as not the ordinary expectancy c but the effective expectancy $c\sqrt{\sigma}$ must be used. Because of interference,

adjacent periods are infected by quasi-persistence. Intermittent quasi-persistence indicates sub-periods of longer periods. In comparison with random conditions, and with respect to tests for persistence as well as the uncertainty of average sine-waves derived from a large material, quasi-persistence acts like a reduction of the number N of available observations to the effective number N/σ.

(*g*) The methods for testing geophysical phenomena with respect to periodicities are generalized for waves of other form than sine-waves. Typical examples are given for the characteristic, a diagram demonstrating, for an assumed length of period, in which way this period is contained in the observational material.

(*h*) In the appendix, the formulae for harmonic analysis of equidistant ordinates are derived, including the effects of smoothing and a new general formula for non-cyclic correction. The relations to correlation, the methods of exhaustion, and persistent waves with periodically changing amplitudes are discussed.

Department of Terrestrial Magnetism,
Carnegie Institution of Washington.
Washington, D. C.

MAGNETIC SECULAR-VARIATION AT APIA, SAMOA, 1905-1929

By P. W. Glover

Abstract—Equations are derived to represent the series of annual means of the terrestrial-magnetic elements at Apia, Samoa, from 1905 to 1929. A definite improvement is effected in the representations by the addition to each equation of a term involving the sunspot-number, with the exception of the case of the vertical intensity. An attempt is made to eliminate the effect of the eleven-year solar periodicity by forming series of means of overlapping groups of eleven years. Equations are deduced to represent these series. The declination (O-C) residuals show a remarkably well-defined periodicity of seven years for which no explanation has yet been established.

The secular variation is also examined by considering the annual changes as due to a variable disturbing force. It is found that the inclination of the direction of this force is rotating slowly from zenithward to nadirward, and that its azimuth is slowly rotating in the direction south to north through east.

The data on which this investigation is based are the annual "all-days" mean values of the terrestrial-magnetic elements, declination (D), horizontal intensity (H), and vertical intensity (Z) determined at the Apia Observatory, Western Samoa, from 1905 to 1929. The values of H and Z used are those as published in the "Annual Report" of the Observatory; but in the case of D, it has been necessary to modify the published values by the application of a correction amounting to —3′.0 from 1905 to 1920. This was necessary since the values from 1905 to 1920 inclusive had not been referred to International Magnetic Standard, whereas those from 1921 to 1929 had been. If the data had been used as published, a sharp discontinuity would have been found between 1920 and 1921.

The sunspot-numbers used are the final relative sunspot-numbers published by Wolfer and later by Brunner in various numbers of this Journal.

Table 1—*Observed relative sunspot-numbers, S, 1905-1929*

Year	S	Year	S	Year	S	Year	S	Year	S
1905	63.5	1910	18.6	1915	47.4	1920	37.6	1925	44.3
1906	53.8	1911	5.7	1916	57.1	1921	26.1	1926	63.9
1907	62.0	1912	3.6	1917	103.9	1922	14.2	1927	69.0
1908	48.5	1913	1.4	1918	80.6	1923	5.8	1928	77.8
1909	43.9	1914	9.6	1919	63.6	1924	16.7	1929	65.0

The investigation falls into three parts: (1) the establishment of equations of the types

$$X = A + B(t-t_0) + C(t-t_0)^2 \quad \ldots\ldots\ldots \quad (A)$$

and

$$X = A + B(t-t_0) + C(t-t_0)^2 + DS \quad \ldots\ldots\ldots \quad (B)$$

to represent the observed series of annual means; (2) the establishment of equations of the type (A) to represent the series of annual means when smoothed by the formation of means of overlapping sets of eleven years; and (3) a discussion of the secular change by the method described by Chree in his "Studies in terrestrial magnetism."

Probable errors have in all cases been computed by means of the formula

$$p = \pm 0.675\sqrt{\Sigma\,(v^2)/(n-m)}$$

wherein p is the probable error, $\Sigma\,(v^2)$ is the sum of the squares of the

$(O-C)$ residuals, n is the number of equations of condition, and m is the number of unknowns.

PART I

Declination—The 25 equations of condition when treated by the method of least squares lead to the equation

$$D=9^\circ\ 33'.23+1'.813\ (t-1905.5)+0'.0323\ (t-1905.5)^2 \ldots\ldots (1)$$

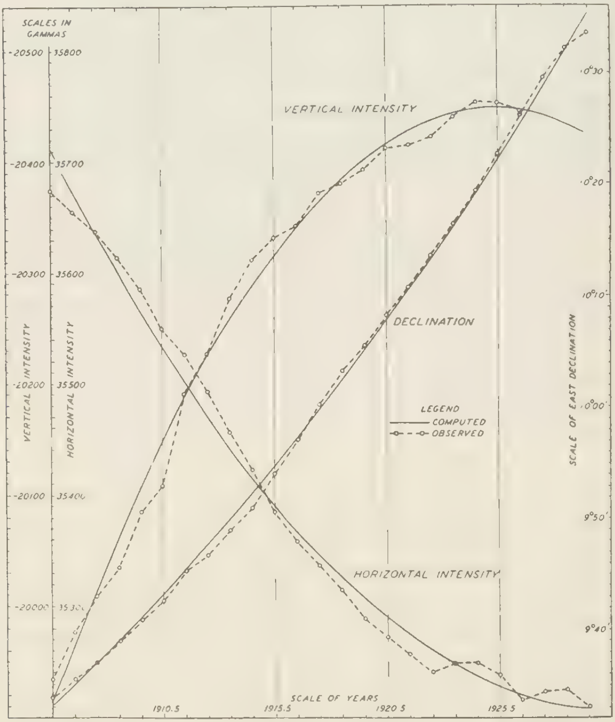


FIG. 1—SMOOTHED AND OBSERVED ANNUAL MEANS OF DECLINATION, HORIZONTAL INTENSITY, AND VERTICAL INTENSITY, APIA OBSERVATORY

where t is the year and 1905.5 is the initial epoch, easterly declination being reckoned positive. The curve of this equation is shown in Figure 1 together with the observational values. The $(O-C)$ residuals from this equation (see Table 4) possess a correlation-coefficient of $+0.45 \pm 0.11$ with the sunspot-numbers. As this coefficient is greater than three times its probable error, it is probably of some significance. If then, we compute the regression-equation, we obtain for the residuals

$$\triangle D= -0'.46+0'.0106S$$

which added to equation (1) gives as an improved representation of the observational series the equation

$$D=9^\circ\ 32'.77+1'.813\ (t-1905.5)+0'.0323\ (t-1905.5)^2+0'.0106S \quad (2)$$

Inspection of the residuals as shown in Table 4 indicates that this represents a slight improvement on the previous equation.

A direct least-square computation with the inclusion of a sunspot-term results in the equation

$$D=9°32'.61+1'.858(t-1905.5)+0'.0301(t-1905.5)^2+0'.0117S \quad \ldots \quad (2a)$$

The residuals from this equation are tabulated in Table 4 and shown graphically in Figure 3. It appears that D should continue to increase easterly for a considerable time.

Horizontal intensity—The 25 equations of condition give

$$H=35712^{\gamma}.9-40^{\gamma}.074\ (t-1905.5)+0^{\gamma}.7928\ (t-1905.5)^2 \ldots\ldots (3)$$

The correlation between the residuals (see Table 5 and Figure 1) and the sunspot-numbers is -0.35 ± 0.12. This is small and, while the probable error is large, is possibly of some significance. The regression-equation is

$$\triangle H=+8^{\gamma}.85-0^{\gamma}.2015S$$

which added to equation (3) gives

$$H=35721^{\gamma}.8-40^{\gamma}.074\ (t-1905.5)+0^{\gamma}.7928(t-1905.5)^2-0^{\gamma}.2015S \quad (4)$$

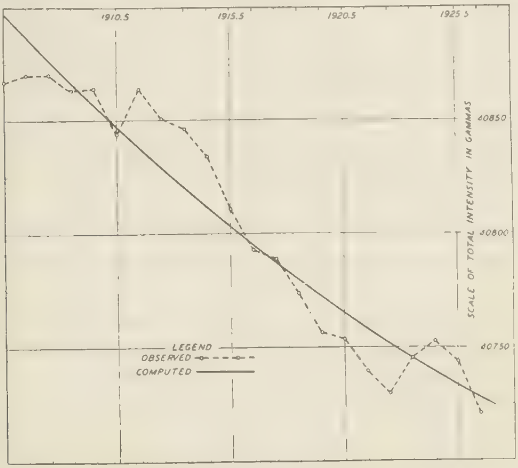


FIG. 2—SMOOTHED AND ACTUAL ANNUAL MEANS OF TOTAL INTENSITY, APIA OBSERVATORY

The residuals from equations (3) and (4) are tabulated in Table 4 and are plotted in Figure 3. It is seen that the inclusion of the sunspot-term introduces a slight improvement in the representation.

Figure 1 shows that the horizontal intensity is in the neighborhood of a minimum, and should soon show definite signs of increase.

Vertical intensity—Owing to the unsatisfactory performance of the Z-variometer, the data have been used only as far as 1926. The 22 equations of condition give the equation

$$Z=-19912^{\gamma}.7-53^{\gamma}.694\ (t-1905.5)+1^{\gamma}.3488\ (t-1905.5)^2 \qquad (5)$$

The curve is plotted in Figure 1, while the residuals are given in Table 4 and are plotted in Figure 3.

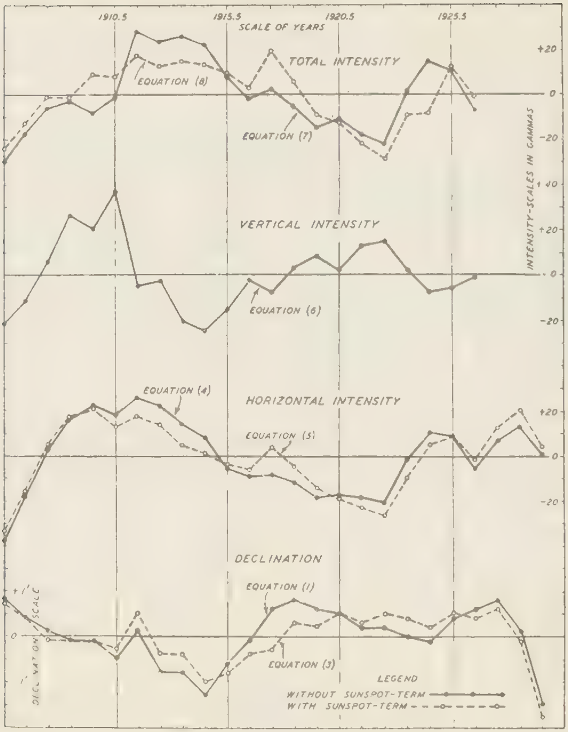


FIG. 3—RESIDUALS, APIA OBSERVATORY

No correlation was found between the residuals and the sunspot-numbers. Z appears to be passing through an algebraic minimum, and should exhibit a definite algebraic increase when satisfactory registration is again in progress.

Total intensity—The annual mean values of F have been computed from those of H and Z, 1905-26. The equation resulting from least-square treatment of the 22 equations of condition is

$$F=40897^{\gamma}.2+10^{\gamma}.707\ (t-1905.5)-0^{\gamma}.1267\ (t-1905.5)^2 \qquad (6)$$

The curve is plotted in Figure 2, and the residuals are given in Table 4. The correlation-coefficient between the residuals and the sunspot-numbers is -0.48 ± 0.11, leading to the regression-equation

$$\triangle F=11^{\gamma}.47-0^{\gamma}.2803S$$

which added to equation (6) gives for an improved representation

$$F=40908^{\gamma}.7+10^{\gamma}.707\ (t-1905.5)-0^{\gamma}.1267(t-1905.5)^2-0^{\gamma}.2803S \quad (7)$$

Reference to Table 4 shows that a slight improvement has been effected by the inclusion of the sunspot-term. The residuals are plotted in Figure 3.

Part II

From the foregoing, it is clear that there is a definite effect of the solar activity on the annual means of the Apia magnetic elements. By forming a series of means of overlapping sets of eleven years as follows, the effect of the eleven-year solar period may be eliminated. The mean for the eleven years 1905.5 to 1915.5 is taken as the mean for the middle date, that is, 1910.5; the mean for the period 1906.5 to 1916.5 is taken as the mean for 1911.5, and so on. This has been done for the elements D, H, Z, X, Y, and F and also for Bauer's[1] "local magnetic constant," $G = (H^2+Z^2/4)^{1/2}$ using the eleven-year means of H and Z. Equations have been deduced by the method of least squares to represent the new series so obtained. The eleven-year means, their values computed from the deduced equations, the residuals $(O-C)$ together with the sums of the squares of the residuals, and also the probable errors are given in Table 5. The residuals are shown graphically in Figure 4.

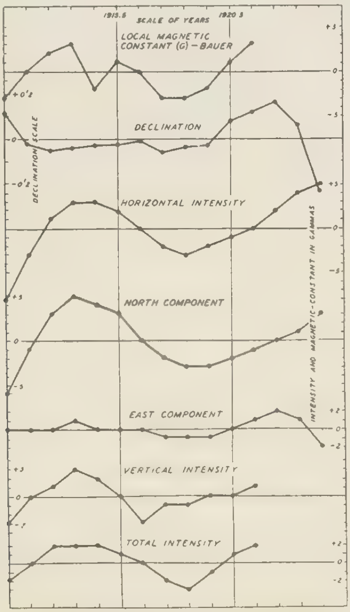


FIG. 4—RESIDUALS IN ANALYSIS OF 11-YEAR MEANS, APIA OBSERVATORY

The deduced equations are as follows

$$D=9° 43'.1+2'.188\ (t-1910.5)+0'.0322\ (t-1910.5)^2 \qquad (8)$$
$$H=35555^{\gamma}-35^{\gamma}.681\ (t-1910.5)+0^{\gamma}.9344\ (t-1910.5)^2 \qquad (9)$$
$$Z=-20134^{\gamma}-41^{\gamma}.40\ (t-1910.5)+1^{\gamma}.461\ (t-1910.5)^2 \qquad (10)$$
$$F=40855^{\gamma}-8^{\gamma}.79\ (t-1910.5)-0^{\gamma}.047\ (t-1910.5)^2 \qquad (11)$$
$$X=35042^{\gamma}-36^{\gamma}.618\ (t-1910.5)+0^{\gamma}.8582\ (t-1910.5)^2 \qquad (12)$$
$$Y=6001^{\gamma}+16^{\gamma}.347\ (t-1910.5)+0^{\gamma}.4665\ (t-1910.5)^2 \qquad (13)$$
$$G=36947-26.694\ (t-1910.5)+0.5402\ (t-1910.5)^2 \qquad (14)$$

[1] L. A. Bauer, Terr. Mag., 19, 113-125 (1914).

Part III

It is of some interest to consider the magnetic data in the manner described by Chree in his "Studies in terrestrial magnetism." For this, the annual mean values of X, Y, and Z, applicable to the middle of the year, are required and are shown in Table 2, those for X and Y not having been published before.

Table 2—*Annual mean values of magnetic elements X, Y and Z and respective yearly changes, Apia Magnetic Observatory, Western Samoa, 1905-29*

Year	X	ΔX	Y	ΔY	Z	ΔZ
	γ	γ	γ	γ	γ	γ
1905	35179		5929		−19935	
		−22		+12		−42
1906	35157		5941		−19977	
		−21		+13		−33
1907	35136		5954		−20010	
		−26		+15		−26
1908	35110		5969		−20036	
		−30		+16		−50
1909	35080		5985		−20086	
		−39		+11		−24
1910	35041		5996		−20110	
		−28		+25		−81
1911	35013		6021		−20191	
		−36		+ 7		−35
1912	34977		6028		−20226	
		−39		+17		−51
1913	34938		6045		−20277	
		−36		+14		−35
1914	34902		6059		−20312	
		−43		+25		−19
1915	34859		6084		−20331	
		−32		+26		−11
1916	34827		6110		−20342	
		−26		+28		−29
1917	34801		6138		−20371	
		−28		+26		− 9
1918	34773		6164		−20380	
		−30		+19		−12
1919	34743		6183		−20392	
		−21		+25		−21
1920	34722		6208		−20413	
		−20		+22		− 1
1921	34702		6230		−20414	
		−21		+27		− 7
1922	34681		6257		−20421	
		+ 2		+28		−19
1923	34683		6285		−20440	
		− 4		+30		−13
1924	34679		6315		−20453	
		−17		+34		0
1925	34662		6349		−20453	
		−29		+29		+ 7
1926	34633		6378		−20446	
		+ 1		+35		+14
1927	34634		6413		−20432	
		− 3		+27		(+ 7)
1928	34631		6440		.....	
		−18		+12		(+ 7)
1929	34613		6452		−20418	

Let $\triangle R$ be the disturbing force producing the secular change, and $\triangle r$ its component in the horizontal plane. Let α be the inclination of the vector $\triangle R$ to the Z-axis (positive towards the nadir) and β the inclination of the vector $\triangle r$ to the X-axis (positive to the north). Then

$$\triangle r = (\triangle X^2 + \triangle Y^2)^{1/2}, \quad \tan\ \beta = \triangle Y / \triangle X$$
$$\triangle R = (\triangle r^2 + \triangle Z^2)^{1/2}, \quad \tan\ \alpha = \triangle r / \triangle Z$$

Applying these formulae to the data in Table 2 we obtain the values given in Table 3.

TABLE 3—*Yearly values of Δr, ΔR, α, and β, Apia Magnetic Observatory, Western Samoa, 1905-29*

Year	Δr	β	ΔR	α	Year	Δr	β	ΔR	α
	γ	°	γ	°		γ	°	γ	°
1905	25	151	49	149	1917	38	137	39	103
1906	25	148	41	143	1918	36	148	38	108
1907	30	150	40	131	1919	33	130	39	122
1908	34	152	60	146	1920	30	132	30	92
1909	41	164	48	120	1921	34	128	35	102
1910	38	138	89	155	1922	28	86	34	124
1911	37	169	51	133	1923	30	98	33	113
1912	43	156	66	140	1924	38	117	38	90
1913	39	159	52	132	1925	41	135	42	80
1914	50	150	53	111	1926	35	88	38	68
1915	41	141	43	105	1927	27	96	(28)	(75)
1916	38	133	48	127	1928	22	146	(23)	(72)
Five-year means					1905-09	31	153	47	138
					1910-14	41	154	62	134
					1915-19	37	138	41	113
					1920-24	32	112	34	104
					1925-28 (4-year mean)	31	116	33	74

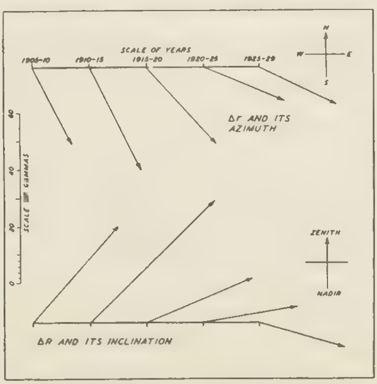


FIG. 5—FIVE-YEAR MEANS VECTORS Δr AND ITS AZIMUTH AND ΔR AND ITS INCLINATION, APIA OBSERVATORY

The five-year means are shown graphically in Figure 5 which clearly indicates the rotational tendency of the disturbing-force vector associated with the secular change.

TABLE 4—*Annual means of observed and adjusted values of*

Year	East declination							Horizontal	
	Obs'd	Comp. (Eq. 1)	$v=$ $(O\text{-}C)$	Comp. (Eq. 2)	$v=$ $(O\text{-}C)$	Comp. (Eq. 2*a*)	$v=$ $(O\text{-}C)$	Obs'd	Comp. (Eq. 3)
	° ′	° ′	′	° ′	′	° ′	′	γ	γ
1905	9 34.0	9 33.2	+0.8	9 33.4	+0.6	9 33.3	+0.7	35675	35713
1906	9 35.5	9 35.1	+0.4	9 35.2	+0.3	9 35.1	+0.4	35655	35673
1907	9 37.1	9 37.0	+0.1	9 37.2	−0.1	9 37.2	−0.1	35637	35636
1908	9 38.9	9 39.0	−0.1	9 39.1	−0.2	9 39.0	−0.1	35614	35598
1909	9 40.9	9 41.0	−0.1	9 41.0	−0.1	9 41.0	−0.1	35587	35565
1910	9 42.6	9 43.1	−0.5	9 42.8	−0.2	9 42.9	−0.3	35550	35532
1911	9 45.4	9 45.3	+0.1	9 44.9	+0.5	9 44.9	+0.5	35527	35501
1912	9 46.7	9 47.5	−0.8	9 47.1	−0.4	9 47.1	−0.4	35493	35471
1913	9 49.0	9 49.8	−0.8	9 49.3	−0.3	9 49.4	−0.4	35457	35443
1914	9 50.9	9 52.2	−1.3	9 51.8	−0.9	9 51.9	−1.0	35424	35416
1915	9 54.0	9 54.6	−0.6	9 54.6	−0.6	9 54.8	−0.8	35386	35391
1916	9 57.0	9 57.1	−0.1	9 57.2	−0.2	9 57.4	−0.4	35359	35368
1917	10 00.2	9 59.6	+0.6	10 00.2	0.0	10 00.5	−0.3	35338	35346
1918	10 03.1	10 02.3	+0.8	10 02.7	+0.4	10 02.8	+0.3	35315	35326
1919	10 05.5	10 04.9	+0.6	10 05.1	+0.4	10 05.3	+0.2	35289	35307
1920	10 08.2	10 07.7	+0.5	10 07.6	+0.6	10 07.7	+0.5	35273	35290
1921	10 10.7	10 10.5	+0.2	10 10.3	+0.4	10 10.4	+0.3	35257	35275
1922	10 13.6	10 13.4	+0.2	10 13.1	+0.5	10 13.1	+0.5	35241	35261
1923	10 16.3	10 16.3	0.0	10 15.9	+0.4	10 15.9	+0.4	35248	35249
1924	10 19.2	10 19.3	−0.1	10 19.0	+0.2	10 19.0	+0.2	35249	35238
1925	10 22.8	10 22.4	+0.4	10 22.4	+0.4	10 22.3	+0.5	35239	35229
1926	10 26.1	10 25.5	+0.6	10 25.7	+0.4	10 25.7	+0.4	35216	35221
1927	10 29.5	10 28.7	+0.8	10 29.0	+0.5	10 28.9	+0.6	35223	35215
1928	10 32.1	10 32.0	+0.1	10 32.4	−0.3	10 32.2	−0.1	35225	35211
1929	10 33.5	10 35.3	−1.8	10 35.5	−2.0	10 35.3	−1.8	35209	35208
$\Sigma\,(v)^2$	.. ....	.. ....	10.54	.. ....	8.21	.. ....	8.15	.....	.....
p	.. ....	.. ....	±0.47	.. ....	±0.42	.. ....	±0.42	.....	.....

TABLE 5—*Eleven-year means, magnetic elements and magnetic constant*

Year	East declination			Hor. intensity			Ver. intensity		
	Obs'd	Comp.	$v=$ $(O\text{-}C)$	Obs'd	Comp.	$v=$ $(O\text{-}C)$	Obs'd	Comp.	$v=$ $(O\text{-}C)$
	° ′	° ′	′	γ	γ	γ	γ	γ	γ
1910	9 43.2	9 43.1	+0.1	35546	35554	−8	−20136	−20133	−3
1911	9 45.3	9 45.3	0.0	35517	35520	−3	−20173	−20173	0
1912	9 47.5	9 47.6	−0.1	35488	35487	+1	−20209	−20210	+1
1913	9 49.9	9 49.9	0.0	35459	35456	+3	−20242	−20245	+3
1914	9 52.3	9 52.3	0.0	35430	35427	+3	−20274	−20276	+2
1915	9 54.8	9 54.8	0.0	35401	35399	+2	−20304	−20304	0
1916	9 57.3	9 57.3	0.0	35374	35374	0	−20332	−20329	−3
1917	9 59.9	10 00.0	−0.1	35348	35350	−2	−20353	−20352	−1
1918	10 02.6	10 02.6	0.0	35326	35329	−3	−20372	−20371	−1
1919	10 05.3	10 05.4	−0.1	35307	35309	−2	−20388	−20388	0
1920	10 08.2	10 08.2	0.0	35290	35291	−1	−20401	−20401	0
1921	10 11.2	10 11.0	+0.2	35275	35275	0	−20411	−20412	+1
1922	10 14.1	10 14.0	+0.1	35263	35261	+2	....	....	
1923	10 17.0	10 17.0	0.0	35252	35248	+4	....	....	
1924	10 19.8	10 20.0	−0.2	35243	35238	+5	....	....	
$\Sigma\,(v)^2$			0.13			159			35
p	.. ....	.. ....	±0.07	.....	.....	±2.5	....	....	±1.3

magnetic elements, Apia Magnetic Observatory, Western Samoa

intensity			Vertical intensity			Total intensity				
$v=$ $(O\text{-}C)$	Comp. (Eq. 4)	$v=$ $(O\text{-}C)$	Obs'd	Comp. (Eq. 5)	$v=$ $(O\text{-}C)$	Obs'd	Comp. (Eq. 6)	$v=$ $(O\text{-}C)$	Comp. (Eq. 7)	$v=$ $(O\text{-}C)$
γ	γ	γ	γ	γ	γ	γ	γ	γ	γ	γ
−38	35709	−34	−19935	−19913	−22	40867	40897	−30	40891	−24
−18	35671	−16	−19977	−19965	−12	40870	40887	−17	40883	−13
+ 1	35632	+ 5	−20010	−20015	+ 5	40870	40876	− 6	40870	0
+16	35597	+17	−20036	−20062	+26	40863	40866	− 3	40864	− 1
+22	35565	+22	−20086	−20106	+20	40864	40872	− 8	40855	+ 9
+18	35537	+13	−20110	−20147	+37	40844	40846	− 2	40836	+ 8
+26	35509	+18	−20191	−20186	− 5	40864	40836	+28	40846	+18
+22	35479	+14	−20226	−20223	− 3	40851	40827	+24	40838	+13
+14	35452	+ 5	−20277	−20256	−21	40846	40820	+26	40831	+15
+ 8	35423	+ 1	−20312	−20287	−25	40834	40811	+23	40820	+14
− 5	35390	− 4	−20331	−20315	−16	40811	40803	+ 8	40801	+10
− 9	35365	− 6	−20342	−20340	− 2	40793	40795	− 2	40790	+ 3
− 8	35334	+ 4	−20371	−20363	− 8	40789	40787	+ 2	40769	+20
−11	35319	− 4	−20380	−20383	+ 3	40774	40779	− 5	40768	+ 6
−18	35303	−14	−20392	−20400	+ 8	40757	40772	−15	40766	− 9
−17	35291	−18	−20413	−20415	+ 2	40754	40765	−11	40766	−12
−18	35279	−22	−20414	−20427	+13	40740	40758	−18	40762	−22
−20	35267	−26	−20421	−20436	+15	40730	40752	−22	40759	−29
− 1	35257	− 9	−20440	−20442	+ 2	40746	40745	+ 1	40755	− 9
+11	35243	+ 6	−20453	−20446	− 7	40753	40738	+15	40761	− 8
+10	35229	+10	−20453	−20447	− 6	40744	40733	+11	40732	+12
− 5	35217	− 1	−20446	−20445	− 1	40721	40728	− 7	40722	− 1
+ 8	35210	+13	....	....	....	.....	.....	....	.....	....
+14	35204	+21	....	....	....	.....	.....	....	.....	....
+ 1	35204	+ 5	....	....	....	.....	.....	....	.....	....
6389 ±11.5	.....	5542 ±11.0	....	....	5083 ±11.0	.....	.....	5614 ±11.6	.....	4190 ±10.3

observed and adjusted, Apia Magnetic Observatory, Western Samoa

Total intensity			North component			East component			Magnetic constant[a]		
Obs'd	Comp.	$v=$ $(O\text{-}C)$	Obs'd	Comp.	$v=$ $(O\text{-}C)$	Obs'd	Comp.	$v=$ $(O\text{-}C)$	Obs'd	Comp.	$v=$ $(O\text{-}C)$
γ	γ	γ	γ	γ	γ	γ	γ	γ	γ	γ	γ
40853	40855	−2	35036	35042	−6	6001	6001	0	36944	36947	−3
40846	40846	0	35003	35004	−1	6017	6017	0	36921	36921	0
40839	40837	+2	34971	34968	+3	6035	6035	0	36898	36896	+2
40830	40828	+2	34938	34933	+5	6055	6054	+1	36875	36872	+3
40821	40819	+2	34905	34901	+4	6074	6074	0	36852	36854	−2
40811	40810	+1	34873	34870	+3	6094	6094	0	36828	36827	+1
40801	40801	0	34841	34841	0	6116	6116	0	36806	36806	0
40789	40791	−2	34811	34813	−2	6137	6138	−1	36784	36787	−3
40779	40782	−3	34785	34788	−3	6160	6161	−1	36765	36768	−3
40771	40772	−1	34761	34764	−3	6185	6186	−1	36749	36751	−2
40763	40762	+1	34739	34741	−2	6211	6211	0	36735	36734	+1
40755	40753	+2	34719	34720	−1	6238	6237	+1	36722	36719	+3
.....	.....	...	34702	34702	0	6266	6264	+2	.....	.....	...
.....	.....	...	34686	34685	+1	6293	6292	+1	.....	.....	
.....	.....	...	34672	34669	+3	6319	6321	−2	.....	.....	
.....	.....	36	.....	.....	133	....	....	14	.....	.....	59
.....	.....	±1.4	.....	.....	±2.2	....	....	±0.7	.....	.....	±1.7

[a]Mean value of magnetic constant, 1905-26, 36823γ.

Between the values of $\triangle R$ and α there is a correlation-coefficient of 0.63 ± 0.09, so that to a considerable extent, these quantities vary together. No significant correlation was established between $\triangle R$ and $\triangle S$, nor yet between α and $\triangle S$. In view of the results obtained in Parts I and II, this is a little surprising, and seems to indicate that the magnitude of the solar effect is less than should have been expected from the preceding parts of the investigation. Between $\triangle r$ and β the correlation-coefficient is 0.30 ± 0.12, which is small; in view of the ratio of the coefficient to its probable error, it is doubtful whether it is of much significance. Furthermore, no definite correlation could be established between the values of $\triangle r$ and $\triangle S$, nor between β and $\triangle S$.

Considering all the results obtained, it would appear that there is a small but definite influence of the sunspottedness on the secular change of the terrestrial-magnetic elements as determined at Apia from 1905 to 1929; certainly the equations to represent the change in the elements are definitely improved by the addition of a term involving the relative sunspot-number.

In conclusion, the author desires to record his thanks to H. F. Skey of the Christchurch Magnetic Observatory, to C. J. Westland, lately of the Apia Observatory, and to Dr. C. Coleridge Farr, Professor of Physics at the Canterbury College, University of New Zealand, with whom he has discussed various points arising during this investigation.

Christchurch, New Zealand,
November 15, 1934

EXPERIMENTAL INTERPRETATION OF MAGNETIC AND GRAVIMETRIC ANOMALIES

By W. P. Jenny

Abstract—A method and apparatus are described, which allow experimental interpretations of magnetic and gravimetric anomalies. The basic difference between this method and other experimental methods so far applied lies in the use of the direction in space of the magnetic lines of force as due to local structures, instead of the magnetic intensity, for interpretative purposes. This method gives due consideration to the inhomogeneous magnetization of structure and to the relation between the magnetization of a geologic structure and its direction of strike and geographical position. Based upon the relationship between magnetic forces and the second derivatives of the gravitational potential as given by Eoetvoes, the direction of the magnetic lines of force as due to local structures may be calculated from the data obtained with torsion-balances. This procedure makes it possible to also interpret gravimetric anomalies by the experimental method under discussion. As an example for the practical application of this method the large magnetic anomaly at Grand Rapids, Michigan, has been interpreted in terms of structure.

Introduction

There are three main reasons, which explain the considerable additional difficulties encountered in the interpretation of magnetic anomalies as compared with the interpretation of gravimetric anomalies.

In the first place the specific gravities of the rocks may be determined with a much higher degree of accuracy than their magnetic susceptibilities; second, the assumption of homogeneous magnetization applies only to bodies limited by surfaces of the second degree; third, the induced magnetic field is so highly dependent upon the angle between the strike of the magnetically active structure and the direction of the inducing field that the same structure, all according to its strike or geographical location, may yield entirely different magnetic anomalies. In our latitudes, for example, the south end of a ridge with higher than average magnetic permeability will show up as a positive, the north end as a negative magnetic anomaly; a dike, dipping towards the north, may show up as a strong positive anomaly, whereas this same dike, dipping towards the south, will be noticeable magnetically through a sequence of one high and one low.

But since, in contradistinction to the gravitational field, the magnetic field may be surveyed by relatively simple means in all three dimensions, the above-mentioned drawbacks are compensated to a great extent. This circumstance has not been taken into due consideration thus far.

If we deduct from the measured absolute values of the vertical and horizontal intensity and from the declination, the respective values of a "normal" field, we may combine the differences between measured and "normal" vertical intensities, south-north intensities and declinations (that is, west-east horizontal intensities) into a magnetic vector in space.[1] Size and direction of this vector are dependent upon continental, regional or local anomalies, all according to the definition of the "normal" field.

We may easily deduce from the formulae for the calculation of the magnetic effect of homogeneous masses that the size of the magnetic

[1]W. P. Jenny, Magnetic vector study of regional and local structures in principal Oil States, Bull. Amer. Ass. Petrol. Geol., **16**, 1177-1203 (1932).

vector in space is proportional to the susceptibility of a given mass, whereas the direction of the vector is dependent upon the shape of such given mass.[2, 3, 4]

This deliberation leads to results of considerable importance for the geologic interpretation of magnetic anomalies, because it is possible to determine by relatively simple experiments the relationship between the directions of the magnetic vectors in space and the shape of a magnetically active structure.

Regional magnetic anomalies will in many instances be the result of the tectonics of the Basement complex. Let us assume the Basement complex as an extended magnetic plate of large thickness and let us cut up this plate into a series of vertical magnetic columns or bar-magnets. If we displace these bar-magnets along the vertical, we may imitate tectonic dislocations of the Basement complex, such as anticlines, synclines, faults, grabens, etc., and examine the influence of such tectonic features upon the directions in space of the respective vectors. In reality the problem will mostly be just reversed, that is, the directions of the magnetic vectors will be determined through measurements in the field and we are to deduce the tectonics of the underground from the directions of these vectors.

Method and apparatus for vertical direction of inducing field

Figure 1 represents a cross-section through an apparatus allowing a qualitative and quantitative interpretation of magnetic anomalies, if the direction of the inducing Earth's magnetic field is assumed to be vertical. Vertical bar-magnets (1) may be displaced along the vertical by means of ribbons (2), rollers (3), and counterweights (4). First, all of the magnets are set at the same depth, so that the positions of the negative poles correspond to the average depth of the basement. The Earth's magnetic field may be neglected relative to the field induced by the bar-magnets, so that the dip-needles (5), (6), (7), all point about perpendicularly downward. The dip-needles are suspended by means of thin threads (8), which are held by stands (9) on a transparent plate (10).

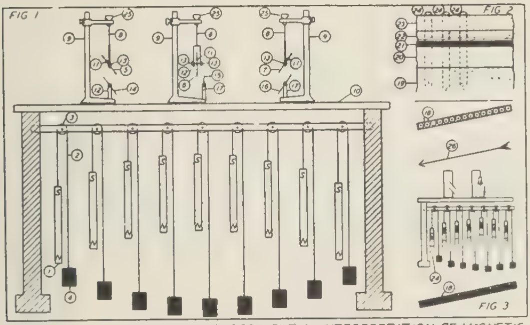
The field induced by the bar-magnets at their normal depth is to be compensated by counterweights (11) on the negative sides of the dip-needles and by the torsions of cross-threads (12) by means of the screws (13). Let us now displace the bar-magnets so that the sequence of their negative poles corresponds to the surface of a tectonically disturbed Basement complex, for example to an anticline as shown in Figure 1, then the different poles will influence the dip-needles all according to their respective directions and distances, so that the needles assume the directions indicated in the figure.

We may on the other hand, measure the directions in space of the local magnetic vectors at the locations of the dip-needles, and indicate these directions of the vectors by the respective indices (14), (15), (16), which are mounted on the stands. Upon compensation of the normal

[2] H. Haalk, Die magnetischen Methoden der angewandten Geophysik, Handbuch der Experimentalphysik, 25, 3. Teil, Angew., Geophysik, Leipzig, 1930.

[3] J. Bartels, Erdmagnetische Aufschlussverfahren, Lehrbuch der Geophysik, Lieferung 3, Berlin, 1926.

[4] A. Nippoldt, Verwertung magnetischer Messungen zur Mutung, Berlin, 1930.



FIGS 1,2,3—APPARATUS FOR EXPERIMENTAL INTERPRETATION OF MAGNETIC AND GRAVIMETRIC ANOMALIES

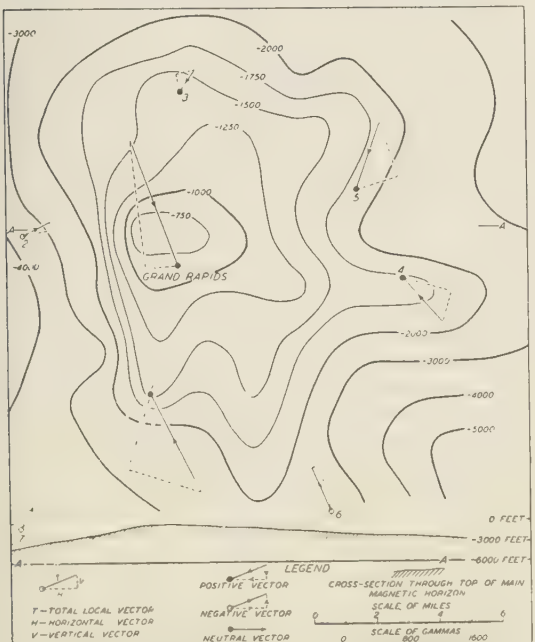


FIG 4—EXPERIMENTAL INTERPRETATION OF LOCAL MAGNETIC VECTORS AT GRAND RAPIDS, MICHIGAN

field by counterweights (11), we shall displace the bar-magnets, until the directions of the dip-needles agree with the directions of the respective indices. The depths of the individual poles may now be read through the transparent plate (10) directly off the scales printed on the ribbons (2).

By setting a number of such cross-sections side by side, we may connect the depth-readings by contour-lines, which will yield directly the depth and shape of the geologic structure.

Method and apparatus for optional direction of inducing field

In practice the interpretation of magnetic anomalies is considerably complicated, first because the direction of the Earth's magnetic field differs from the vertical, as has been assumed for the inducing field of Figure 1, according to geographic latitude, and second because there usually exists above the Basement complex a series of layers with varying magnetic susceptibilities.

We may create an artificial magnetic field approaching the actual conditions by placing the apparatus within an electrically induced magnetic field with its axis parallel to the Earth's magnetic field. The determination of the absolute magnetic susceptibilities of the different layers above the Basement complex offers considerable difficulty, but it is relatively easy to determine their relative susceptibilities. Since we confine ourselves to the determination of the directions of the magnetic vectors, we need only a knowledge of the relative susceptibilities, which circumstance offers great advantage over other methods for undertaking to geologically interpret magnetic anomalies on the basis of relative vertical- or horizontal-intensity measurements.

In the cross-section of Figure 2 through a series of layers, (19) represents the Basement complex with a magnetization 1, (20) represents a non-magnetic layer, (21) and (22) represent layers with magnetizations ½ and ¼ respectively, (23) represents a non-magnetic layer. Let us mix a plastic material, plastilin for example, with corresponding amounts of small iron particles, roll the material into layers of respective thicknesses and arrange them as shown in the cross-section. Tubes (24) are now pushed through this series of layers in such a way that they contain cores of the series after again being pulled out. Replacing the bar-magnets (1) in Figure 1 by these tubes, as shown in Figure 3, we may imitate the various geologic structures by displacing the tubes along the vertical. The magnetic effect of such structures along the Earth's surface (10) may then be studied by means of the dip-needles.

The induced magnetism of the Earth's magnetic field will as a rule not yield sufficient deflections of the dip-needles, so that the Earth's field must be intensified by a homogeneous artificial field. The homogeneous artificial field will, however, also influence the dip-needles and this "normal" influence must be eliminated at an average depth of the tubes. The "normal" horizontal components are compensated by twisting the suspended threads of the dip-needles by means of the screws (25) until the needles assume the positions corresponding to the respective magnetic vectors, which have been surveyed in the field and which are indicated by the indices (14), (15), (16); hereupon the "normal"

vertical components are compensated by counterweights (11) and torsions of the cross-threads (12), until the directions of the needles correspond to the directions of the respective measured vectors.

By displacing the tubes a combination of them has to be found so that the directions of the needles remains unchanged.

If, on the other hand, the magnetic influence of a given structure is to be determined, we first compensate only the "normal" vertical components, then adjust the tubes according to the structures and turn the indices so as to agree with the horizontal deflection of the needles. Upon replacing the tubes to their normal positions, the needles will assume their starting positions. By means of the screws (25) we twist the needles, so as to agree with the directions of the indices (14), (15), (16). Upon restoration of the structure, the needles will again be deflected. The new directions of the needles will again be marked by the indices, the tubes replaced to their normal positions, the needles twisted to their new directions, and so forth, until the restoration of the structure does no more bring about a horizontal deflection of the needles.

Hereupon the respective inclinations must be adjusted by a similar procedure.

In Figure 3 the tubes are arranged so as to form an anticline; the coil for the induction of the artificial field is indicated by two cross-sections (18) and the direction of the homogeneous inducing field by an arrow (26).

Once the geologic conditions of the underground are cleared up by the above-described apparatus, the results may easily be checked and their accuracy increased by models, which preserve the continuity of the layers; the sequence of layers (Fig. 2) may be cut into segments with but one millimeter interspace, and these segments moved up or down in the place of the tubes. The accuracy of the results obtained is in direct proportion to the density of the poles.

Method for the experimental interpretation of vertical magnetic anomalies

This apparatus also allows the measuring of horizontal and vertical intensities, as due to structures, relative to the strength of the homogeneous inducing field. To this end the torsion-coefficient of the cross-thread (12) is determined, the dip-needle displaced from the direction of the local vector by turning the screw (13) and the displacement observed, first with the structure, second without the structure. The relative horizontal and vertical intensities as due to structures may then be figured through mathematical analysis of the two observations.

Such determinations are of special interest for the interpretation of magnetic anomalies, for which only the vertical component has been measured and for the interpretation of gravimetric anomalies, in so far as the influence of a relatively thin bed, corresponding to the sequence of the upper poles, is looked for.

Method for the experimental interpretation of gravimetric anomalies

For the interpretation of optional gravimetric anomalies, we shall have to recur to the formulae deduced by Eoetvoes[4] for the relation

[4]R. v. Eoetvoes, Bestimmung der Gradienten der Schwerkraft und ihrer Niveauflaechen mit Hilfe der Drehwage, XV Allgemeine Konferenz der Intern. Erdmessung, Budapest, 1906.

between gravimetric and magnetic anomalies on the basis of the theorem of Poisson. The final formulae are

$$\beta X - \alpha Y = \frac{1}{g(\sigma - \sigma_0)} \left\{ -\alpha\beta\, U_\Delta + (\beta^2 - \alpha^2)\, Uxy + \beta\gamma\, Uxz - \alpha\gamma\, Uyz \right\}$$

$$2\gamma X - \alpha Z = \frac{1}{g(\sigma - \sigma_0)} \left\{ -\alpha\gamma\, U_\Delta + 2\beta\gamma\, Uxy + (\alpha^2 + 2\gamma^2)\, Uxz + \alpha\beta Uyz \right\}$$

where X, Y, Z are the magnetic forces in the respective directions of a Cartesian coordinate system, Uxz, Uyz, U_Δ, and Uxy the second derivatives of the gravitational potential as determined by the torsion-balances, α, β, γ the direction-cosines for the induced magnetization, multipled by its intensity I, g the gravitational constant, σ the density of a body embedded in material with a density σ_0.

If the direction of the artificially induced magnetization forms an angle of 45° with the three axes of the coordinate system, then $\alpha = \beta = \gamma = 0.7071\ I$, and the formulae assume the following form

$$X - Y = \frac{0.7071}{g(\sigma - \sigma_0)}\, I \left\{ -U_\Delta + Uxz - Uyz \right\}$$

$$2X + Z = \frac{0.7071}{g(\sigma - \sigma_0)}\, I \left\{ -U_\Delta + 2Uxy + 3Uxz + Uyz \right\}$$

These formulae hold true only for a homogeneous magnetization of the body, which causes the gravitational anomaly. For the sake of an experimental interpretation of gravimetric anomalies it is by no means necessary that the body causing the gravitational anomaly is in reality also magnetized, but it should be of such a shape that the magnetization would be uniform, if it were magnetized.

Though it is impossible to determine the three unknowns X, Y, Z from the two equations, it is possible to determine the direction of the magnetic local vector by assuming a value for Z. The solution of the two equations will then furnish the values for X and Y in relation to the assumed Z, in other words the direction of the local vector, but not its intensity. Since we are in need, for the experimental interpretation of magnetic anomalies by means of the before-described apparatus, of the directions of the local vectors only, this method therefore allows the experimental interpretation of torsion-balance data.

The basically new principle embodied in the method and apparatus described above consists in the use of the directions of the local magnetic vectors for the interpretation of structures. This method offers the only possibility to experimentally interpret gravitational anomalies and further enables us to interpret regional and local magnetic anomalies, with due consideration to the relation between the magnetization of a geologic structure and its direction of strike and geographical position, and also with due consideration to the inhomogeneous magnetization, which cannot be analyzed mathematically. The method has mainly been devised for the solution of problems encountered in structural geology, though it may prove of value also to many mining problems. In case the geological conditions cannot be approached by vertical displacements of segments, these may be replaced by plastic models.

Application of experimental interpretation-method

As an example for the practical application of the experimental interpretation-method we have chosen the large magnetic anomaly at Grand Rapids, Michigan.

A large uplift is indicated there by a magnetic anomaly of more than 1500 gammas.[5] The United States Coast and Geodetic Survey has made observations of the magnetic elements at seven auxiliary stations near Grand Rapids. In order to obtain the directions in space and the density of the magnetic lines of force as due to the local structure, we have deducted the "normal" values of the Earth's magnetic field from the absolute measurements at the United States Coast and Geodetic Survey's stations, of the declinations and of the vertical and horizontal magnetic intensities.

The differences between the absolute and "normal" vertical and horizontal intensities have been combined into a vector triangle, indicating the direction and intensity of the local magnetic force. The vector-triangles have been plotted on the map at their respective stations, by turning the triangles through 90° around the horizontal component (which is the dashed line starting at the station-point) into the plane of the map. A black station-point means that the magnetic force is directed towards the station; a hollow station-point means that the magnetic force is directed away from the station, as indicated by the arrows in the legend.

In Figure 4 we notice the local magnetic vectors at Grand Rapids and auxiliary stations. These vectors all point toward a positive anomaly of more than 1500 gammas with its center slightly to the east of Grand Rapids. According to other experiences in this State it seems safe to interpret this anomaly as a large domal uplift.

We have experimentally interpreted this magnetic anomaly in terms of structure under the following assumptions: The main magnetically active horizon is the Cambrian or pre-Cambrian at a normal depth of about 6000 feet. The direction of the inducing Earth's magnetic field is vertical.

By using an apparatus very similar to the one shown in Figure 1, we have adjusted the vertical bar-magnets in such a way that the directions of the magnetic needles set up at the eight stations agreed with the respective directions in space of the local vectors. The procedure of this experiment is fully explained above, under the heading "Method and apparatus for vertical direction of inducing field."

After the needles had been removed, the depths to the upper poles of the bar-magnets could be read through the glass plate from the scale printed upon the ribbons.

The contour-lines shown in Figure 4 represent the depths to the poles or to the top of the magnetically active horizon. We notice that a large, gentle uplift of the Cambrian or pre-Cambrian from a normal depth of about 6000 feet to about 800 feet would account for the magnetic anomaly. Since the primary assumption as to the normal depth of the main magnetic horizon and as to the vertical direction of the Earth's

[5]W. P. Jenny, Magnetic-vector study of Kentucky and Southern Michigan, Bull. Amer. Petrol. Geol., **18**, 97-105 (1934).

magnetic field may be considered as good approximations to actual conditions, the structure outlined in Figure 4 should represent a fair approach to the actual structure.

We may conclude from the close agreement of the vectors that the United States Coast and Geodetic Survey's data are fairly reliable, with the possible exception of auxiliary station 3, where the intensity of the vector appears to be too small, if the direction of the vector is correct. The writer has, however, not had the opportunity to recheck the field-data.

Houston, Texas

THE ELECTRICITY OF RAIN AND THUNDERSTORMS

By Ross Gunn

Abstract—Electricity of rain: It is shown that the observed properties of rain-drops of very different histories are readily described if each water-droplet and the surrounding ionized water-vapor is considered an electrical concentration-cell. The potential difference between the drop and points several radii outside approximate 0.06 volt. The equilibrium-charge on each droplet is proportional to the radius and is positive if the droplet is evaporating and negative if it is condensing. These droplets grow by association into large rain-drops and thus build up large charges of the order of 1/50 e. s. u. which discharge by conduction in a time of the order of 10^3 seconds. The drops thereafter take on an equilibrium-charge depending upon the evaporation or condensation of the drop and its size. The calculated charges are in agreement with observation. A laboratory experiment verifying certain aspects of the predicted effects is described.

A theory of thunderstorm-electricity: It is shown that in the presence of a rapidly rising current of air and the formation of rain-drops greater than a critical size, separation of charge takes place. At levels of condensation the atmospheric ions (usually positive) are swept to great heights while the newly-formed rain (usually negative initially) falls. Both charge-distributions discharge by conduction at different rates, leaving the charge in the zone of rain-formation in excess (usually negative). The excess charge induces charges on the Earth or on the high conducting atmosphere of a magnitude equal to a large fraction of the excess charge. Rain-drops formed by the association of droplets below the saturation-level impart a normally positive space-charge to the region. The charge-distribution resulting from the separation is quantitatively calculated from physical data and agreement obtained with observation in regard to the potential differences, electrical field, electric moment, charging current, recovery-time, and the average charge on the precipitated rain.

Maintenance of the Earth's charge: Some objections to Wilson's thunderstorm-theory of the maintenance are noted. It is shown that special thunderstorm-conditions in mountainous regions are such that a total storm-area of 0.01 per cent of the Earth's surface would precipitate sufficient negative charge on rain and transfer enough positive charge upward by convection to maintain the field. The possible importance of forced vertical convection over natural barriers in "fair-weather" regions is considered and it is shown that such a systematic convection may account for an appreciable part of the necessary replenishing current.

Distribution of free space-charge: The factors determining the free space-charge are considered and the resulting distribution of charge, electric field-intensity, and potential are found.

Detailed studies of atmospheric electricity have served primarily to emphasize the relative complexity of the processes which produce the observed electrical distribution within a thunderstorm and over the Earth. Several investigators have proposed theories to account for certain observed electrical properties of a thunderstorm. The most notable of these, perhaps, include that of Simpson[1], which attributes the developed electrical charges to breaking rain-drops, and the not very definitely formulated one of C. T. R. Wilson[2] which attributes the large free-charges on the rain-drops to an unusual and peculiar type of ion-capture which requires the presence of an initial electric field of a particular sign that is difficult to account for. It is not the purpose of this paper to criticize the existing theories of thunderstorms but rather to bring together new evidence and ideas of a fundamental nature which, sys-

[1]Proc. R. Soc., **114**, 376-401 (1927).

[2]J. Frank. Inst., **208**, 1-12 (1929); Phil. Trans. R. Soc., **221**, 73-115 (1921).

tematically interpreted, permit one to calculate from well-founded data, unrelated to direct thunderstorm-observation, the observed electrical properties and magnitudes of a typical thunderstorm. The study represents a frontal attack upon an intricate problem and one cannot hope to describe in detail all the variations possible in nature, but the typical storm selected for study does reproduce with reasonable completeness the outstanding electrical features of most observed thunderstorms.

FACTORS INFLUFNCING THE DISTRIBUTION OF FREE-CHARGE

The distribution of free-charge in the Earth's atmosphere determines completely, through Poisson's equation, the resulting electrical state and hence it is of the greatest importance to determine what free-charges can exist in the atmosphere, where they are located, and how long they will persist. Free charges in the Earth's atmosphere result from (*a*) electrical separating forces produced by currents traversing the non-homogeneous conducting atmosphere, (*b*) mechanical separation of charge by the accelerating forces acting on rain, dust, or snow.

It is convenient to discuss first the space-charge arising from the conduction-current which observation shows to be flowing toward the Earth under fair-weather conditions. Following an earlier paper[3], it is assumed that the fundamental independent variable is the electrical conductivity of the atmosphere. This increases with altitude and in order that, under equilibrium-conditions, the current-density be maintained constant and independent of altitude (thus satisfying the equation of continuity), it is necessary that the electric field also be non-uniform. This, in turn, implies a space-charge given very nearly by

$$\mathrm{d}E/\mathrm{d}z = 4\pi\rho_1 \tag{1}$$

where E is the electric field-intensity, ρ_1 is the space-charge density, and z is the altitude.

The conduction current-density i is given by

$$i = \sigma E \tag{2}$$

where σ is the electrical conductivity. We are here considering steady-state conditions so that (1) becomes by aid of (2)

$$4\pi\rho_1 = -(i/\sigma^2)\ (\mathrm{d}\sigma/\mathrm{d}z) \tag{3}$$

where it is evident that the free-charge ρ_1, is maintained so long as i has finite values and hence this is the free-charge most important under fair-weather conditions.

Consider next an enclosed unit-volume immersed in a conducting region and containing a free-charge ρ_2. Then noting that the current-density across the boundary of the region is the product of the conductivity σ and the electric field E, we may write, using Gauss' theorem, that

$$-(\mathrm{d}\rho_2/\mathrm{d}t) = \sigma\int\int E\mathrm{d}S = 4\pi\sigma\rho_2 \tag{4}$$

[3] R. Gunn, Terr. Mag., 38, 303-308 (1933).

whence upon integration it is found quite generally that

$$\rho_2 = \rho_{02}[\exp(-4\pi\sigma t)] \tag{5}$$

The initial free-charge density ρ_{02} is therefore reduced to $1/\epsilon$ times its value in a time (relaxation-time) $\tau = 1/4\pi\sigma$. Thus, in the absence of special mechanism, like a current supported by external means, any free-charge left to itself will be shortly neutralized by conduction-processes.

Table 1 gives the electrical relaxation-time for a number of altitudes using conductivities given by Benndorf[4]. It is considered significant that at moderate altitudes the relaxation-velocities (defined rather arbitrarily as the ratio of the altitude to the relaxation-time at that altitude) are considerably less than the vertical wind-velocities often observed. Thus it is quite possible to transfer free-charge to moderate altitudes by convection even in the absence of special means to maintain the free-charge.

TABLE 1

Altitude	Conductivity, σ	Relaxation-period	Relaxation-velocity
cm		*sec*	*cm/sec*
0	2×10^{-4}	398	.....
5×10^{3}	4.4×10^{-4}	180	27
1×10^{5}	5.7×10^{-4}	139	714
3×10^{5}	9.6×10^{-4}	82	3660
6×10^{5}	20×10^{-4}	40	15000
9×10^{5}	45×10^{-4}	17	52900

Consider again a distribution of free-charge density such that its value at some reference-level is ρ_{03}. Suppose that the mass of air containing the free ions responsible for ρ_{03} is suddenly transferred to a different level. What will be the magnitude of the new density of free-charge, assuming that the molecular density n at any level z above the reference-level is given by the well-known relation

$$n = n_0[\exp(-mgz/kT)] \tag{6}$$

where $mg/kT = \alpha$ is assigned its observed value $\alpha = 1.4\times10^{-6}$. In this expression m is the mean mass of the atmospheric molecule, g is the acceleration due to gravity, k is the Boltzmann constant, and T is the temperature of the isothermal atmosphere. Since the transition from the original state is considered instantaneous, it is clear that the total free-charge is constant and that the ratio of the number of free ions to the number of molecules is also constant. Thus the density of free-charge decreases with increasing altitude precisely as the molecular density decreases or

$$\rho_3 = \rho_{03}[\exp(-\alpha z)] \tag{7}$$

Such a distribution of ρ_3 is evidently the equilibrium-distribution for a rapidly mixed atmosphere in which discharge by conduction is very small.

[4]Handbuch der Experimentalphysik, 25, I, 262 (1928).

If the charge-density at a selected level is maintained at a value ρ_{03} by any means whatever, then (7) gives the approximate free-charge density at any other level, provided only, that the air-circulation is rapid. In general, however, even the rising currents of air encountered in thunderstorms do not circulate rapidly enough to satisfy the conditions imposed in deriving (7) and it becomes necessary to allow for the loss of free-charge by conduction. This can be done approximately by combining the variation expressed by (5) with that of (7). The free-charge density, ρ_4, at any level may therefore be represented by

$$\rho_4 = \rho_{00} [\exp(-\alpha z - 4\pi\sigma_1 t)] \tag{8}$$

where ρ_{00} is the maintained free-charge density at the reference-level due to mechanical separation-effects and σ_1 is an average value of σ taken for convenience to be a constant.

Consider now a current of air rising with a velocity whose average value is a constant v_1, then t may be eliminated and (8) written

$$\rho_5 = \rho_{00} [\exp\{[-\alpha - (4\pi\sigma_1/v_1)] z\}] \tag{9}$$

If one imposes the rather severe condition that the final value of the free-charge density delivered to any given level z shall be at least $1/\epsilon$ times that which would be delivered were there no dissipation of charge by conduction, it follows that

$$v_1 \geq 4\pi\sigma_1 z \tag{10}$$

whence substituting $\sigma_1 = 5.7 \times 10^{-4}$ and $z = 1$ km, it is found that the mean required vertical velocity in the first kilometer must approximate 714 cm/sec or about 7 meters per second.

It is therefore concluded that free-charge generated near the surface of the Earth and quite unsupported by auxiliary means can be transported in important quantities to quite moderate elevations provided only that the vertical air-velocity approximates some meters per second.

The space-charge in the Earth's atmosphere may therefore be considered to be made up of two components. The first component arises from the observed world-wide conduction or fair-weather current and the non-homogeneous conductivity of the Earth's atmosphere. The second component of the space-charge may be considered to be due to the mechanical separation of charge and this is distributed throughout the atmosphere by air-motions. The resulting space-charge density ρ may accordingly be represented by

$$\rho = -\frac{i}{4\pi\sigma^2}\,\frac{d\sigma}{dz} + \rho_{00} [\exp(-\alpha z - 4\pi\sigma_1 t)] \tag{11}$$

In case the entire atmospheric region is expanding upward at a uniform velocity v_1, this may be put in a more useful but very approximate form or

$$\rho = -\frac{i}{4\pi\sigma^2}\,\frac{d\sigma}{dz} + \rho_{00} \{\exp[(-\alpha - \frac{4\pi\sigma_1}{v_1})z]\} \tag{12}$$

from which we may calculate by the usual methods that the electric field-intensity

$$E_z = 4\pi \int_0^z \rho dz = \frac{i}{\sigma} - \frac{4\pi\rho_{00}}{(\alpha + 4\pi\sigma_1/v_1)} \{\exp[(-\alpha - \frac{4\pi\sigma_1}{v_1})z]\} \quad (13)$$

where we have imposed the condition that $E \to 0$ as $z \to \infty$. Further, the potential of any level with respect to the Earth is

$$\phi_z = -\int_0^z E dz = -i\int_0^z \frac{dz}{\sigma} - \frac{4\pi\rho_{00}}{(\alpha + 4\pi\sigma_1/v_1)^2} \{\exp[(-\alpha - \frac{4\pi\sigma_1}{v_1})z] - 1\} \quad (14)$$

Numerical calculations employing the foregoing relations show that potential differences and electric fields set up in the Earth's atmosphere will depend almost completely upon the fair-weather conduction-current density *unless* meteorological conditions are developed which produce *simultaneously* large values of ρ_{00} and v_1. It will be shown presently that the required unusual conditions are produced in a thunderstorm.

THE ELECTRICITY OF RAIN

The successive changes of state of a rain-drop and the importance of these changes in influencing the electrical properties of rain have been ignored by earlier investigators. In a thunderstorm the humid air is first cooled until the associated water-vapor condenses into rain, and subsequently is re-evaporated in part or completely before the rain-drop reaches the Earth. The problem is therefore one of dynamic equilibrium rather than a static one as heretofore assumed. The accompanying changes of state will be shown to play a very important rôle in describing the observed electrical conditions within a thunderstorm.

Some preliminary and not very well-controlled experiments were devised and carried out as a result of certain theoretical considerations to determine the electrical effect of condensing and evaporating water. A single-fiber electrometer was set up inside a metal vessel and arranged so that it could be easily observed. A small "pill-box" of thin sheet-tin was arranged so that an iron disc about 2 inches in diameter could be easily introduced into it. This entire assembly was connected to the central system of the electrometer. Water was present in the large vessel containing the metal pill-box and electrometer so that the vapor-pressure within was appreciable. The iron disc was reduced to the temperature of carbon-dioxide snow and quickly introduced inside the pill-box. Polarizing potentials were then applied to the electrometer and as the metal parts cooled and water-vapor condensed on the outside of the pill-box it took on a negative potential of about 0.1 volt. Then as the pill-box warmed up, condensation ceased and the charge on the box gradually became positive as the water already condensed was evaporated and finally the central system was left with a positive charge that gradually leaked off as a result of ordinary electrical conduction. Some typical plots of the data obtained are shown in Figure 1. Attention is called to the fact that the charge- and discharge-rates are apparently quite large when the evaporating or condensing rates are large, whereas the discharge-

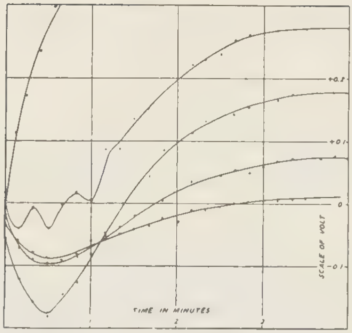


FIG 1—POTENTIALS DUE TO CONDENSATION AND EVAPORATION

rate without either is the normal small discharge-rate for any charged system.

Now the preliminary experiments just described, the observed electrical charge on fog-particles[5], the observed electrical charges on rain-drops of quite different histories[8] and certain properties of breaking rain-drops may all be described quantitatively if it is assumed *that each water-droplet and the moisture-ladened atmospheric ions surrounding it constitute an electric concentration-cell.*

It is well known that many ions exist in the atmosphere and that water-vapor has a marked tendency to associate with both the positive and negative ions. These ionized water-particles are in dynamic equilibrium with the adjacent free-surface of the rain and if the vapor-pressure adjacent to the free-surface is different than that some distance away, then there will be a general migration of the carriers toward the region of least concentration. Since the equilibrium is a dynamic one, the faster moving and more mobile ions will determine the electrical state. Moreover, as is well known, the mobility of the fastest moving negative ions is greater than the positive ones and therefore as a result of the dynamic equilibrium *regions of low concentration will necessarily become negatively charged with respect to the regions of higher concentration. This is the fundamental process of a concentration-cell.* More complete discussion of these matters, as related to electrolytes at least, may be found in any good advanced textbook on Physical Chemistry[6].

Consider any two rain-drops in the Earth's atmosphere and suppose that one of these drops evaporates while the other condenses. The difference of potential between the drops is made up of three components; two of these appear at the water ionized-water-vapor interface while the third is due to the difference in the mean concentrations of the ions adjacent to the two drops[6]. The drops are both of pure water, whence on rigorous theoretical grounds we have[6] that the potential difference between the two drops is

$$V = \frac{2u_-}{u_- + u_+} \frac{kT}{e} \log_e(c_2/c_1) \tag{15}$$

[5]A. Wigand and E. Frankenberger, Phys. Zs., 31, 204-215 (1930).

[6]For example see Getman, Outlines of theoretical chemistry, 552 (1928).

where c_2 and c_1 are the ionized water-vapor concentrations just outside the surface of each drop, k is the Boltzmann constant, T is the absolute temperature, e the ionic charge, and u_- and u_+ are the mobilities of the faster moving negative and positive ions, respectively. Now it is convenient to refer to the potential of a drop with respect to the space between the drops and it is clear that the potential of the drop is given by (15) if c_2 and c_1 are the concentrations adjacent to the drop and at a point many radii away from the drop, respectively.

The preliminary experimental data suggest that the potential difference between the evaporating and condensing water-surfaces are of the order of 0.1 volt so that by aid of (15) it is found that the ratio of the concentrations must have approximated 100. Therefore the ratio of the concentrations at the surface of a rain-drop and at a point many radii away is presumed to approximate 10 in typical cases.

Employing electrostatic units throughout unless otherwise noted, taking $u_- = 1.1\ u_+$ for moist air[7] and $\log (c_2\ c_1) = 1$ it is found from (15) that

$$V = 6.92 \times 10^{-7} T \log_{10} (c_2/c_1) \text{ e.s.u.} \tag{16}$$

or

$$V = 2.08 \times 10^{-4} T \log_{10} (c_2/c_1) = 0.063 \text{ volt (roughly)} \tag{16a}$$

Upon applying this relation to a typical rain-drop it may be seen that the charge on the drop will depend on what the drop is doing. If the drop is evaporating the concentration just outside its surface obviously must be greater than it is out several radii away and $c_2 > c_1$ so that the drop takes on a positive charge of an amount

$$Q_e = CV = 6.92 \times 10^{-7}\ a\ T \log_{10} (c_2/c_1) \tag{17}$$

where C is the electrostatic capacity and is approximately equal to the radius a of the rain-drop. On the other hand, it is clear that a drop which is condensing will have the lowest concentration at its surface or $c_2 < c_1$ and it will acquire a negative charge. Thus *rain-drops or snow-flakes in electrical equilibrium carry positive or negative electrical charges roughly proportional to their radii according as the drop or flake is evaporating, or condensing.* Under normally existing conditions the equilibrium-charge in elementary electronic units is of the order of $3 \times 10^5 a$, where a is the radius of the drop in cm. This agrees well in magnitude with the charges on fog-sized particles actually measured by Wigand and Frankenberger[5], who found that in most cases the charge was positive (corresponding to evaporation) but in some instances was negative, the latter indicating that the fog was still forming. It is well to point out here that except under special conditions the charges on the rain-drops or fog-drops are exactly neutralized by the presence of ions of opposite sign in the air surrounding each droplet.

The foregoing discussion is somewhat limited because it considers only electrical-equilibrium states and ignores such important factors

[7] J. Zeleny, Phil. Trans. R. Soc., A, 195, 193-234 (1900).

as the union of several small drops to make a large one or the breaking up of a drop. These processes result in extraneous and non-equilibrium charges being placed on the drop which will subsequently discharge by conduction in a time of the order of the electrical relaxation-time of the region and will ultimately reach the electrical equilibrium-value. We now consider the more general electrical effects of rain-formation.

In a later paragraph a critical sized water-droplet will be introduced as the elementary droplet of which all larger drops are composed. Let the mass and radius of this elementary droplet be n and r, respectively, further let M be the mass of a large rain-drop formed from M/n elementary drops. Then if the large drop is formed in a time short compared to the relaxation-time, all the charges on the small drops will unite into a single total charge Q_t on the large drop of magnitude

$$Q_t = 6.92 \times 10^{-7} \ (M/n) \ rT \log_{10} \ (c^1{}_2/c^1{}_1) \tag{18}$$

where $c^1{}_2$ and $c^1{}_1$ refer to the associated concentrations of the elementary droplet. Normally this free-charge is far greater than the equilibrium-value and as soon as the drop is formed the charge starts to leak off by electrical conduction. From earlier sections it is clear that if there is no further association or disruption, the charge on the drop after a time t will be

$$Q_t = 6.92 \times 10^{-7} (M/n) \ rT \ log_{10} (c^1{}_2/c^1{}_1) \ [\exp \ (-t/\tau)] + Q_f \tag{19}$$

where Q_f is the final equilibrium-charge of the drop and as before $\tau = 1/4\pi\sigma$. Except the ions be swept away this drop and all others will be surrounded with an equal and opposite neutralizing charge in the immediately adjacent atmosphere.

The charge on a rain-drop depends rather critically on where and under what conditions it was formed. The newly-formed droplets are always formed by condensation and therefore initially carry negative equilibrium-charges. The rain-formation or condensation-zone is of moderate thickness and perhaps most of the droplets fall through it as they coalesce to form larger drops. Such drops will carry large negative charges because they were condensing during association. Certain other newly-formed droplets formed perhaps near the lower surface of the condensation-zone will fall into air of different characteristics and begin to evaporate. These droplets will take on positive charges and as they fall towards the Earth, they will coalesce with similar droplets to form a positively charged rain-drop. It thus appears that rain falling quietly some distance to Earth may carry down electrical charges of both kinds and of magnitude intermediate between that given by (17) and by (19). It must also be noted that a drop falling from a condensation-zone usually must fall farther and for a longer time than one falling from a region of evaporation. Thus at the Earth's surface the negative rain will be more nearly discharged than will the positive rain formed at a lower level. This effect and the evaporation both tend to make falling rain predominantly positive.

It seems further possible that the ordinary rain-drops may associate and build up superdrops which are known to break up as a result of the

air-blast which they encounter. The magnitude of the changes on such drops cannot be foretold without a complete and detailed history of each drop, and this, of course, is not known. However, unless the free-charge on a drop dissipates very rapidly as a result of rapid evaporation or condensation somewhat as our preliminary experiments suggest, it appears that the process of break-up and reformation will not, on the average, greatly influence the resulting space-charge. A more detailed study is desirable but must be postponed.

Special meteorological conditions favor the production of certain types of rain-drops. For example, if the droplets are condensed near the surface of the Earth and if they coalesce to form a mist, they will usually be precipitated while still condensing and therefore should carry down a large negative charge. This is in agreement with observation.

Light rain formed under quiet conditions at a considerable altitude and precipitated into regions of evaporation would, as we have seen, probably carry down moderate charges of both signs but with an excess of positive charges, because evaporation predominates in the lower zones. Further, in agreement with the findings of Gschwend[8], the smallest drops which fall the slowest and hence spend the longest time in the evaporating region are predominantly positive.

The rain-drops formed within the thundercloud usually condense and coalesce in the rain-formation zone at a considerable altitude and therefore usually carry an initially large negative charge. As the drop falls toward the Earth, however, it loses part of its charge by conduction and usually begins to evaporate, thus tending to take on a moderate positive charge. Such drops as are formed from droplets that coalesce in the evaporating regions may bring down large positive charges. Moreover, we have just noted that since the condensing zones are above the regions of evaporation, the positive drops formed in the latter zone will have less time to discharge than will the negative drops from above. Thus even in a thunderstorm the average charge precipitated may be positive. It is also apparent that rising currents of air will slow up the drops and thus accentuate the differences between the final positive and negative charges, and tend to make the falling rain more positive. Conversely, downcoming winds near the back of the storm will sweep the negative charges down from condensation-regions in a short time and they will have insufficient time to discharge. Under these circumstances the negative charge on each drop will be greater than normal and the rain on the whole can deposit a negative charge.

The Lenard effect or electrification of water-drops by disruption[9] appears to have an important relation to the thesis that rain-drops are concentration-cells. Lenard is of the opinion that the phenomenon is due to an electrical double layer *inside* the drops but from the available experimental evidence it appears that our new interpretation is also allowable. A water-drop in the lower atmosphere or in the laboratory is usually evaporating and hence the drops will appear to have a positive charge in accordance with Lenard's experimental findings. A crucial test can be applied when experimental details are worked out; for according to the present study the rain-drops would be negatively charged

[8] Jahrb. Radioakt. u. Elektronik, **17**, 62, 192 (1921).

[9] Ph. Lenard, Ann. Physik, **47**, 463-524 (1915); **65**, 629-639 (1921).

if they were broken up a reasonable time after they were formed and while still condensing. Lenard's hypothesis apparently suggests no such change in sign.

The amount of charge resulting from the disruption of a rain-drop in electrical equilibrium is readily calculated from the developed relations. Consider a rain-drop of mass M and therefore of radius $(3M/4\pi d)^{1/3}$ where d is the density of the drop. Its equilibrium charge will be

$$Q_e = (6.92 \times 10^{-7})\ (3M/4\pi d)^{1/3} T \log\ (c_2/c_1) \qquad (20)$$

and it will be surrounded by a space-charge of $-Q$. Let the initial drop be broken up into N equal-sized drops. The new total charge on all drops *for equilibrium* will evidently be

$$Q_n = (6.92 \times 10^{-7})\ N\ (3M/4\pi N d)^{1/3} T \log(c_2/c_1) \qquad (21)$$

so that if the loosely bound ions outside the drops are swept away by some means (such as a blast of air) the ratio of the initial and final charges will approximate

$$Q_n/Q_0 = N^{2/3} \qquad (22)$$

Thus a considerable free-charge may be temporarily generated whose apparent charge on the water is positive if evaporating or negative if condensing. The calculations apply only to the equilibrium-charges. If a drop carries such a large charge that the charges on the divided droplets greatly exceed their equilibrium-values and if break-up occurs the charge will obviously be divided somehow between the resulting droplets without an appreciable change in the total charge. The effect of a superposed charge in modifying the charge generated by break-up should be investigated in the laboratory.

If the above analysis of breaking rain-drops is essentially correct, then one might conclude that usually the rain in Simpson's thunderstorm which is breaking up is *negatively* charged and not positive as he has supposed.[1] This will materially improve the agreement of his theory with observation. However, the analysis suggests that the break-up of an already highly charged drop will not contribute much to the total charge and therefore, since Simpson's theory requires successive break-ups of the rain-drops before they can reasonably be expected to discharge, it appears that the mechanism cannot play an important part in thunderstorm-electricity. There are other serious objections to the theory[9a]. In the following we have accordingly ignored the part that breaking rain-drops might play in a thunderstorm, although, it would have somewhat assisted in describing special properties of a storm.

The foregoing analysis is useful in predicting the sign and approximate magnitude of the electrical charges associated with actual precipitation. The analysis predicts (*a*) that rain-drops or droplets over 1000 seconds old (say) will carry a small equilibrium-charge which will be positive if the drop is evaporating and negative if condensing; (*b*) that newly formed large rain-drops may carry a charge of the order of

[9a] A. von Hippel, Erdfeld, Gewitter, und Blitz, Naturw., 22, 701-712 (1934).

1/50 e.s.u. which results from the association of equlibrium-charges of many small droplets; (*c*) that the sign of the charges on the large drop is initially the same as on the droplets forming them at the time of their association; (*d*) that rain or mist formed and condensing very near the surface of the Earth will usually be negatively charged because of its youth; (*e*) that snow falling into a region of such temperature and humidity that water-vapor condenses on the flakes, will usually be negatively charged but if the flakes are evaporating the charge will usually be positive; (*f*) that rain-drops falling in the front of a heat thunderstorm will encounter rising currents of warm humid air thus slowing up the rate of fall and producing evaporation of the rain-forming droplets (both factors conspire to communicate a positive charge to the rain); (*g*) that rain-drops blown rapidly downward from the condensation-zones by winds at the back of a storm will, because of their youth, often still possess much of their originally large and usually negative charge.

We have calculated the equilibrium and total charges on the various types of rain-drops from (17) and (18) assuming that $\log c_2/c_1 = 1$. These values are tabulated in Table 2, columns 6, 7, 8, and 9. The resulting charge-density on the rain formed by association is given in column 10 and the rain-convected precipitation current-density when the drops are all of one sign is given in the last column.

In the foregoing paragraphs the mechanisms of liquid concentration-cells have been carried over to the case of gaseous cells without appreciable change because of the scarcity of experimental data. Further work is desirable to substantiate the assumption made. Perhaps the absorption of ions on the surface of the drop due to chemical affinity or contact-potentials may play a part in the electrification of rain and it is not impossible that evaporation or condensation would determine the sign of the charge on the drop. Irrespective of the fundamental mechanism by which the elementary drops become charged, the process of building up a large charge on a rain-drop by association of many small drops will still take place. Thus the processes and mechanisms considered in this paper may still be applied to the thunderstorm-problems after making appropriate adjustment of the numerical magnitudes. The gaseous concentration-cell has been used as a guide throughout because of its definite nature.

A THEORY OF THUNDERSTORMS

Thunderstorms develop only under unstable atmospheric conditions which induce strong vertical convections of humid air. The vertical convection may be brought about in various ways but it is usual to classify storms as "heat-thunderstorms" and "squall-thunderstorms". The heat thunderstorm is essentially the only type of storm occurring in the tropics and is produced through heating of the air near the surface by intense insolation. The squall-type of storm induces convection by the coming together of large masses of air of radically different temperature and humidity.

The height and velocity of penetration of humid air into the higher levels naturally depend somewhat on the latitude. Tropical storms in general have their regions of rain-condensation and great electrical activity at comparatively great heights, while the same type of storm

in temperate latitudes develops its active centers at such low altitudes that discharges to Earth are relatively common.

It is convenient to discuss an idealized storm having the essential characteristics of all types and we have therefore illustrated in Figure 2 a type of storm common in temperate latitudes. The storm is supposed

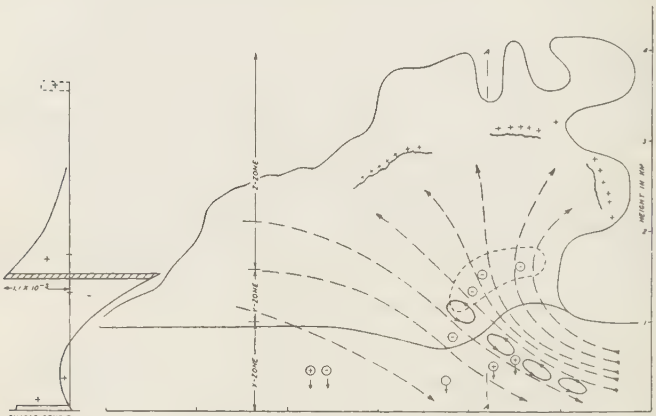


FIG. 2—CROSS-SECTION OF TYPICAL HEAT THUNDERSTORM

to be moving toward the right. Warm moist air moves in from the right at low levels and is deflected upward along the dashed stream-lines as shown. Cooler air moving with the storm is deflected downward somewhat as shown. It is characteristic of such a storm that a nearly adiabatic or superadiabatic temperature-gradient is established throughout the active region and the rising warm humid air very quickly reaches a saturation-level where the moisture starts to condense. Nothing of especial electrical interest happens at first because the initial charges on the fog-sized water-droplets are always accompanied by a neutralizing charge just outside, and these together with the fog-droplets are swept upward with identical velocities. However, as the temperature drops, more and more moisture condenses out and these fog-sized droplets unite to form larger ones. (We need not consider the exact mechanism at this time because we know that it *does rain.*) Now just as soon as the growing droplets attain such a size that they can move with a sufficient velocity with respect to the upward moving air, the ions which heretofore were always associated with the droplets but were outside it in a disperse sheath-like distribution are carried away by the upward moving air. It will be shown quantitatively in another section that separation of electrical charge by this means can actually be effected and that the separation will produce important electrical effects.

For convenience in discussion the storm is divided into three different zones. The X-region extends from the Earth's surface to the saturation-level, the Y-region extends from the saturation-level to roughly the middle of the rain-formation zone, and the Z-region extends roughly from the

middle of the rain-formation zone outward. We shall now examine the different regions separately. First, consider the phenomena in the rain-formation regions and above (the Z-region). Storms duplicating the effects in this region undoubtedly occur in high mountains where the rising current of air sweeps up the mountainside and rain-condensation takes place at all levels right down to the ground. This type of storm is quite simple. The vertical motion of the humid air brings about the condensation of great numbers of droplets. When these reach a critical size they start to fall and move with appreciable velocity with respect to the updraft of air. The droplets are condensing and therefore carry a negative charge specified by (17). They associate with similar droplets to form a full-sized rain-drop which carries a negative charge specified by (19). At first each droplet is surrounded by a sheath of positive ions which just neutralize the charge of the drop proper. But, as will be shown, when the rain-drop reaches a certain size it falls so fast with respect to the ion-ladened air that the positive ions can no longer follow the drop and they remain where formed or are swept to higher levels by the updraft of air. Thus a separation of charge is produced. The negatively charged rain-drops fall toward the Earth and produce a volume-distribution of negative charge. Now as the positively charged air rises and the negatively charged rain falls, both discharge by conduction, usually at a different rate, in such a manner that after a time τ each unit-volume (say) has been reduced to approximately $1/\epsilon$ times its initial charge. It is clear that the free-charge density is readily calculated if the charge on each drop, the number of drops per unit-volume, and the conductivity are known. Moreover, at the place where separation of charge is accomplished the free-charge density of the positive ions must be the same as that on the rain. Thus, since the free-charge density specified the electrical state completely in the absence of induced surface-charges, it is clear that the observed physical quantities can be calculated from independent data. The calculations are reserved for a later section.

The condensing mountain-type storm is beautifully simple in its electrical aspects and is thought to describe well the phenomena of an ordinary storm in regions well above the condensation-level. If we now permit the condensing level to be a kilometer or so above the Earth as it usually is in flat or rolling country and assume (properly) that on the average a droplet or drop in this lower region (the X-region) is evaporating, then the problem becomes complicated.

The effects in the condensing zone and above are nearly identical with those of a mountain-type storm except that the negative rain falls not on the Earth but into the regions of evaporation. The new-born droplets which are produced by condensation are always negatively charged. These droplets fall slowly and before they coalesce with other droplets, they may often encounter conditions which are favorable to evaporation rather than condensation. If the droplets coalesce in regions of condensation, just as one usually would expect, the final charge on the large drops will be negative. However, the droplets fall into regions of higher temperature and it appears that large numbers of the droplets, particularly those formed near the lower surface of the condensing region, will be evaporating at the time they coalesce to form the rain-drops. Rain formed under such conditions will be positively charged. Therefore, in the evaporating or X-regions the raindrops are

of two types—those that coalesced above in the Y- and Z-regions and those that were formed within the zone by droplets falling into it from above. The former usually carries a negative and the latter usually a positive charge.

Now as the positive rain is formed in the X-region, negative ions surrounding each drop are swept upward into the condensing (Y) zone and this augments the negative charge already present. Some of the negative ions may even be swept to levels above the negative rain partially neutralizing the positive space-charge there. However, as the negative charges move upward they constantly discharge and the negative space-charge carried into the normally positive Z-region is probably quite small, certainly much smaller than the positive space-charge generated locally that has not had time to discharge. Conversely the negative rain that falls into the X-region is partially discharged while the positive rain formed within the region has substantially its initial charge. These two effects cooperate to impart a positive space-charge to the lower part of the evaporation-zone. Further, in this same region, but not too close to the condensation-zone, one would normally expect to find rain-drops carrying both kinds of charges but with an excess of positive space-charge. The exact situation is seen to depend somewhat on the relative number of droplets actually uniting to form raindrops in the Y- and Z- and in the X-regions.

In another section it will be shown that the difference in discharge-rates between the positive charge high above the rain-formation zone and the negative charge within it, is considerable. After the cloud has been charging for some time the negative charge on the condensing region notably exceeds the positive charge elsewhere. This excess negative free-charge induces positive charges on the Earth and on the high atmosphere; the relative amounts induced, of course, depend somewhat upon geometrical considerations.

The air-currents in the front of a typical thunderstorm are usually upward and a rain-drop falling to Earth from this region will be both aged and evaporating. Thus a typical drop would be expected to carry a positive charge. On the other hand a drop near the back of the active part of the storm may be moderately close to the Earth and still be in a condensing region. Moreover the winds here are usually downward and this tends to bring the drop to Earth in its youth. Therefore, one might expect negatively charged rain to reach the Earth most often in this region. The inference has been verified in more instances than could be attributed to chance, by comparing the occurrence of negative precipitated rain with the presence of considerable atmospheric turbulence or with irregular and sudden increases in the barometric pressure. The latter is presumed to result from downward air-gusts. The correlation, however, requires more study.

As a result of the action of all the agencies of transfer, the distribution of free-charge in the atmosphere just prior to a lightning-discharge is something like that shown in Figure 2. Usually regions above the rain-formation zone are positively charged; the rain-formation zone itself carries a large negative charge and the regions below normally carry a moderate or small positive space-charge with a moderate induced positive surface-charge on the Earth. Thus a typical thunderstorm with

its associated free and induced charges may be characterized as a quadrapolar distribution. One pair of poles being the negative rain and the positive ions high above and the other pair of poles being the excess negative charge on the rain-formation zone and the positive free and induced charges below. The electric moment of the upper pair of poles is usually called positive (positive charge above) while the electric moment of the lower pair is negative. Such a distribution of charge, with appropriate discharges to adjacent zones can obviously reproduce the fluctuations of almost any type of observed electric field. Discharges may occur between the Y- and Z-regions, between Y- and X-regions or between the Y-region and the ground. It is worthy of note that the electric moment of the upper (Z) zone is positive and since, in the tropics, this is the active component, the usual distribution inferred here is in agreement with the observations of Schonland and Craib.[10]

We may finally summarize the qualitative requirements for the production of lightning. All factors depend on the existence of a high-velocity vertical current of air to sweep the humid air to high levels where it can condense and to transport the free-charges a kilometer or so before they can discharge by conduction. In the condensing region it is necessary that the droplets grow to sufficient size so that, due to gravity or centrifugal force, the droplets can be separated from the associated neutralizing ions surrounding the droplets which are then usually carried to high levels by the vertical current of air. To account for the magnitude of the charge that must exist on the rain-drops it is necessary to suppose that the smaller drops coalesce into big ones, that is; it must rain at the high levels. Moreover all this must take place in a time comparable to the electrical relaxation-time of the region or otherwise the discharges are likely to be non-disruptive. In this entire discussion it must be noted that electrical conduction minimizes the effects and discharges the free electricity at all times; it is the great stabilizing influence. Moreover from the form of (12), (13), and (14) it is apparent that the fields and potentials set up in the atmosphere depend critically upon the velocity of transport of the charge. Speed is essential for the development of important electrical effects— there is no such thing as a quiet, well-mannered thunderstorm.

QUANTITATIVE ESTIMATES AND COMPARISON WITH OBSERVATION

(A) *Free-charge and electric fields*

It will be shown presently that in the presence of a strong updraft of air the ions associated with the rain-drops are swept away as fast as the charge is developed on the larger rain-drops. Thus, a calculation of the rain-drop free-charge density will also approximately provide the charge-density of the atmospheric free ions at the level of separation. The charge-density depends upon the rate of precipitation and it becomes necessary to select the type of precipitation and the appropriate size for the elementary droplets which combine to form the larger ones. The permissible choice is extremely limited and as a guide it is convenient to refer to Table 2 which lists some of the most important properties of rain-drops. The first part of this Table is computed from a useful one-

[10] Proc. R. Soc., A, 114, 229-243 (1927).

TABLE 2—*Properties*

Type	Mass M	Radius a	Drop-density D	Velocity fall	Distance apart
	gram	*cm*	*drops/cc*	*cm/sec*	*cm*
Fog	5.2×10^{-10}	$5\ \times10^{-4}$	11	0.3	0.3
Elementary	3.4×10^{-8}	$2\ \times10^{-3}$	....	5	....
Mist	5.2×10^{-7}	$5\ \times10^{-3}$	$1\ \times10^{-2}$	25	2
Drizzle	4.1×10^{-6}	$1\ \times10^{-2}$	$2\ \times10^{-3}$	75	3
Light rain	4.4×10^{-5}	2.2×10^{-2}	$3\ \times10^{-4}$	200	10
Medium rain	5.2×10^{-4}	$5\ \times10^{-2}$	5.3×10^{-4}	400	13
Heavy rain	1.7×10^{-3}	7.5×10^{-2}	5.3×10^{-4}	500	13
Excessive	4.1×10^{-3}	$1\ \times10^{-1}$	5.3×10^{-4}	600	13
Cloudburst	$3\ \times10^{-2}$	$2\ \times10^{-1}$	$2\ \times10^{-4}$	700	16
Snow (aged)	$2\ \times10^{-4}$	1.5×10^{-1}	....	...	.

given by Humphreys.[11] According to this Table, fog-sized particles of radius 5×10^{-4}cm fall very slowly under gravity whereas mist-particles of radius 5×10^{-3} cm fall at the rate of 25 cm/sec. Now the elementary droplets which unite to form the larger rain-drops must fall through the rising air at an appreciable velocity if they are to grow in size. Thus one must calculate the equilibrium-charge on the critical size of droplet which is in a state rapidly to combine. It is clear from Table 2 that the critical droplet must be intermediate in size between a fog-droplet and a mist-droplet, the exact value depending somewhat on circumstances. The radius of the critical-sized droplet r is here assumed to be an average of that of fog and mist or $r=2\times10^{-3}$ cm. Such a drop has a mass n of 3.4×10^{-8} gm and falls with a velocity of 5 cm/sec. By use of (17) and taking $T=300°$ and $c_2/c_1=1/10$ it is found that the equilibrium-charge on this condensing, critical-sized, elementary drop is

$$Q_{crit}=q=-4.15\times10^{-7}\text{ e.s.u.} \tag{23}$$

This result is relatively insensitive to the selected value of c_2/c_1 which may be somewhat in error.

The critical-sized elementary drops immediately unite to make larger drops and if this process takes place in a short time compared to the relaxation-time it follows that the charges on all the newly formed droplets are combined onto the final large drops. Therefore the total charge on a newly formed "heavy-rain" drop of mass 1.7×10^{-3} gm[11] is by (18)

$$Q_{total}=(M/n)q=-2.1\times10^{-2}\text{ e.s.u./drop} \tag{24}$$

With "heavy rain" falling normally, Table 2 shows that at the Earth there are $D=5.3\times10^{-4}$ of these drops per cubic centimeter so that the space-charge density in the region approximates

$$MqD/n=(\rho_{00})_{rain}=-1.1\times10^{-5}\text{ e.s.u./cm}^3 \tag{25}$$

[11]Physics of the Air (1929).

of rain-drops

Equilibrium-charge		Total charge		Charge-density	Precipitation current-density
Per drop	Per gram	Per drop	Per gram		
e.s.u.	*e.s.u.*	*e.s.u.*	*e.s.u.*	*e.s.u./cc*	*e.s.u./cm²*
1.04×10^{-7}	200	1.04×10^{-7}	200	....	....
4.15×10^{-7}	12.2	4.15×10^{-7}	12	....	....
1.1×10^{-6}	2	6.4×10^{-6}	12	6.4×10^{-7}	1.6×10^{-5}
2.1×10^{-6}	0.5	5×10^{-5}	12	1.0×10^{-6}	7.5×10^{-5}
4.5×10^{-6}	0.1	5.4×10^{-4}	12	1.6×10^{-6}	3.2×10^{-4}
1.1×10^{-5}	0.02	6.4×10^{-3}	12	3.4×10^{-6}	1.35×10^{-3}
1.6×10^{-5}	0.009	2.1×10^{-2}	12	1.1×10^{-5}	5.5×10^{-3}
2.1×10^{-5}	0.005	5.0×10^{-2}	12	2.6×10^{-5}	1.5×10^{-2}
4.2×10^{-5}	0.0015	3.7×10^{-1}	12	7.4×10^{-5}	5.2×10^{-2}
3.1×10^{-5}	0.5	....	....	....	....

and since equal amounts of charge are separated in any given interval

$$(\rho_{00})_{air} = +1.1\times10^{-5}\ \text{e.s.u./cm}^3 \tag{26}$$

As long as the rain is being formed this free-charge density is maintained in the rain-formation region and the positive free-charges are constantly swept high in to the atmosphere by the rising current of air which also stirs the region. This is just the condition assumed in deriving (12), (13), and (14) and therefore these equations may be directly applied to the thunderstorm-problem.

The density of charge on the rain, computed above, is an underestimate because negative ions from the evaporating region are swept into the region of rain-formation. Further, there is some accumulation of the rain and the drop-density D is considerably greater than the value adopted from Table 2 which refers to conditions at the surface. It is difficult to estimate the factor to be introduced to allow for the negative ions and the accumulation, but in general, assume that

$$(\rho_{00})_{actual} = f\,(\rho_{00})_{rain} \tag{27}$$

where f is an unknown factor depending on the velocity of the rising air-current, the size of the drops, and the electrical forces acting on the drops (simple physical ideas suggest that it may approximate 2).

Adopt the rain-formation level of the atmosphere as one of the reference-levels. Measure z upward from this level and y downward and refer to the levels above the reference as the Z-region and the zones below as the Y-region. The distribution of free-charge in the Z-region is given by (12). The first term is zero in the absence of an externally impressed current, and ρ_{00} is given by (26). The exponential term depends on the average electrical conductivity of the region and the average velocity of transport of the ions by the rising current of air. Both quantities increase with increasing altitude but we cannot be much in error if we take σ_1 as the conductivity at the 3-km level or $\sigma = 10^{-3}$ and $v_1 = 10$ meters/sec or 10^3 cm/sec. The latter value is thought to be conservative since

vertical velocities of 3×10^3 cm/sec are required to form moderate sized hailstones. Adopting these figures it is found that

$$\rho_z = +1.1\times10^{-5}\,[\exp(-14.8\times10^{-6}z)] \tag{28}$$

and therefore the charge-density is reduced by expansion and conduction to $1/\epsilon$ times its initial value by the time it has been transferred 0.67 km.

In the Y-region, where rain is falling, a somewhat similar distribution is produced except that in this region the great velocity of the vertical air-current results in a small velocity of fall relative to the reference-level and therefore the extent of the charged region is not great. In (19) we adopt for τ_1, its value at the 1-km level or 139 seconds and set $t=y/v_R$ where v_R is the velocity of fall of the rain-drops. This latter quantity also has an influence on f which cannot be easily evaluated. A reasonable velocity for v_R is taken to be 400 cm/sec. Employing hese values it is found that

$$\rho_y = -1.1f\times10^{-5}\,[\exp(-18.8\times10^{-6}y)] \tag{29}$$

The total charge in the two regions is readily calculated by use of (13)

$$\int\rho_z dz = +1.1\times10^{-5}/14.8\times10^{-6} = +0.75 \text{ e.s.u./cm}^2 \tag{30}$$

and

$$\int\rho_y dy = -1.1f\times10^{-5}/18.8\times10^{-6} = -0.59f \text{ e.s.u./cm}^2 \tag{31}$$

The expression for the total charge in the Y-region is not considered especially reliable and is an underestimate because the effect of negative ion-transport and ion-accumulation cannot be accurately specified. Further, the rising air-current which together with the electrical forces are responsible for the accumulation of rain also modifies the rate of fall of the drops and thus affects the exponential term. The order of magnitude of (31), however, is correct.

The process of separation of charge here considered is such, of course, as to produce equal amounts of charge within a given interval of time. In the Z-region the usually positive free-charge is transferred to high levels where the electrical conductivity at a representative level of 3 km is greater than it is at the level of rain-formation. Now from Table 1 the usually positive charge will be reduced to $1/\epsilon$ times its value in a time $\tau_3=83$ sec while the usually negative charge on the rain will be similarly reduced in a longer time $\tau_1=139$ sec. Therefore after the atmosphere has been charging for a moderate interval the charge on the negative will be much in excess.

The ratio of the free-charges in any two regions after any assumed charging interval is readily evaluated. Let Q be the total electrical charge in any region and let the charge be delivered to the region at a constant rate $(dQ/dt)_0$, then by aid of a procedure equivalent to that used to derive (4) we have

$$\mathrm{d}Q/\mathrm{d}t = (\mathrm{d}Q/\mathrm{d}t)_0 - 4\pi\sigma Q \qquad (32)$$

Integrating and imposing the condition $Q=0$ when $t=0$ it is found that

$$Q = (1/4\pi\sigma)\ (\mathrm{d}Q/\mathrm{d}t)_0 \{1-[\exp(-4\pi\sigma t)]\} \qquad (33)$$

whence, since the rate of supply of charge to the two zones is always the same, the charge built up in each, after a period of the order of the relaxation-time, is inversely proportional to the conductivity of the region. To a fair approximation one may write

$$Q_1/Q_2 = \sigma_2/\sigma_1 = \tau_1/\tau_2 \qquad (34)$$

where σ and τ refer to average values for the region containing the free-charge Q.

Substitution of the numerical values shows that in the rain-formation regions the free usually negative charges are always greater than the free positive charges at a much higher altitude and the ratio approaches $\tau_1/\tau_3 = 139/83$ or 1.68. This is an important result because it shows that simple bipolar thunderclouds are unstable and do not long exist. *Actual thunderclouds are characterized by the superposition of a unipolar and a bipolar distribution supplemented as circumstances demand by induced poles on the surface* of the Earth or on the highly conducting layers of the high atmosphere.

The difference in discharge-rates at the different levels is of particular importance in the rain-formation zone. The situation in the region below the saturation-level is very complicated because of the presence of rain-drops with large charges on them which are of both signs. The newly formed positive drops at 0.5 km (say) discharge in about 150 seconds while the negative ions from this region transferred to 1 km (say) discharge in something like 139 seconds. The difference is not great and therefore the excess positive charge produced below the saturation-level is appreciable but seldom so great that it can overwhelm the much larger negative charge above.

Except for geometrical factors which cannot change the order of magnitude of the result, the total free-charge per unit-area in a prism extending from the surface proper to the high atmosphere must approximate zero in any quasi-steady state. We may therefore write

$$\int \rho \mathrm{d}x + \int \rho \mathrm{d}y + \int \rho \mathrm{d}z + \Sigma + \triangle = 0 \qquad (35)$$

where Σ and $\triangle$ are the charges per unit-area induced on the surface of the Earth and on the high conducting layer respectively. In tropical storms $\triangle$ may be conceivably much larger than Σ because of geometrical differences, but in the type of storm considered above, $\triangle$ is presumed to be small compared to Σ. Therefore we may write approximately

$$\int \rho \mathrm{d}x + \Sigma = -\int \rho \mathrm{d}y - \int \rho \mathrm{d}z \qquad (36)$$

Now we have just shown that to a first approximation

$$-\int \rho dz < \int \rho dy < -1\ 68\int \rho dz$$

Thus the sum of the free positive charges in the X-region and that induced on the Earth's surface is less than, but comparable to $0.68\int \rho dz$ or 0.51 e.s.u./cm². Now since the surface electric field E_s is given by

$$E_s = 4\pi\Sigma \qquad (37)$$

it follows that the surface electric fields will usually be smaller than, but still comparable to 6.4 e.s.u. or 1900 volts/cm. Observation shows that surface-fields about one-third this value are frequently measured. It will be difficult to estimate the free space-charge actually present in the X-region but comparison of the above estimate with observation shows that it is comparable to the surface-charge and this, in turn, is of the same order as the total charge in the Z-region. Therefore the magnitudes determined from the calculation of the electrical quantities in the Z-region will also be descriptive of the magnitudes in others, since the dimensions are also comparable. We therefore proceed to the detailed calculation of the electrical quantities of the Z-region.

The electric field existing at any level in the Z-region is given by (13). As before take $\sigma_1 = 10^{-3}$, $v_1 = 10^3$, $a = 1.4\times 10^{-6}$, and $\rho_0 = 1.1\times 10^{-5}$ then

$$E_z = 9.4\ [\exp\ (-14.8\times 10^{-6}z)] \qquad (38)$$

Whence the maximum value is 9.4 e.s.u. or 2800 volts/cm. It is therefore to be assumed that the region is on the verge of electrical breakdown and a resulting disruptive discharge.

(B) *Distribution of potential*

It is convenient to calculate the potentials with respect to the level $z=0$. The potential due to the convection of electricity, therefore, is given by the second term of (14). Substantiation of numerical values employed in the previous section yield

$$\phi_z = -6.3\times 10^5\ [\exp\ (-14.8\times 10^{-6}z) - 1] \qquad (39)$$

whence the potential at great altitudes is $\phi_\infty = (6.3\times 10^5)\ 300 = 1.9\times 10^8$ volts. The total potential difference between the upper reaches of the Z-region and the middle of the Y-region is something like twice that just given or $\phi_{max} = 3.8\times 10^8$ volts. Such are the potentials necessary and sufficient to produce a typical lightning-discharge.

(C) *Electric moment*

The electric moment per unit-area of the Z-region by aid of (12) is approximately

$$\Omega = \int_0^\infty \rho z dz = \rho_{00}/(a + 4\pi\sigma_1/v_1)^2 \qquad (40)$$

which is $1.1\times 10^{-5}\ (14.8\times 10^{-6})^2 = +5\times 10^4$ e.s.u. cm. The total electric

moment is evidently ΩS where S is the effective area of the storm. We adopt as representative a storm 5 km square and therefore $S = 25\times10^{10}$, whence the total electric moment of a storm in the Z-region is 1.25×10^{16} e.s.u./cm or 41 coulomb/kilometer. The electric moments actually observed in nature are of just this magnitude.[10]

(*D*) *Charging current and recovery-time*

The charging current-density to the Z-region is

$$i = \rho_0 v = 1.1\times10^{-5}\times10^{3} = 1.1\times10^{-2}\ \text{e.s.u./cm}^2 \qquad (41)$$

and the total charging current for the storm $I = iS = (1.1\times10^{-2})\ (25\times10^{10}) = 2.7\times10^{9}$ e.s.u. $\doteq$ 1 ampere. The recovery-time τ_R after a complete discharge is

$$\tau_R = \int \rho dz/\rho_0 v_1 = 1/(\alpha v_1 + 4\pi\sigma_1) = 72 \text{ seconds} \qquad (42)$$

Thus the recharging time is of the order of a minute if the cloud for any region is completely discharged. In general a cloud does not completely discharge and the partial recovery-time is therefore some fraction of 72 seconds. The recovery-time will also decrease if the conductivity of the region is materially increased by lightning-discharges.

(*E*) *Electrical forces on the rain-drops*

The electrical force on a negatively charged rain-drop in the region of rain-formation is upward and it is of interest to determine if the rain-drop could be supported until a discharge released the forces and precipitated the particles in a "rain-gush."

The total charge on the rain-drop is given by (24). Thus if F is the upward force on the drop, Mg is the gravitational force, and E is the electric field in condensation-zones given by (38), then

$$F/Mg = (q/n)\ (E/g) = [(4.15\times10^{-7})9.4]/[(3.4\times10^{-8})980] = 0.17 \qquad (43)$$

Therefore, the electrical force at maximum contributes a large fraction of the support. Considering the approximate nature of the data it is possible that the electrical forces might be sufficient completely to support the drop and increase to large values the accumulation of rain in the rain-formation zone. Upon the annihilation of the electric field by a disruptive discharge the rain would be released all at once and therefore the phenomenon of "rain-gush" may actually result from the acting electrical forces. It is also evident that this effect can act as a very definite limiting mechanism to the separation of charge.

(*F*) *Precipitation-charge*

It has been shown in earlier paragraphs that the initial charge on a "heavy" rain-drop may be either negative or positive, according to the region in which it is formed, and its magnitude by (24) is something like

2.1×10^{-2} e.s.u. This drop falls for an indeterminate time and constantly discharges by conduction. If it falls long enough it will completely discharge and take on an equilibrium-value specified by (17). It is impossible to specify how long it will take a given drop to reach the Earth, but it is usually sufficient for the drops to dissipate a large fraction of their initial charge. The faster the drops fall and the shorter the distance they fall, the larger the fraction, in general, of the initial charge that will be precipitated. The result is expressed quantitatively by (19). If we take the mean velocity of fall of a "heavy" rain-drop to be 500 cm/sec (see Table 2) and assume that the negative drop falls 1.5 km (say), while a typical positive drop falls only 1 km, it is found that the charge on the negative drop is 20 per cent of its initial value while the positive charge is 30 per cent of its initial value. If these results are typical, the expected charge precipitated on heavy rain-drops will approximate 25 per cent of their original value (see Table 2) or 5×10^{-3} e.s.u. This agrees well with the results of Gschwend[8] who made extended measurements on the charge brought down by single rain-drops.

One further test can be applied to the results of this study. Schindelhauer and Simpson have published values[12] for the mean charge brought down by rain over a very long series of observations; the former gives the precipitation-charge as $+0.029$ e.s.u./gm, while the latter gives $+0.176$ e.s.u./gm. Now with a sufficiently extended series of observations, it is possibly correct to suppose that the association of charges occur at random and therefore the final average charge on the rain should be something like the equilibrium-charge on the rain-drops were there no electrical effects arising from association whatever. Calculating the equilibrium-charges by (17), noting that the drops are usually evaporating, and adopting the masses given in Table 2, it is found that the equilibrium-charge per gram for drizzle is $+0.5$ e.s.u./gm, for light rain it is $+0.102$ e.s.u./gm, and for moderate rain it is $+0.02$ e.s.u./gm. This is considered to be in numerical agreement with the results of Schindelhauer and of Simpson mentioned above.

(G) *The separation of rain-drops and associated ions*

It is necessary to examine the circumstances that permit the physical separation of the falling rain and the neutralizing ions that form the ion-sheath enclosing each drop. It has been noted that the elementary droplets that combine to form the larger rain-drops must move with respect to the rising air in order that they may grow in size. In general, the newly formed drops do not move sufficiently fast to escape their neutralizing ions but after they have grown appreciably it is found that the velocity of the drop relative to the rising air-current exceeds the maximum systematic attractive velocity of the ions toward the charged rain-drops.

Consider the newly formed rain-drop having a charge given by (18) and let the equilibrium-charge [given by (17)] on each elementary droplet be q then the electric field at the surface of the drop will be $E_s = (M/n)(q/R^2)$ where R is the radius of the large rain-drop. Now let the mobility of the fastest moving ions, of charge opposite in sign to that on the rain-drop, be represented by $u = u_0\,[\exp(az_1)]$ where u_0 is the mobility of the same class of ions at the surface of the Earth and u is its value at

[12]See B. F. J. Schonland, Atmospheric electricity, 64 (1932).

a distance z_1 above the Earth's surface. Then the maximum possible systematic velocity that the charged drop can impose on one of its neutralizing ions urging it toward itself is $E_s u_0 [\exp(\alpha z_1)]$

The rain-drop falls with a velocity with respect to the moving air given by the familiar relation

$$v = (2/9)\ (gR^2w/\mu) \tag{44}$$

where μ is the coefficient of viscosity, g the acceleration due to gravity, and w the density of the drop diminished by the density of the surrounding air. Now it is clear that all the ions associated with the rain-drop will be stripped off the drop if they cannot keep up with it as it falls. Therefore for complete separation of charge to take place it is sufficient that

$$V \geq E_s u_0 [\exp(\alpha z_1)] \tag{45}$$

and this combined with (18) gives the inequality

$$Rr^2 \geq (3.12\times10^{-6})\ (T \log c_2/c_1)(\mu/wg)\ u_0 [\exp(\alpha z_1)] \tag{46}$$

Adopting the values of these quantities given above and taking $\mu = 1.8\times10^{-4}$, $u_0+ = 410$, $w = 1$, and $g = 980$ it is found that

$$Rr^2 \geq 6.9\times10^{-8} \tag{47}$$

In this investigation we have taken (from Table 2) for "heavy rain" $R = 7.5\times10^{-2}$ cm and for the elementary droplet $r = 2\times10^{-3}$ cm, whence $Rr^2 = 30\times10^{-8}$ which satisfies the requirements of (46). In a similar manner $Rr^2 = 20\times10^{-8}$ for moderate rain, 8.8×10^{-8} for light rain and 4×10^{-8} for drizzle-size droplets. Therefore *except for snowflakes and the smallest sized droplets it is proper to assume that* all of the associated ions are stripped off the regions surrounding the rain-drop and that *separation of charge actually takes place.* It must be noted that in the absence of some vertical current of air the light ions are at first left more or less in the region of separation but the falling rain shortly sets the associated air in motion and carries the ions downward with it. Hence heavy rain *can* fall without accompanying lightning.

MAINTENANCE OF THE EARTH'S CHARGE

An important contribution of Wilson's somewhat nebulus and incomplete theory of thunderstorms was the recognition of the part that thunderstorms might play in maintaining the fair-weather electrical field. Wilson further notes that the lightning-discharges will contribute to the ionization within the storm and therefore materially aid in transferring by conduction the charges to the upper conducting layers. The great difficulty with the theory is the apparent lack of sufficient conductivity in the lower atmosphere to transform the required quantity of electricity with the fields available. For example the maximum observed electric fields at the Earth's surface are not long applied and amount to a maximum of only about 3 e.s.u. cm. The conductivity is 2×10^{-4} e.s.u. so

that the total current flowing would approximate (6×10^{-4}) (25×10^{10}) $=1.50\times10^{8}$ e.s.u. or 0.05 ampere. Therefore even if the field is maintained constant, without the reversal of sign known to take place, yet the quantity of positive electricity transferred from the Earth is still too small to be significant unless the number of storms occurring over the surface of the Earth at any instant is 15 times that usually assumed or unless the area of each storm is much greater than that assumed. The transfer of positive charge to the high conducting layers in adequate amounts seems barely possible as Wilson has shown[2], but this is not enough because the current-circuit must be *completely closed*, not partially so. Thus conduction-processes alone seem inadequate.

The present analysis at first suggested that positive charge might be transferred to the high conducting layer by convection and to Earth by disruptive electrical discharge in sufficient amounts to be of interest. It has been noted that the excess negative charges in the rain-formation zones *induce* positive charges on the surface of the Earth, which are of considerable magnitude. Conduction in the lower zone seems inadequate but disruptive discharges can apparently transfer an average of 20 coulombs of negative charge to the Earth per discharge, hence completing the electrical circuit through the fair-weather regions of the rest of the Earth. Even this mechanism is barely adequate because the fair-weather conduction-current is more than 1200 coulombs per sec over the whole Earth and therefore observation demands that 60 discharges shall be made to Earth every second, each discharge carrying to the Earth a negative charge of 20 coulombs. Assuming the favorable condition that in a given storm, discharges occur once every 30 seconds, then it follows that 1800 storms must be going on simultaneously over the Earth and that each one must periodically discharge to the Earth. Brooks[13] estimates that 1800 storms are actually in action at all times and the agreement might be considered satisfactory save for one thing. The storms must be of such a nature that discharges to Earth are frequent and therefore tropical storms cannot all be counted because, as Schonland has noted[12], only 10 per cent of the storms in South Africa discharge to Earth. Since most of the estimated 1800 storms occur in the tropics it appears that thunderstorms alone are inadequate to account for more than a small part of the maintenance of the Earth's charge. The data are unreliable, however, and the matter cannot be considered settled.

The present study of rain-drop and thunderstorm-electricity suggests that the Earth's charge may be maintained by rain-precipitation processes combined with vertical ion-convection. The considerations discussed in the foregoing sections show that if the negatively charged rain can be precipitated on the Earth before it discharges itself by conduction, then the precipitation current-density may be considerable. For example, we have seen that the space-charge density in the rain-formation zone is roughly -1.1×10^{-5} e.s.u. for "heavy rain" and this charge is swept earthward with a velocity of 500 cm/sec (see Table 2) so that the precipitation current-density is 5.5×10^{-3} e.s.u./cm.^{2} Now the total current necessary to maintain the fair-weather field is 1200 amperes or 3.6×10^{12} e.s.u. Therefore the above precipitation of charge alone over an area of $3.6\times10^{12}/5.5\times10^{-3}=6.5\times10^{14}$ cm^2 will maintain the observed

[13]London Met. Office, Geophys. Mem. No. 24 (1925).

distribution. This area corresponds to a square 250 km or 150 miles on a side and is but 1.3×10^{-4} or 0.013 per cent of the total Earth's surface. Thus, if special regions and thunderstorms exist on the Earth where the rain precipitates almost immediately and does not have to fall a kilometer or so earthward and if simultaneously rising currents of air necessary to produce the thunderstorm sweep the usually positive ions to high levels, then these regions will contribute in an important manner to the maintenance of the Earth's charge. The requirements may be stated simply by noting that the necessary type of storm is one in which part of the Y-region and all of the X-region considered earlier do not exist because of the presence of the ground in or immediately below the rain-formation zone.

It is clear that thunderstorms in high mountainous regions of tropical or extra-tropical latitudes will satisfy the required conditions and because of the relatively small total area of storms necessary to produce the required transfer, the mechanisms seem of considerable importance. The suggestion is susceptible of quantitative verification in part because it requires that the charge carried down by condensing-type rain in high mountainous regions shall be large and predominantly negative. As far as is known, there are no data available at this time which bear on this important question.

The convection by rain in the above cases will be supplemented by the conduction-current which is established by the negative atmospheric charge and the induced positive charge on the Earth below. Taking the conductivity at the assumed high level of perhaps 2 km as 7×10^{-4} e.s.u. and the mean electric field as 3 e.s.u. the conduction-current density is 2.1×10^{-3} e.s.u./cm². This is roughly the same as the convection-current and in the same direction. Its consideration will materially reduce the estimated required storm-area.

The analysis in earlier sections has shown that in the presence of the fair-weather current a positive space-charge is always present, which results from the non-homogeneous nature of the atmospheric conductivity. This space-charge is represented by the first term of (11) and if vertical convection takes place on a suitable scale it is clear that positive charge will be carried upward and contribute to the maintenance of the observed negative charge.

Assume that a given fraction, F, of the average fair-weather electrical conduction-current-density, i, is to be maintained by convection, then

$$\rho V + Fi = 0 \tag{48}$$

where ρ is the normally positive average space-charge at a given level and V the average vertical transport-velocity. Whence, combining (3) and (15), we have

$$(V/F) = -4\pi\sigma \Big/ \left(\frac{1}{\sigma}\frac{d\sigma}{dz}\right) = \left[\frac{\tau}{\sigma}\frac{d\sigma}{dz}\right]^{-1} \tag{49}$$

At the surface of the Earth $(1/\sigma^2)(d\sigma/dz)$ approximates 0.47 so that (V/F) must approximate 25 cm/sec at the surface and, because $(1/\sigma)(d\sigma/dz)$ is nearly a constant, must increase with altitude at a rate roughly proportional to σ.

Earlier investigations have been directed to an examination of the general circulation of the air and they appear to show that such convection-effects are unimportant.[14]

However, certain *systematic* vertical motions of the atmosphere should be considered and these depend in an important manner upon the surface features of the Earth.

It is well known that the heating of a mountainside warms the air adjacent to the surface and it sweeps up the side of the mountain like warm air up a chimney, only to be discharged near the ridge with appreciable velocity. The integrated result of such motions in a great mountain range may be important. Further, a few active and many comparatively inactive volcanic regions discharge locally tremendous quantities of heat into the atmosphere and must be expected to induce considerable systematic vertical convection. The Katmai region, for example, discharges, even now, more than 10^{10} calories/sec[15]. A large fraction of this undoubtedly goes to heat the surrounding air. It is not difficult to see how a few such regions could initiate steady vertical air-currents which would produce important electrical convection-effects.

Forced convection on a large scale occurs whenever winds blow over extended mountain ranges. Of particular interest is the great Pacific chain of mountains that extend from Alaska to southern South America. The prevailing winds blow nearly perpendicular to this great natural barrier and there is little doubt that vertical convection is induced on a large scale. It is suggested that this systematic "barrier-convection," acting simultaneously with the "chimney-type" of convection, may play an important part in maintaining the Earth's electric charge.

Little reliable information is available for estimating the average vertical convection-velocities over mountainous regions. However, the land-surface of the Earth is about 30 per cent of the whole and if it is assumed that as little as 15 per cent of the land-surface (4.5 per cent of the whole) is mountainous and effective in producing vertical air-motions, it follows from (49) that the average vertical velocity at the surface needs to be $25F/0.045 = 550F$ cm/sec or about $12F$ miles per hour. This estimate is to be modified somewhat by an unknown factor of the order of unity because it has been assumed that the average electric field intensity and its space-gradient over the mountains approximates that over level country.

We do not feel that the meteorological data now available are complete enough to warrant a more detailed calculation and do not permit conclusions to be drawn, but an average vertical velocity of possibly 4 miles/hr throughout the mountainous regions might readily exist. Such a velocity might transfer sufficient charge by convection to the high atmosphere to account for roughly one-third of the observed transfer by conduction. It appears therefore that the systematic convections produced by great natural barriers, etc., may play a moderate part in maintaining the Earth's electric charge.

[14]W. Schmidt, Physik. Zs., **27**, 472-473 (1926).

[15]E. G. Zies, Nation. Geog. Soc., Contrib. Tech. Papers, Katmai Ser., **1**, No. 4 (1929).

GENERAL REMARKS

The present discussion would be incomplete without some reference to the difficulties of correctly measuring the charge on precipitated raindrops. The rough experiments described in an earlier section show that evaporation of any water collected on the central system of the measuring electrometer will cause it to take on a positive charge. Since most experiments are carried on under conditions which are favorable to evaporation, it appears that most measurements made heretofore may have accentuated the measured amount of positive charge and minimized the quantity of negative charge precipitated on rain. The correction to be applied to much of the present data may be large.

It is a well-known fact that lightning discharges to Earth with resultant damage are relatively infrequent in the tropics, whereas at the latitude of Washington, thunderclouds that discharge to Earth are perhaps the rule. Only about two thunderstorms per year in this vicinity could be classed as typically tropical. These occasional storms are characterized by relatively quiet surface-conditions and frequent lightning between clouds of an extremely meandering type. The active layer is high and in at least one electrically very active storm the amount of rain reaching the Earth's surface was small, suggesting that the rain-formation level was high and the drop-evaporation was large.

The common type of storm in this vicinity is one in which the active condensation and rain-formation regions are at a comparatively low altitude. Such a storm appears to be of the type considered typical in an earlier section. Discharges not only occur frequently at high levels, but discharges to ground are usual rather than exceptional.

The difference between the tropical and temperate type of storm arises primarily from the difference in meteorological conditions. In the tropics the water-vapor content of the atmosphere is greater than elsewhere and since the amount of water-vapor determines the degree of instability and the velocity of the rising air-currents, one must expect these currents to extend to greater heights. Moreover, the level of condensation occurs at greater heights than in temperate latitudes. These factors and others in the tropics conspire to place the rain-formation levels at such great heights that the falling rain is usually fairly well discharged before it reaches the Earth and because the drops usually evaporate, many will take on a positive-equilibrium charge. Moreover, the actively charged region is relatively far away from the Earth's surface and relatively nearer the high conducting-layer so that the charge induced on the high conducting-layer by the excess free-charge (usually negative) in the active region is larger or at least comparable to that induced on the Earth. Further, with a given potential difference and induced charge on the Earth, it is clear that the maximum electric fields which precede break-down will be less if the active and induced charges are well separated. All these factors cooperate to make discharges to Earth relatively infrequent in tropical-type storms. On the other hand, if the active region with its excess of free-charge is at relatively low levels the geometry is different and the larger charges induced on the Earth's surface may readily set up electric fields of sufficient magnitude to cause break-down. From the form of the expression for the electric field and potential it follows that the electrical activity of the Z-region increases very rapidly

as the vertical convection-velocity is increased, so that, in the tropics, one would expect the effects in the Y- and Z-regions to predominate while in temperate latitudes the effects in the X- and Y-regions are the most important. It is therefore clear that the observed differences between the tropical and non-tropical types of storm are not of a fundamental nature.

Meteorological effects are so complicated and involved that it is quite impossible to describe a thunderstorm in every detail. The foregoing discussion considers only a typical distribution and wide variations from it must be expected. Conditions below the rain-formation zone are particularly involved and more study of the electrical behavior of the ions and rain-drops in this region should be undertaken.

The theory of thunderstorms proposed is far more detailed and specific than any of the earlier theories but in spite of this the agreement with observation obtained without forcing is satisfactory. Perhaps, the principle value of the theory lies in the suggestions that it makes for future investigation.

United States Naval Research Laboratory,
Washington, D. C.

VILHELM CARLHEIM-GYLLENSKÖLD, 1859-1934

By Gustaf S. Ljungdahl

Dr. Vilhelm Carlheim Gyllensköld died in Stockholm December 13, 1934. Up to the last few weeks before his death, he was actively engaged on the work which had claimed his chief interest during life—the Earth's magnetism and allied phenomena.

He was born in Stockholm October 17, 1859. In 1877 he became a student at the University of Uppsala, where he received his doctor's degree in 1897. He interrupted his studies to participate in the First International Polar Year 1882-83 as a member of the Swedish expedition to Cape Thordsen, Spitzbergen. Carlheim-Gyllensköld's part in this Expedition, which was under the leadership of Ekholm, was to carry out the auroral observations, while Solander made those of terrestrial magnetism. Considering the technical difficulties with which an auroral observer of that time had to deal, it must be recognized that the results of this, his first geophysical work, were most remarkable, especially because of the care and devotion which thoroughly distinguish it. In particular, his investigations of the spectrum of the aurora seem to be among the most accurate ever made. During this Expedition his lifelong interest in the science of the Earth's magnetism appears to have been aroused.

Having completed the discussion of the material from the Polar-Year Expedition, he spent several years as an assistant astronomer in Pulkova. Thereafter he began a series of magnetic surveys in southern Sweden, where only a few magnetic observations had been made. The results of these surveys were published in a series of treatises of which the "Mémoire sur le magnétisme terrestre dans la Suède meridionale," published in 1895, is perhaps the most important since he not only presented the results of his own observations but also those of all known earlier magnetic measurements from all parts of Sweden. All these data were reduced to the common epoch 1892. Much of the material which he thus gathered and examined had never been published and very probably would have been forgotten had it not been for his arduous efforts. Using this material, he tried to derive formulas for the secular variation in Sweden, by the aid of which he could reduce all the available observational values to a common epoch.

In 1896 he published the treatise which probably did the most to make his name known, namely, "Sur la forme analytique de l'attraction magnétique de la Terre exprimée en fonction du temps," in which he computed the potential for the epochs 1600, 1700, 1787, 1858, etc. He based his calculations on the assumption that the field of terrestrial magnetism is not only a function of latitude and longitude but also of time, that is, that the Gaussian coefficients $A_n^{(1)}$ and $B_n^{(1)}$ change with the time. It must still be acknowledged that this work is a particularly valuable contribution to the analysis of the Earth's magnetic field. The computation of the coefficients was again made in 1906.

During the years 1894-97 he served at the Astronomical Observatory in Stockholm. In 1898 he took part in the Swedish Expedition to Spitzbergen, the object of which was to measure an arc of meridian. At the same time he made magnetic measurements. That same year he became

secretary of the Swedish-Russian Committee for the measurement of an arc of meridian. He was lecturer in physics at the University of Stockholm during 1907-10 and in 1911 received the title of professor of the University.

During 1900-01 he directed the prospecting of the iron-ore deposits in Kiirunavaara in Swedish Lapland, based on magnetic researches; in this he assumed that the iron-ore was approximately similar to a very oblate ellipsoid. These researches made it possible for the first time to obtain an estimate of the great depths of the mass of ore in this region. His method has undoubtedly been a great contribution to the later development of magnetic prospecting methods in mining.

About 1900 he made great effort to promote the establishment of a Swedish institute for terrestrial magnetism and of a "central observatory," as well as of a detailed net of magnetic stations embracing the whole country. His efforts were then unsuccessful but in later years, with the assistance of the Swedish National Committee of Geodesy and Geophysics, they led to the great magnetic survey of Sweden by the Geological Survey during 1928-34. The organization of that enterprise is entirely the work of Carlheim-Gyllensköld but he did not live to see its results.

His last contribution to terrestrial magnetism was the preparation of the Swedish Expedition to Spitzbergen during the Second International Polar Year 1932-33. He is thus one of the very few who participated in the First Polar Year 1882-83 and whose experience could be utilized in organizing the Second Polar Year 1932-33.

As a member of the Swedish National Committee for Geodesy and Geophysics and of the International Commission of Terrestrial Magnetism and Atmospheric Electricity, he was personally well known to participants of most of the geophysical conferences held during the last decades. From 1927 he was vice-president of the Association of Terrestrial Magnetism and Electricity of the International Geodetic and Geophysical Union, and he presided at the sessions during the Stockholm meeting in 1930.

In a Swedish newspaper, Dr. Gavelin, Director of the Geological Survey of Sweden, writes: "It would be a very incomplete picture of Carlheim-Gyllensköld, that only dealt with his activity in the fields of physics and geophysics. His was a singularly many-sided talent, and he did not like to restrain himself. His interests transcended the limits of the physical sciences and embraced also other natural sciences, as well as, to a certain extent, the humanistic studies. He had, for instance, attacked the problem of the magnetic measures of older times, a study, which he, as far as the writer of these lines can judge, approached in a most original manner. He had collected copious material on the subject and it is regrettable that he did not have time to arrange it for publication. It was perhaps so that the many-sidedness of his intelligence and his great enthusiasm for tasks of varying nature and lying in different spheres, became a hindrance to him. They caused him to divide his forces. A number of valuable schemes are left incomplete at his death and others have not been followed up as he himself intended and, to the very last, hoped to do. Another characteristic of his broad intelligence was his keen love of literature. As a young man he wrote dramas, and there existed a warm and unbroken friendship between him

and August Strindberg. After the death of the latter he published two volumes of his posthumous works.

"Carlheim-Gyllensköld's personality was not easily approached. Rather, to those who did not belong to the close circle of his friends, he may have seemed harsh and forbidding. He was a man of pronounced sympathies as well as antipathies. He could show anger and his answer could display a sharp rigour. Those who had the pleasure of knowing him more closely knew, that this rough surface concealed a noble man, a deep and lofty refinement, a personality that never bargained or compromised with his conviction—one, that with a singular disinterestedness, without thought of personal profit, devoted all his powers to science."

Scientific publications by Vilhelm Carlheim-Gyllensköld

Flyttfoglarnas ankomsttider i Sverige åren 1873-1877, Svenska sällsk. för antropologi och geografi, skrifter, 1878-80, Stockholm, 1879.

Aurores Boréales, Exploration internationale des Régions Polaires 1882-1883, Observations faites au Cap Thordsen, Spitzberg, par l'expédition Suédoise, **2**, No. 1, Stockholm, 1886.

Phénomènes optiques de l'atmosphère avec quelques remarques sur les étoiles filantes, le spectre de Copépodes, l'accroisement des brins d'orge. Exploration Internat. des régions polaires 1882-1883. Obs. faites au Cap Thordsen, Spitzberg, par l'Expédition Suédoise, Stockholm 1887.

Détermination des éléments magnétiques dans la Suède méridionale, Kongl. Svenska Vetensk. Akad. Handl., **23**, No. 6, Stockholm, 1889.

Magnetiska deklinations-observationer utförda på svenska kuster av svenska sjöofficerare åren 1852-1855. Öfvers. af Kungl. Vetensk. Akad. förhandl., **51**, No. 2, Stockholm, 1894.

Mémoire sur le magnétisme terrestre dans la Suède méridionale, K. Svenska Vetensk. Akad. handl., **27**, No. 7, Stockholm, 1895.

Observations magnétiques faites par Th. Arwidsson sur les côtes de la Suède pendant les années 1860-1861, K. Svenska Vetensk. Akad. Handl., **27**, No. 8, Stockholm, 1895.

Déterminations des éléments magnétiques effectuées sur la glace de quelques lacs en Suède pendant l'hiver 1889, Bihang till K. Svenska Vetensk. Akad. Handl., **20**, I, No. 8, Stockholm, 1895.

Déterminations des éléments magnétiques dans la Suède méridionale pendant l'année 1892, Nova acta Reg. Soc. scient., Ser. 3, Upsal, 1896.

Sur la forme analytique de l'attraction magnétique de la Terre, exprimée en fonction du temps, Astron, iaktt. och unders. på Stockholms Observatorium, **5**, No. 5, Stockholm, 1896.

Om jordmagnetiska kraften i Sverige, Teknisk Tidskrift. Avd. Kemi och Bergvetenskap, **27**, No. 48, 1897, Stockholm, 1897.

Om jordmagnetiska kraftens förändringar, Teknisk Tidskrift. Avd. Kemi och Bergvetenskap, **27**, No. 52, 1897, Stockholm, 1897.

Verzeichniss der Schriften von H. Gyldén, Astr. Ges., Vierteljahrschr., **32**, Leipzig, 1897.

Catalogue des oeuvres et écrits de Hugo Gyldén, Bull. Astronom., 1897, Paris, 1897.

Om norrskenet, Ord och Bild, **7**, Stockholm, 1898.

Travaux de l'expédition Suédoise au Spitzberg en 1898 pour la mesure d'un arc du méridien.
1. Déterminations du temps et d'une longitude fondamentale.
2. Geologiska anteckningar.
3. Déterminations des éléments magnétiques.
4. Déterminations de latitudes et de longitudes.
5. Meteorologiska observationer.
6. Iakttagelser på tidvattnet.
Övers. av K. Vetensk. Akad. Förhandl., Stockholm, 1899-1900.

På attionde breddgraden. En bok om den svensk-ryska gradmätningen på Spetsbergen, Stockholm, 1900.

Uppmätningen av en meridianbåge på Spetsbergen genom en svensk-rysk expedition, Ymer, **20**, Stockholm, 1900.

Sur l'état actuel de nos connaissances des variations séculaires du magnétisme de la Terre, Congrès internat. de météorol., Proc. verb. et Mém., Paris, 1900.

Proposition faite à la Conférence Internationale du Magnétisme terrestre à Innsbruck sur l'organisation d'observations magnétiques temporaires dans les régions arctiques et antarctiques, Internat. met. Direktoren-Konferenz, Innsbruck 1905, Ber., App. 28, Wien, 1906.

Note sur le potentiel magnétique de la Terre exprimé en fonction du temps. Arkiv. Mat., Astr., Fys., **3**, No. 7, Stockholm, 1906.

Sur le choix des stations polaires pour l'étude des variations périodiques du champ magnétique terrestre, Congrès internat. pour l'étude des régions polaires, Rapport d'ensemble, Bruxelles, 1906.

Sur les latitudes qu'il convient de choisir pour les observatoires magnétiques. Konferenz Intern. met. Direkt. Ber., App. 27, Wien, 1906.

Sur l'origine des spectres de bandes, K. Vetensk. Akad. Handl., N. F., **42**, No. 8, Uppsala and Stockholm, 1907.

Sur les luminiscences causées par les rayons-canaux, Kungl. Vetensk. Akad., Ark. f. Mat., **4**, No. 33, Uppsala and Stockholm, 1908.

Ernst Danielson, K. Vetensk. Akad., Årsbok, 1910, Uppsala and Stockholm, 1910.

A brief account of a magnetic survey of the iron-ore field of Kiirunavaara: Scientific and practical researches in Lapland arranged by Luossavaara-Kiirunavaara Aktiebolag, Stockholm, 1910.

Berättelse i fysik, våren 1907-1909, Stockholms Högskola, Stockholm, 1910.

Sur une cause possible de l'influence des éclipses de soleil sur le magnétisme terrestre, Arkiv Mat., Astr., Fys., **10**, No. 9, Uppsala and Stockholm, 1914.

Résultats sommaires d'une investigation du gisement de fer à Kiirunavaara en Laponie, Arkiv Mat., Astr., Fys., **10**, No. 13, Stockholm, 1914.

Sur la diffusion des radiations dans l'atmosphère du Soleil, K. Vetensk. Akad., Arkiv f. Mat., **10**, No. 12, Uppsala and Stockholm, 1914.

De l'influence des minérais de fer dans l'écorce terrestre sur les perturbations magnétiques, Arkiv Mat., Astr., Fys., **11**, No. 20, Uppsala and Stockholm, 1916.

Nobelpristagern i fysik 1920 Charles-Édouard Guillaume, Teknisk Tidskr., **50**, Stockholm, 1920.

Om den tychoniska foten, Pop. astr. tidskrift, **2**, Stockholm, 1921.

Om bevarandet av ruinerna efter Tycho Brahes observatorium på Hven, Pop. astr. tidskrift, 1921, Stockholm, 1921.

Magnus Nyrén, Några minnesord, Pop. astr. tidskrift, 1921, Stockholm, 1921.

450-de årsdagen av Copernicus födelse, Pop. astr. tidskrift, Stockholm, 1923.

Hydrographic Service,
Stockholm, Sweden.

PROVISIONAL SUNSPOT-NUMBERS FOR DECEMBER, 1934, TO FEBRUARY, 1935

(Dependent alone on observations at Zürich Observatory and its station at Arosa)

Day	Dec.	Jan.	Feb.	Day	Dec.	Jan.	Feb.
1	28	35	..	17	0	14[a]	10
2	21	..[d]	..	18	0	12	7
3	12	27	..	19	..	10	*M*19[c]
4	14	..	*M*[c]	20	*E*15[c]	9	28
5	*W*14[c]	..	..[a]	21	..	7	21
6	10	21	29	22	21	14	7?
7	13	19[a]	46	23	..	16[ad]	*M*17[c]
8	11	11	29	24	27[a]	31	17[a]
9	0	11	23	25	*E*50[cc]	32	..[d]
10	0	9	28?	26	39	31	18?
11	0	19[d]	21	27	29	26	20
12	0	..	*W*26[cd]	28	34[a]	24[a]	20
13	0	24	18	29	32	..	
14	0	..	..	30	37[a]	9	
15	..	16	*M*22[c]	31	30	10	
16	0	16	20				
				Means	16.2	18.1	21.2
				No. days	27	25	21

Mean for quarter October to December, 1934: 10.1 (84 *days*)

Mean for year 1934: 8.8 (343 *days*)

[a]Passage of an average-sized group through the central meridian.
[b]Passage of a large group or spot through the central meridian.
[c]New formation of a center of activity: *E*, on the eastern part of the Sun's disc; *W*, on the western part; *M*, in the central-circle zone.
[d]Entrance of a large or average-sized center of activity on the east limb.

EIDGEN. STERNWARTE,
Zürich, Switzerland

W. BRUNNER

AMERICAN *URSI* BROADCASTS OF COSMIC DATA[1] OCTOBER TO DECEMBER, 1934

The data for terrestrial magnetism, sunspots, solar constant, and auroræ are the same as given in previous tables.

The first three columns of the Table give (1) the magnetic character according to the scale 0-2 of the International Commission of Terrestrial Magnetism and Atmospheric Electricity, (2) the type featuring the day other than normal by the letters *b*, *p*, *o*, and *i* for days marked by bay, rapid pulsations, long-period oscillations, and irregular oscillations, respectively, and (3) the hour and minute of Greenwich mean time marking the beginning of a storm, the end of the storm being indicated in the footnote to the Table. The next two columns give the data relating to sunspots: (1) the number of groups of spots and (2) the total number of spots. It is to be noted that sunspot-numbers such as those from Zürich can be obtained from the number of groups and spots given in the Table by the formula $N=k\ (10g+s)$, where k for Mount Wilson is about 0.7.

[1]For previous announcements see Terr. Mag., **35**, 184-185 and 252-253 (1930); **36**, 54, 141, 258-259, and 358-360 (1931); **37**, 85-89, 189-192, 408-411, and 484-487 (1932); **38**, 60-63, 148-151, 262-265, 335-339 (1933); **39**, 73-77, 159-163, 244-247, and 353-356 (1934).

Mount Wilson Observatory is now supplying corrections and additions to the sunspot-data which are broadcast in the *URSI*gram. So far as possible, these additional and corrected values will be used in this tabular summary and will be designated as such in footnotes to the Table.

Beginning January 1, 1934, the magnetic information for the *URSI*-gram is for Cheltenham, Maryland, instead of Tucson, Arizona. In addition to this change in observatory, the data cover the 24 hours ending 8 A. M., 75° west meridian mean time, instead of the 24 hours ending at 7 A. M., 105° west meridian mean time.

The columns headed solar constant show (1) the value in calories of the solar constant, and (2) by letters *s*, *f*, and *u* whether the determination was satisfactory, fair, or unsatisfactory, respectively.

In accordance with information received from Dr. C. G. Abbot, Secretary of the Smithsonian Institution, transfer from Table Mountain to Montezuma solar-constant values was made as of October 23, 1934. Table Mountain for a considerable time has been 0.012 calorie above

Summary American URSI daily broadca

October

Date	Magnetism: Character	Magnetism: Type	Magnetism: G. M. T. begin. distur.	Sun-spot: Groups	Sun-spot: Number	Solar constant: Value	Solar constant: Character	Aurora: Character	Aurora: Duration	Aurora: Cloudiness	Aurora: Form: Without rays	Aurora: Form: With rays	Aurora: Area covered	Aurora: Av. altitude	Aurora: Position	Aurora: G.M.T. greatest distur
			h m			*cal*			*hrs*					°		*h*
1	0	...	.. ..	2*	2*	1.958	*f*	1	5	3	*HV*	.....	0.4	35	*W-N-E*	10
2	0	...	.. ..	0	0	1.959	*s*	9	0	10	.....	.....	...	...	....	....
3	0	...	.. ..	0	0	......	...	1	2	6	*HA*	.....	0.2	25	*NW-N-E*	10
4	0	...	.. ..	0	0	1.959	*f*	1	6	3	*HV*	.....	0.6	20	*NW-N-E*	11
5	0	...	.. ..	0	0	1.952	*s*	3	6	5	*HV*	*RB*	0.6	40	*W-N-E*	9
6	0	...	.. ..	...	...	......	...	1	6	1	*HV*	.....	0.2	15	*NW-N-E*	9
7	0	...	.. ..	0	0	1.951	*s*	0	0	8	.....	.....	...	...	....	....
8	0	...	.. ..	1	2	1.949	*f*	1	4	4	*HV*	.....	0.2	20	*NW-N-E*	10
9	0	...	.. ..	0	0	1.953	*s*	9	0	10	.....	.....	...	...	....	....
10	0	...	.. ..	0	0	1.963	*f*	9	0	10	.....	.....	...	...	....	....
11	0	...	.. ..	1*	1*	......	...	9	0	10	.....	.....	...	...	....	....
12	0	...	.. ..	1	1	......	...	1	3	6	*HV*	.....	0.4	45	*NW-N-E*	12
13	1	...	.. ..	1	1	1.961	*s*	3	5	4	*HV*	.....	0.4	25	*NW-N-E*	11
14	1	...	.. ..	...	...	1.957	*f*	9	0	10	.....	.....	...	...	....	....
15	0	...	.. ..	1	2	1.967	*f*	3	4	5	*HV*	.....	0.2	25	*NW-N-E*	7
16	0	...	.. ..	...	...	1.969	*f*	9	0	10	.....	.....	...	...	....	....
17	0	...	.. ..	...	...	......	...	9	0	10	.....	.....	...	...	....	....
18	0	...	.. ..	...	...	......	...	1	3	8	*HV*	.....	0.2	20	*NW-N-E*	12
19	0	...	.. ..	0*	0*	1.950	*s*	9	0	10	.....	.....	...	...	....	....
20	0	...	.. ..	...	...	1.954	*f*	9	0	10	.....	.....	...	...	....	....
21	0	...	.. ..	0	0	1.955	*s*	9	0	10	.....	.....	...	...	....	....
22	0	...	.. ..	1	2	1.949	*s*	1	1	8	*HB*	.....	0.2	20	*NW-N-E*	8
23	0	...	.. ..	1	2	1.950[a]	*s*	0	0	7	.....	.....	...	...	....	....
24	1			0	0	1.951	*s*	9	0	10						
25	1	...	.. ..	1[b]	1[b]	1.949	*s*	9	0	10	.....	.....	...	...	....	....
26	1	...	.. ..	1[b]	1[b]	......	...	3	6	2	*HV*	.....	0.6	25	*W-N-SE*	9
27	0	...	.. ..	1[b]	1[b]	1.947	*s*	1	1	9	*HB*	.....	0.2	15	*NW-N*	10
28	0	...	.. ..	1[b]	1[b]	1.958	*f*	1	2	4	*HV*	.....	0.2	30	*NW-N-E*	14
29	0	...	.. ..	1[b]	1[b]	......	...	0	0	9	.....	.....	...	...	....	....
30	0	...	.. ..	1[b]	2[b]	1.955	*u*	1	3	5	*HV*	.....	0.2	15	*NW-N-E*	8
31	0	...	.. ..	...	...	1.959	*u*	1	3	6	*HB*	.....	0.2	10	*NW-N-NE*	9
Mean	0.2	...	.. ..	0.6	0.8	1.955	...	4.3	2.0	7	.....	.....	...	...	....	10

November

Date	Magnetism: Character	Magnetism: Type	Magnetism: G. M. T. begin. distur.	Sun-spot: Groups	Sun-spot: Number	Solar constant: Value	Solar constant: Character	Aurora: Character	Aurora: Duration
			h m			*cal*			*hrs*
1	0	...	.. ..	1	2	1.954	*f*	0	0
2	0	...	.. ..	1	9[c]	1.949	*f*	1	1
3	0	...	.. ..	1	7	1.960	*f*	3	6
4	0	...	.. ..	1	5	......	...	9	0
5	0	...	.. ..	1	3	1.956	*s*	9	0
6	0	...	.. ..	1	7	1.954	*f*	9	0
7	0	...	.. ..	1	2	1.946	*f*	1	2
8	1	...	.. ..	1	2	1.944	*s*	1	3
9	1	...	.. ..	1*	3*	......	...	1	5
10	0	...	.. ..	1	1	1.973	*f*	1	7
11	0	...	.. ..	1	1	......	...	1	1
12	0	...	.. ..	1	1	......	...	3	8
13	0	...	.. ..	0	0	......	...	1	2
14	0	...	.. ..	0	0	1.949	*s*	1	1
15	0	...	.. ..	1	2	1.952	*f*	1	5
16	0	...	.. ..	...	...	......	...	5	11
17	0	...	.. ..	...	...	......	...	3	4
18	0	...	.. ..	...	...	......	...	9	0
19	0	...	.. ..	...	...	1.958	*f*	1	1
20	0	...	.. ..	0	0	1.958	*f*	0	0
21	0	...	.. ..	0	0	1.950	*s*	1	1
22	0	...	.. ..	0*	0*	1.948	*f*	0	0
23	0	...	.. ..	0	0	1.945	*f*	1	1
24	0			0	0			0	0
25	0	...	.. ..	2	8	1.951	*f*	3	5
26	0	...	.. ..	1	3	1.953	*f*	1	2
27	0	...	.. ..	2	4	1.951	*f*	1	5
28	0	...	.. ..	2	9	1.950	*s*	3	5
29	0	...	.. ..	3	20	1.950	*u*	9	0
30	0	...	.. ..	3	17	......	...	9	0
31									
Mean	0.0	...	.. ..	1.0	4.1	1.953	...	2.9	2.5

Greenwich mean time for ending of storms: 4[h], Decembr 2; 8[h], December 4. *A revision of value originally broadcast.

Montezuma, and above the scale of 1913 to 1930. Hence the value of October 23 and succeeding values are on a scale 0.012 calorie lower than those of recent months.

Under the general heading of aurora in the Table, the first column gives the character of the day: 0 indicates no aurora; 1, faint; 3, moderate; 5, strong; 7, brilliant; and 9, no observation or no observations possible on account of cloudiness. The second column gives the number of hours during which aurora was present. The third column indicates the amount of sky covered by cloud on a scale of 0-10, where 0 means cloudless, and 10 completely overcast.

Columns four and five describe by letters the form of the aurora, column four indicating forms without ray structure and column five, forms with ray structure. The letters employed are the same as those used in the Photographic Atlas of Auroral Forms published by the International Union of Geodesy and Geophysics, Oslo, 1930, so far as it was possible to use those letters. For forms without ray structure *HA*

f cosmic data, October to December, 1934

November						December																	Date
Aurora						Magnetism				Sun-spot		Solar constant		Aurora									
Form: With-out rays	Form: With rays	Area covered	Av. altitude	Position	G.M.T. greatest distur.	Character	Type	G. M. T. begin. distur.		Groups	Number	Value	Character	Character	Duration	Cloudiness	Form: with-out rays	Form: with rays	Area covered	Av. altitude	Position	G. M. T. greatest distur.	
			°		h			h	m			cal			hrs					°		h	
....					....	0	...	..	..	3	8	1.941	f	5	3	5	HV	R	0.6	50	W-N-SE	9	1
HA	RA	0.2	15	NW-N-NE	12	0	b	3	15	2	5	1.950	f	1	2	5	HA	.....	0.2	25	NW-N-NE	14	2
HV	.....	0.4	30	W-N-E	12	0	...	..	..	1	7	1.944	s	1	1	9	HA	.....	0.2	12	NW-N-NE	6	3
....	.....	...	..	....	....	2	...	22	00	1	5	1.949	f	1	1	10	HB	.....	0.2	90	NW-SE	5	4
....	.....	...	..	....	....	0	...	..	..	...	...	1.945	f	1	1	9	DS	.....	0.2	25	N-NE	10	5
....	.....	...	..	....	....	0	...	..	..	1	4	1.947	f	1	6	3	HV	.....	0.2	12	NW-N-NE	8	6
HV	.....	0.2	20	NW-N-NE	9	0	...	..	..	1*	5*	......	...	3	9	1	HV*	RB	0.4	30	NW-N-E	10	7
HV	R	0.4	50	W-N-SE	10	0	...	..	..	...	...	1.951	s	1	1	9	HA	.....	0.2	10	NW-N-NE	12	8
HB	.....	0.2	18	NW-N-E	7	0	...	..	..					9	0	10							9
HV	.....	0.2	10	NW-N-E	9	0	...	..	..	...	...	1.947	u	1	8	1	HV	RB	0.2	15	NW-N-E	9	10
HB	.....	0.2	10	NW-N-NE	6	0	...	..	..	0	0	1.959	s	9	0	10	.....	.....	...	..	....	....	11
HV	.....	0.2	20	NW-N-E	10	0	...	..	..	...	...	1.951	f	9	0	10	.....	.....	...	..	....	....	12
HV	.....	0.2	30	NW-N-E	11	0	...	..	..	...	...	1.950	s	9	0	10	.....	.....	...	..	....	....	13
HA	.....	0.2	15	NW-N-E	14	0	...	..	..	...	...	1.952	u	0	0	5	.....	.....	...	..	....	....	14
HV	.....	0.4	25	NW-N-E	11	0	...	..	..	0*	0*	......	...	3	8	2	HV	RB	0.4	25	NW-N-E	11	15
HV	R	0.2	30	NW-NE-SE	9	0	...	..	..	0	0	......	...	1	1	1	HA	.....	0.2	15	NW-N-NE	12	16
HV	RB	0.2	10	NW-N-NE	6	0	...	..	..	0	0	1.947	f	1	2	0	HV	.....	0.2	12	NW-N-NE	10	17
....	.....	...	..	....	....	0	...	..	..	...	...	1.945	f	1	2	0	HB	.....	0.2	10	NW-N-NE	10	18
HA	.....	0.2	30	NW-N-E	14	0	...	..	..	0	0	1.953	f	1	1	1	HB	.....	0.2	15	NW-N-E	10	19
....	.....	...	..	....	....	0	...	..	..	2*	8*	1.954	s	0	0	0	.....	.....	...	..	....	....	20
HA	RB	0.2	15	NW-N-NE	11	0	...	..	..	3	16	......	...	1	2	0	HB	RB	0.2	20	NW-N-E	11	21
....	.....	...	..	....	....	0	...	..	..	2*	4*	1.952	f	1	2	0	HV	.....	0.2	15	NW-N-E	8	22
HA	RB	0.2	6	N-NE	10	0	...	..	..	3	12	......	...	1	1	0	HB	.....	0.2	10	N-NE	10	23
....	.....	...	..	....	....	0	...	..	..	6*	19*	......	...	3	6	0	HV	R	0.2	30	NW-NE-E	16	24
HV	RB	0.6	30	W-N-E	10	1	...	..	..	8	30	......	...	1	6	0	HV	.....	0.2	15	NW-N-E	12	25
HV	.....	0.2	10	N-NE	8*	1	...	..	..	6	34	1.951	u	1	2	6	HB	RA	0.2	20	NW-N-E	11	26
HV	.....	0.2	20	NW-N-E	9	0	...	..	..	...	...	......	...	1	1	7	HB	.....	0.2	8	NW-N-NE	11	27
HV	.....	0.6	35	W-N-E	9	0	...	..	..	*	*	......	...	9*	0*	10*	.....	.....	...	..	....	....	28
....	.....	...	..	....	....	0	...	..	..	3*	21*	......	...	5	10	2	HV	R	0.6	60	W-N-SE	12	29
....	.....	...	..	....	....	1	...	..	..	3*	13*	......	...	5	11	0	HV	R	0.4	60	SW-N-SE	13	30
						1	...	..	..	4*	13*	......	...	1	7	0	HV	.....	0.2	15	NW-NE-SE	9	31
.....	.....	...	..	....	10	0.2	...	..	..	2.3	9.7	1.949	...	2.8	3.0	4	.....	.....	...	..	....	10	Mean

[a]Change from Table Mountain to Montezuma. [b]Old cycle, [c]New cycle.

Kennelly-Heaviside Layer heights, Washington, D. C., October to December, 1934
(Nearest hour, Greenwich mean time, of all observations is 17)

Date	Frequency	Height	Date	Frequency	Height	Date	Frequency	Height
1934	*kc/sec*	*km*	*1934*	*kc/sec*	*km*	*1934*	*kc/sec*	*km*
Oct. 3	2,500	120	Oct. 31	3,000	*	Nov. 28	4,300	270
" "	3,100	130	" "	3,030	230	" "	4,500	250
" "	3,110	*	" "	3,100	200	" "	5,000	250
" "	3,200	300	" "	3,930	290	" "	5,700	240
" "	3,400	200	" "	4,500	250	" "	6,000	270
" "	4,300	370	" "	5,500	270	" "	6,400	440
" "	4,500	330	" "	6,000	280	" "	6,500	*
" "	5,700	340	" "	6,300	270, 350	Dec. 5	2,500	130
" "	5,900	330, 430	" "	7,000	320	" "	2,800	150, 330
" "	6,600	440	" "	7,600	370	" "	3,350	220
" "	6,700	*	Nov. 7	2,500	130	" "	4,300	310
" 10	2,700	120	" "	2,920	330	" "	4,500	240
" "	3,100	180	" "	3,100	180	" "	5,300	250
" "	3,150	*	" "	3,190	320	" "	5,900	270
" "	3,200	210	" "	3,500	230	" "	6,300	310
" "	3,400	190	" "	4,200	300	" "	6,500	420
" "	4,100	360	" "	4,500	240	" "	6,700	*
" "	4,500	300	" "	5,100	250	" 12	2,500	140
" "	5,000	280	" "	5,300	280	" "	2,600	*
" "	5,500	260	" "	7,300	300	" "	2,700	240
" "	6,000	270	" "	7,500	310, 430	" "	3,100	160
" "	6,400	300, 420	" "	8,300	340	" "	3,130	280
" "	7,000	320	" "	8,700	700	" "	3,300	240
" "	7,300	410	" 13	2,500	110	" "	3,900	270
" "	7,400	*	" "	2,800	120, 250	" "	4,500	250
" 17	2,780	110	" "	3,150	110, 190	" "	5,300	240
" "	2,800	170	" "	3,310	310	" "	6,300	250
" "	3,000	190	" "	3,550	240	" "	7,300	270
" "	3,050	240	" "	4,000	280	" "	7,500	340
" "	3,170	200	" "	4,400	270	" "	7,700	*
" "	3,220	300	" "	4,900	250	" 19	2,800	110
" "	3,450	190	" "	6,900	280	" "	3,200	120, 230
" "	4,200	300	" "	7,300	280, 350	" "	4,200	120, 270
" "	4,700	260	" "	7,700	310	" "	4,500	250
" "	5,100	270, 490	" "	7,900	370	" "	5,500	250
" "	6,000	290	" 21	2,500	130	" "	6,700	250
" "	6,700	410	" "	3,000	170	" "	7,900	250
" "	7,100	780	" "	3,180	180, 220	" "	8,300	310
" 24	2,800	130	" "	3,400	250	" "	8,500	*
" "	2,850	130, 270	" "	4,000	280	" 26	2,700	130
" "	3,150	180	" "	4,600	280	" "	2,890	270
" "	3,380	370	" "	5,000	260	" "	3,100	170
" "	3,600	280	" "	5,500	270	" "	3,200	270
" "	4,030	560	" "	5,900	280, 370	" "	3,450	220
" "	4,500	350	" "	6,200	300, 490	" "	3,750	280
" "	4,900	370	" "	6,600	360	" "	4,400	260
" "	5,100	480	" "	7,000	430	" "	4,500	230
" "	5,200	400, 610	" "	7,100	*	" "	5,100	250
" "	5,500	400	" 28	2,500	130	" "	6,100	240
" "	5,800	430	" "	2,800	160	" "	6,900	270
" "	6,000	720	" "	2,920	290	" "	7,500	280
" "	6,100	*	" "	3,080	190	" "	7,900	320
" 31	2,900	130	" "	3,180	270	" "	8,100	*
			" "	3,300	240			

* = No value obtained.

indicates homogeneous quiet arcs; *HB*, homogeneous bands; *PA*, pulsating arcs; *DS*, diffuse luminous surfaces; *PS*, pulsating surfaces; *G*, feeble glow; *HV*, varied forms; *HF*, flaming aurora; and *HVF*, varied forms with flaming. For forms with ray structure *RA* indicates arcs; *RB*, bands; *D*, draperies; *R*, rays; *C*, corona; *RV*, varied forms; *RF*, flaming aurora; and *RVF*, varied forms with flaming.

Column six gives the maximum area of sky covered in tenths of the whole sky, column seven the average altitude in degrees, and column eight the general position of the aurora, being reckoned for included positions in a clockwise direction with *Z* representing zenith and *A* the whole sky. The final column gives the Greenwich mean hour of the observed greatest display in the preceding 24 hours of the Greenwich day.

All indicated revisions of auroral data as originally broadcast are in accordance with written report to the Department from Professor Veryl R. Fuller, Alaska Agricultural College and School of Mines, Fairbanks, Alaska.

On January 1, 1935, Professor Fuller reported by radio: "Since today's auroral data conclude the period covered by the original grant of funds for auroral study and as there have been no further grants made for continuation of the study, today's message will be the last until such time as additional funds are made available."

The table of Kennelly-Heaviside Layer heights is self-explanatory.

C. C. Ennis

Department of Terrestrial Magnetism,
Carnegie Institution of Washington,
Washington, D. C.

PROVISIONAL SOLAR AND MAGNETIC CHARACTER-FIGURES, MOUNT WILSON OBSERVATORY, OCTOBER, NOVEMBER, AND DECEMBER, 1934

Greenwich mean time							Range
Beginning			Ending				Hor. int.
1934	*h*	*m*	*d*	*h*	*m*		γ
Oct. 24	0	..	24	17	..		117
Dec. 3	21	..	4	16	..		148
Dec. 29	5	..	30	22	..		153

No exceptionally bright hydrogen flocculi were observed during or just prior to any of the storms recorded during the last quarter of 1934. No spots were seen on October 24, but a small group was observed on October 22 and 23 in an area which crossed the central meridian on October 24. On December 3 the only spots on the Sun were included in a group in latitude 28° south which was then 65° west of the central meridian. On December 29 three groups were observed. The positions of the two largest were 14° west and 31° south and 15° east and 24° south.

On December 11 at $10^h\ 34^m.5$, G. M. T., the horizontal intensity increased suddenly by 16γ in a manner similar to a "sudden commencement" but no magnetic storm followed.

Day	October 1934							
	K_2		$H\alpha B$		$H\alpha D$		No. groups	Mag'c char.
	A	B	A	B	A	B		
1	1.5	0.5	1	0	1	1.5	2	0.5
2	1	0.5	1	0	1	2	0	0
3	1	1	1	1	1	2.5	0	0
4	1	1	1?	1?	1?	2.5?	0	0
5	1	1	1	1.5	1	2	0	0
6	...	...	...	...	...	...		0
7	1	1	1	1	1	0.5	0	0.5
8	0.5	0.5	1.5	1.5	1	0	1	0
9	0.5	0.5	1	1	1.5	0	0	0
10	0	0	...	...	...	...	0	0
11	0	0	0	0	0.5	0	1	0
12	0	0	0	0	1	0	1	0
13	0	0	0	0	1	0	1	0
14	...	...	...	...	...	...	...	0
15	0.5	0	0	0	1.5	0	1	0.5
16	...	...	...	...	...	...	...	0
17	...	...	...	...	...	...	...	0
18	...	...	...	...	...	...	...	0
19	1	0.5	0	0	1	0	0	0
20	1	0	0.5	0	0.5	0	0	0
21	1	0.5	1	0	0.5	0	0	0
22	1	0.5	1	0	0.5	0	1	0
23	1	0.5	0.5	0	0.5	0	1	0
24	0.5	0.5	0	0	0.5	0	0	1
25	0.5	1	0.5	0	0.5	0.5	1	0.5
26	...	...	0.5	0	0.5	0.5	1	0.5
27	0.5	0	0.5	0	0.5	0	1	0
28	0.5	0	0.5	0	1	0	1	0
29	0.5	0.5	0.5	0.5	1	0	1	0
30	0.5	1	0.5	1	0.5	1	1	...
31	...	...	...	...	...	...	...	0
Mean	0.7	0.5	0.6	0.4	0.8	0.5	0.6	0.1

Day	November 1934							
	K_2		$H\alpha B$		$H\alpha D$		No. groups	Mag'c char
	A	B	A	B	A	B		
1	...	...	...	...	...	...	1	0
2	...	...	...	...	...	...	1	0
3	0.5	0	1	0	1	0	1	0
4	0.5	0	1	0	1	0	1	0
5	0.5	0.5	0.5	0.5	1	0	1	0
6	1	1	1	1	2	0	1[a]	0
7	1	1	1	1	2.5	0	1	1
8	1	0.5	1	0.5	2.5	0	1	0.5
9	...	...	...	...	...	...	1	0.5
10	1	0	1	0	2.5	0	1	0
11	1	0	0.5	0	2	0	1	0
12	0.5	0.5	0.5	0	2	0	1	0
13	0.5	0.5	0	0	2	0	0	0
14	0	0	0	0	2	0	0	0
15	0	0.5	0	0	2	0	1	0
16	...	...	...	...	...	...	...	0
17	...	...	...	...	...	...	...	0
18	...	...	...	...	...	...	...	0
19	...	...	...	...	...	...	...	0
20	0	0	0	0	0.5	0	0	0
21	0.5	0	0.5	0	1	0	0	0
22	0.5	0	0.5	0	0.5	0.5	0	0
23	0.5	0	0.5	0	0.5	0	0	0
24	0.5	0.5	1	0.5	0.5	0	0	0.5
25	1	1	1	1	0.5	0.5	2	0
26	1	1.5	1.5	1.5	1	1	1	0
27	...	...	...	...	...	...	2	0
28	2	0	2	0	0.5	0	2	0
29	...	...	...	...	...	...	3	0
30	1	0	1.5	0	0.5	0	3	0
31								
Mean	0.7	0.4	0.8	0.3	1.3	0.1	1.0	0.1

Day	December 1934							
	K_2		$H\alpha B$		$H\alpha D$		No. groups	Mag'c char.
	A	B	A	B	A	B		
1	1	0.5	1	0	0.5	0	3	0.5
2	1	0.5	1	0.5	0.5	0	2	0.5
3	1	0.5	1	0	0	0	1	0.5
4	0.5	0.5	0.5	0	0.5	0	1	1.5
5	...	...	...	...	...	...	...	0
6	0.5	0	1	0	0.5	0	1	0
7	...	...	...	...	...	...	1	0
8	...	...	...	...	...	...	...	0
9	...	...	...	...	...	...	...	0
10	...	...	...	...	...	...	...	0
11	...	...	...	...	...	...	0	0.5
12	...	...	...	...	...	...	...	0
13	...	...	...	...	...	...	...	0
14	...	...	...	...	...	...	...	0
15	...	...	...	...	...	...	0	0.5
16	0	0	0	0	1	0	0	0
17	0	0	0	0	0.5	0	0	0
18	...	...	...	...	...	...	...	0
19	0.5	0	0.5	0	1	0	0	0
20	1	0	1	0	2	0	2	0
21	1	0	2	0	1.5	0	3	0.5
22	...	...	...	...	...	...	2	0
23	2	1	2	1	1.5	0.5	3[b]	0
24	2	1.5	...	...	...	...	6	0
25	2.5	2	2.5	2	2	0.5	8[c]	0.5
26	2.5	1.5	2.5	1.5	0.5	0	6[c]	0.5
27	...	...	...	...	...	...	...	0
28	...	...	...	...	...	...	...	0
29	...	...	...	...	...	...	3	1.5
30	2	2	2	2	0.5	0	3[a]	0.5
31	2	2	2	2	0.5	0	4	0.5
Mean	1.2	0.8	1.3	0.6	0.9	0.1	2.3	0.3

NOTE—For an explanation of these tables see this JOURNAL, **35**, 47-49 (1930).

?Indicates an uncertain value which should be given low weight.

[a, b] Passage of an average-sized group through the central meridian within 20°, 30° of the center of the disc, respectively. [c, d] Passage of a small group through the central meridian within 25°, 35° of the center of the disc, respectively.

AURORA OF JANUARY 27, 1935, AT ALEXANDRIA BAY, NEW YORK

As the aurora has been a rather rare occurrence for several months, I thought the following report might be of interest.

Between 19^h and 20^h 30^m January 27, a quite brilliant and active aurora was observed at Alexandria Bay, New York (latitude 44° 20′ north, longitude 75° 55′ west) beginning promptly at 19^h. It started as a pale green arch, the lower segment of which was about 20° exactly over the northern horizon, and could be designated as *IIA* (2) according to the Photographic Atlas of Auroral Forms, published by the International Union of Geodesy and Geophysics. It gradually became brighter and higher and was soon accompanied with streamers of greenish hue, *RA* (25-26 of the Atlas). At this time patches of intense green light would develop with tinges of pink and yellow, such bundles of rays seeming to turn and move rapidly about and to the west. All this time this aurora was working higher towards the zenith but never higher than 35° to 40°. At one time a patch of whitish light was seen in the east near the zenith but this soon died away. After considerable activity with constantly changing rays kindling and dying out with at times intense colors of greenish hue, the aurora gradually spread all over the northern sky and then finally died out entirely. At 20^h 30^m there was only a faint glow. At the time of observation the sky was absolutely clear with light northwest winds and the temperature −18°F.

U. S. Weather Bureau,
Alexandria Bay, New York

Douglas F. Manning

THE GREEN FLASH

The phenomenon of the green flash was looked for on many occasions during the Polar Year 1932-33 by the Canadian Party at Chesterfield, Northwest Territories, but it was observed only once. This was at sunrise September 1, 1933, at 4:45 a. m. (90° west meridian time). It was seen by J. Rea and the undersigned from the deck of a small schooner in Hudson Bay. A vivid green point changing quickly to a fairly sizable ball of green fire preceded the first glimpse of the Sun. The whole occurrence lasted from two to three seconds. There was hardly any wind and the sea was calm. Long billows of alto-cumulus clouds running in a northwest to southeast direction covered nine-tenths of the sky, but for three or four degrees above the eastern horizon the sky was clear.

F. T. Davies

Department of Terrestrial Magnetism,
Carnegie Institution of Washington,
Washington, D. C.

PRINCIPAL MAGNETIC STORMS

Sitka Magnetic Observatory
October to December, 1934[1]

(*Latitude 57° 03'.0 N., longitude 135° 20'.1 or 9^h 01^m.3 W. of Gr.*)

October 24—The disturbance, which began gradually at about 4^h, G. M. T., was marked by slow fluctuations of the elements until about 8^h 40^m, when the amplitudes abruptly became much larger and the periods shorter. In about two hours the violent fluctuations ceased, and by 17^h the storm had ended. The ranges were: Declination, 69′; horizontal intensity, 338γ; vertical intensity, 697γ.

December 3-4—This storm also began gradually, about 23^h, G. M. T., December 3. The first part was characterized by small, short-period oscillations of the declination. All the elements began to fluctuate violently about 1^h 45^m, December 4, but became relatively quiet again within half an hour, and remained so until about 4^h, when another period of rapid fluctuations began, lasting until about 8^h. The storm ended about 16^h. The ranges were: Declination, 108′: horizontal intensity, 645γ; vertical intensity, 590γ.

John Hershberger, *Oberver-in-Charge*

Cheltenham Magnetic Observatory
October to December, 1934[1]

(*Latitude 38° 44'.0 N., longitude 76° 50'.5 or 5^h 07^m.4 W. of Gr.*)

There was only one disturbance of note during the quarter, namely, that recorded during December 3-4. It began at 22^h, G. M. T., December 3 and ended the following day at 8^h. The ranges were: Declination, 47′.9: horizontal intensity, 148γ: vertical intensity, 147γ.

Eoline R. Hand, *Oberver-in-Charge*

[1] Communicated by R. S. Patton, Director, United States Coast and Geodetic Survey.

Huancayo Magnetic Observatory
October to December, 1934

(*Latitude 12° 02'.7 S., longitude 75° 20'.4 or 5^h 01^m.4 W. of Gr.*)

Greenwich mean time						Range		
Beginning			Ending			Decl'n	Hor. int.	Ver. int.
1934	*h*	*m*	*d*	*h*	*m*	′	γ	γ
Nov. 7	1	..	7	23	..	8	193	27
Dec. 1	4	56	1	18	40	7	110	31
Dec. 3	11	50	4	20	..	10	232	56
Dec. 11	10	34	11	22	..	6	182	27
Dec. 29	12	..	30	22	..	9	360	69

November 7—A mild disturbance (character 1) of short duration began with minor fluctuations of rather long period in *H* but was not perceptible in *D* or *Z*. Perturbations in *H* were rapid and comparatively large between 13^h and 18^h, there being five bays with from 50γ to 100γ

range from crest to trough during that interval, the period of each bay (crest to crest) being about 30 minutes. Rapid, small oscillations were superposed on these bays. D and Z experienced minor perturbations during the same interval, not greatly affecting the general diurnal variation. The following day, especially during daylight-hours (12^h to 21^h, G. M. T.), was somewhat disturbed also, but not unusually so, considering the character of November as a whole.[2]

December 1—A mild disturbance (character 1) of short duration began with a sudden commencement in all three elements at 4^h 56^m, G. M. T., with increase in H of 29γ in seven minutes, decrease in D of $0'.3$, and increase in Z of 3γ. Following the commencement, oscillations of small amplitude in H were superposed on four bays each of period 50 to 60 minutes, having a range from crest to trough of about 50γ. December 2, between 1^h and 18^h, G. M. T., was irregular without affecting the normal diurnal trend.

December 3-4—A mild disturbance (character 1) of considerable duration was recorded with inconspicuous commencement in H, characterized by minor irregularities of rather long period. Conspicuous features are suppression of the normal diurnal maxima in H on December 3 and 4, a bay commencing at $21^h.0$, G. M. T., on December 3, and arriving at the minimum at $2^h.0$ on December 4, with a range of 200γ. The recovery was gradual thereafter to the end of the disturbance, with values of H much lower than normal. Between 12^h and 17^h on December 4, there were several bays of period about 30 to 40 minutes (crest to crest) with range of about 50γ from crest to trough. D and Z were but slightly disturbed throughout, the normal diurnal trend being followed.[2]

December 11—There was a mild and very brief disturbance (character 1) chiefly noteworthy by a sudden commencement in all three elements at 10^h 34^m, G. M. T., H increasing 29γ in five minutes, D decreasing $0'.6$, and Z increasing 3γ. The disturbance of eleven hours was thereafter characterized by numerous bays and peaks in H with changes of 25γ to 50γ in intervals of twenty to thirty minutes, these being superposed upon the normal diurnal maximum. D and Z showed minor perturbations not affecting the diurnal trend.

December 29-30—This was a disturbance without conspicuous commencement, chiefly characterized by a great and almost continuous decrease in H of 360γ from 16^h 54^m to 20^h 50^m, G. M. T. Prior to that period there were a few bays of range 50γ to 100γ; after the period, recovery almost to normal values occurred in about three hours. Thereafter, there was little disturbance in H until 12^h, G. M. T., December 30, at which time a series of bays commenced, the bays having ranges of 25γ to 50γ with period of thirty to forty minutes, completely eliminating the usual daily maximum. These persisted until 19^h, G. M. T. D showed minor perturbations during the disturbance, and Z a larger diurnal range than usual, but otherwise normal trends in these elements were maintained.

O. W. Torreson, *Observer-in-Charge*

[2]It will be noted that the particulars regarding times for this disturbance as recorded at Huancayo do not agree with the reported times at other observatories. As stated, however, the disturbances were not marked by conspicuous commencements or endings at Huancayo and thus the times of these features are not very definitely established.

Apia Observatory
October to December, 1934

(*Latitude 13° 48′.4 S.; longitude 171° 46′.5 or 11^h 27^m.1 W. of Gr.*)

November 7—A slight disturbance began at 4^h 50^m, G. M. T., with a gradual increase of 6γ in H. It reached a maximum increase of 32γ at 8^h 45^m and afterwards fell by 24γ between then and 9^h 10^m. The corresponding movements in Z were much smaller, namely, increases of 2γ and 5γ and then a fall of 6γ.

December 1—A sudden increase of 22γ occurred in H at 4^h 57^m, G. M. T., and an increase of 5γ in Z.

December 11—A sudden increase of 15γ took place at 10^h 34^m, G. M. T., with an increase in Z of 3γ.

J. Wadsworth, *Director*

NOTES

(See also page 123)

1. *Potsdam-Niemegk Magnetic Observatory*—In connection with the reorganization of the meteorological service in Germany, the Potsdam-Niemegk Magnetic Observatory has been separated from the former Prussian Meteorological Institute and attached to the University of Berlin as a university-institute with the designation "Magnetisches Observatorium der Universität Berlin in Potsdam-Niemegk." The directorship of the new Institute has been entrusted to Prof. Dr. A. Nippoldt.

2. *Toyohara Observatory*—Readers of this Journal will be interested to learn that, through the efforts of Professor T. Okada, Director of the Central Meteorological Observatory of Japan, the continuation of the work at the Toyohara Observatory, which was established for work during the International Polar Year, 1932-33, has been assured.

3. *Agincourt Observatory*—We have been informed by J. Patterson, Director of the Meteorological Service of Canada, that a new room is to be added to the Agincourt Observatory where two la Cour instruments, previously used at the Polar-Year Station at Chesterfield Inlet, will be installed. As these include one quick-run and one ordinary-sensitivity magnetograph, it is not felt necessary to install an insensitive set also, as the wide range on the Observatory's regular magnetograph will take in practically everything required.

4. *Publications*—Revised editions of two publications, "Magnetic Declination in 1935, Florida" and "Magnetic Declination in 1935, California and Nevada" are in preparation by the United States Coast and Geodetic Survey.

5. *Sixteenth annual meeting of American Geophysical Union*—The sixteenth annual meetings of the American Geophysical Union and of its sections will be held in Washington, D. C., April 25 and 26, 1935. The program of the Section of Terrestrial Magnetism and Electricity on the morning of April 25 will deal with instruments for, observations of, and discussion of magnetic anomalies, earth-current results, correlations of auroral display with other geophysical phenomena, atmospheric-electric results, and preliminary reports on International Polar-Year work. The general session of the Union on the afternoon of April 26 is to be devoted to a symposium on "The Earth's outer atmosphere" and will include papers in various geophysical fields bearing on this subject.

6. *Magnetic work in Africa*—R. H. Mansfield, Assistant Observer of the Department of Terrestrial Magnetism, who has been engaged in field work in Africa for several months obtaining secular-variation data, recently completed his work in Portuguese East Africa and in the Transvaal and is now in Southern Rhodesia. After securing observations at a few stations there he will proceed to Aden, Port Sudan, and Suez.

REVIEWS AND ABSTRACTS

DEHALU, M., ET MARIE MERKEN: *Nouvelle carte magnétique de la Belgique.* Bruxelles, Mem. Acad. roy., Cl. sci., Sér. 2, T. 10, 1931 (125 avec 7 cartes). HOGE, E.: *Nouvelle contribution à la carte megnétique de la Belgique.* Bruxelles, Mém. Acad. roy., Cl. sci., Sér. 2, T. 11, 1934 (50 avec 1 carte). 30 cm.

The work of Prof. Dehalu and Dr. Merken is based primarily on the magnetic survey of A. Hermant (Levé magnétique de la Belgique au 1er janvier 1913. Ann. Obs. roy. Belgique, **6**, fasc. 3, 1920). The instruments used were the standards in declination, horizontal intensity and inclination at the Uccle Observatory, and were returned to the Observatory each week for the regular absolute observations. This removes from the field of doubt the question of standards which so often arises to trouble the compiler of results. Of course in the use of modern instruments, the differences of standards are small, and for many purposes are considered negligible. On the other hand, Dr. Van Rijckevorsel found the differences quite appreciable when he compared his instruments at Kew, Parc Saint-Maur, Utrecht, and Wilhelmshaven, before beginning his magnetic survey of Holland (1889-92). If magnetic observations of three elements over extended land-areas are to be of great importance in revealing hidden geological features, the question of standards must be considered. A more complete knowledge of the distribution of diurnal and secular variation, in fact, increased information on all changes, periodic and non-periodic, as well as more accurate observations at closely spaced stations, will also be necessary.

Hermant used a Wild-Edelmann earth-inductor to determine inclination at his field-stations. This is probably the earliest case of regular field-use of this instrument in land-survey work. (The Department of Terrestrial Magnetism of the Carnegie Institution of Washington began the use of its light, portable, land-survey earth-inductor in Australia in September 1914.) The gain in accuracy that the earth-inductor gave was partly offset, at least, by the fact that there was no trace in vertical intensity to use in reducing the field-values of inclination to epoch. The vertical-intensity variometer at Uccle was abandoned in 1909 on account of disturbances by electric-car lines. This is particularly unfortunate, as the inclination or vertical intensity is the important element in the use that has been made of these observations in the study of local anomalies. The declination and horizontal-intensity variometers had, with special precautions, been kept in operation. It was accordingly possible to reduce the field-observations in these two elements to the adopted epoch, January 1, 1913. There were 136 stations in Hermant's survey which gave an average of one station to an area of 216 square kilometers.

In 1904 Prof. Dehalu observed declination at 100 stations, inclination at 81, and horizontal intensity at 78, in the vicinities of Liége, Namur, Charleroi, and Mons. The object of these observations was the determination of local anomalies. The publication by him and Dr. Merken, gives the results of these observations in full with discussion of instruments and methods. A Chasselon magnetometer was used for declination and horizontal intensity, and the intensity-constants were redetermined in part. A Dover dip-circle with two needles was used in observing the inclination. All observations were reduced to epoch through Uccle. This was done by comparing the values obtained with his instruments at the Cointe absolute piers with both the Cointe traces and the published Uccle hourly values. It does not appear from the published results and discussions that it could be said that the field-values are on the Uccle standards. A comparison of the two dip-needles with an earth-inductor both before and after the field-observations would have been especially desirable in view of the important part the inclination plays in the determination of anomalies.

The deduction of the magnetic anomalies from the results of the Dehalu and Hermant surveys was made by Dr. Merken, following, in a general way, methods used by Van Rijckevorsel and by Rücker and Thorpe. As there was some variation in detail the method may be briefly described.

All of Belgium was divided into "districts" with sides of 12′ in latitude and 12′ in longitude. There were 95 of these "districts" which contained at least one magnetic station. In case of more than one station in a "district," the mean latitude, longitude, and magnetic value were taken to represent that "district." This amounts to weighting according to area as applied by Rücker and Thorpe but not by Van Rijckevorsel. For convenience of applying the method, only those stations were used at which all three

magnetic elements had been observed. This gave 73 stations from the Dehalu survey and 136 from that of Hermant, reduced to 95 "district" stations.

It was assumed in the first place that the "normal" magnetic element varied uniformly with latitude and longitude, that is, that the normal isomagnetic lines were parallel and equally spaced. In other words the normal magnetic value was assumed to be a linear function of the latitude and longitude. To escape the labor of a least-s uare method, the following assumptions were also made: (1) The "ideal" station, whose latitude is the mean of the 95 latitudes of the individual "districts" and whose longitude is the mean of all the longitudes, has a magnetic value equal to the mean of the 95 "district" values, which magnetic value is free from anomaly; (2) the sum of the anomalies north of the "ideal" station is equal to the sum of those south of it; (3) the sum of the anomalies east of the "ideal" station is equal to the sum of the anomalies west of it. By means of these assumptions the solution for each element was reduced by mere summations to a pair of linear equations.

The adjustments thus made gave normal values at each station and from these the residuals or anomalies were computed by differences from observed values. These anomalies are tabulated for declination, inclination, horizontal intensity, and also for the usual three rectangular components of the total force. A chart is given for each of these six elements showing the normal curves along with curves of the actual observations. A seventh chart in colors shows by shades variations in vertical-intensity anomalies at intervals of 100γ. The vector of the horizontal-intensity anomaly is also shown on this chart in direction and relative magnitude.

It may be asked how the method applied by Dr. Merken compares with the least-square adjustment. In the latter the anomalies are treated as if they occur in accordance with the normal law of error; in the former certain additional arbitrary assumptions are made besides the assumptions common to the two methods of linear variation of magnetic value with latitude and longitude. Rücker and Thorpe adjusted one of their nine districts, Scotland, by both methods and found that the differences were negligible. If this is true in general, the method used by Dr. Merken is much to be preferred, for the amount of the computation is much less. On the other hand Angot (Cartes magnétiques de la France au 1er janvier 1901) used the least-square method and assumed that the value of the magnetic element varies according to a second-degree expression in latitude and longitude.

Dr. Hoge extends the work of Dr. Merken by the addition of stations from the surrounding countries. Dr. Merken's original 209 stations had been reduced to 195 by her rejection of 14. With these 195, 46 from French surveys, 30 from Holland, and six from Germany, 277 in all, 47 "districts" were added to the original 95 and the adjustment carried through in exactly the same way. The derived anomalies are shown on a colored chart constructed in the same way as Chart 7 of the publication of Prof. Dehalu and Dr. Merken. The two charts of the anomalies are very much alike as was to be expected.

The inclusion in this way of surveys from different countries brings up with increased emphasis the question of absolute standards and in addition, the more difficult question of reduction to epoch. Also, in this more extended area, the method of Angot of assuming curvature in the normal isomagnetic lines might be considered superior. Where the boundary lies between normal value and anomaly rests on arbitrary decision in all cases. It would be difficult to say how great a part such decisions play in the chart maker's work, and it is these charts that furnish the data for the spherical harmonic analysis of the Earth's field with its questions of inner and outer fields and potential and non-potential parts.

The work of Prof. Dehalu, Dr. Merken, and Dr. Hoge is a valuable contribution to the knowledge of magnetic anomalies in Belgium, and hence of importance to the geologist. It also provides well worth-while material for the chart-maker and thus for the analyser of the Earth's field. Continuation in the future of such studies will advance the solution of the question of changes in anomaly.

Rücker and Thorpe and Van Rijckevorsel concluded from their discussion of land-anomalies that there were also anomalies at sea. The correctness of this conclusion may be tested by future sea-observations, denser and more accurate than in the past. Reduction to absolute standards offers no serious difficulty either for sea or land instruments, and observations with both should be made with the greatest care. If in the future we arrive at a satisfactory solution of the problem of magnetic variation, the value of present observations will be in proportion to their accuracy.

C. R. Duvall

NOTES

(See also page 120)

7. *Transpacific radio transmission*—Important scientific messages from the Watheroo Magnetic Observatory in Western Australia to the Department of Terrestrial Magnetism are transmitted through radio station VK5HG in Southern Australia to amateur stations in the United States whence they are received by the Department. For brief periods during the southern summer the Australian station is transferred to Kangaroo Island where no electric power is available. During the present temporary sojourn at Kangaroo Island, it was found necessary to use "B" batteries in place of the generator which had burned out. Nevertheless, with a power not exceeding 10 watts, reliable communication has been maintained with American amateur stations, a feat which must be regarded as remarkable in view of the recent unfavorable static conditions.

8. *International Polar Year 1932-33*—A further grant of $15,000 has been made by the Rockefeller Foundation to the International Commission for the Polar Year 1932-33.

9. *Geophysical studies to locate underground water*—An allotment of $4,950 to the Bureau of Mines, Department of the Interior, for geophysical studies to locate sources of underground water to supply gold-mining operations was announced by the United States Public Works Administration. The purpose of this project is to help those gold mines that lack water for milling operations to find a source of water near their properties making it possible for them to become productive.

10. *Franklin-Adams chart of the sky*—The Royal Astronomical Society proposes to undertake the publication (provided sufficient support is forthcoming) of a third edition of the Franklin-Adams chart of the sky, in 206 sheets, each covering an area 15° by 15°, on a scale of $1° = 15$ mm. The chart is in three sections: (1) North Pole to declination $+22°$; (2) declination $+22°$ to $-22°$; (3) declination $-22°$ to South Pole. It will be issued as each section is completed and the whole within twelve months. The price of the complete set, in three cases, has been fixed at £27, including carriage. Should only one or two of the sections be desired, subscriptions will be received for the part required at a corresponding price. Further particulars, and application forms, may be obtained from the Assistant Secretary, Royal Astronomical Society, Burlington House, London, W. 1, England.

11. *Oxford University Arctic Expedition*—An expedition consisting of nine men, has received the financial support of the University of Oxford and the Royal Geographical Society of London and plans to leave England in July to spend twelve months on the north coast of Northeast Land. The staff will consist of three surveyors, two physicists, a wireless operator, a geologist, a doctor-biologist, and a glaciologist. The sealer M. S. *Polar* of Tromsö has been chartered and it is hoped that the north coast will be reached before the beginning of August. The base is to be established at Rijps Bay, where, in addition to a three-roomed, double-walled hut, two smaller huts will be built, one for the magnetic instruments and the other for part of the wireless equipment to be used in the investigations of the ionosphere. This work, together with the remainder of the physics program, will be under the direction of R. A. Hamilton, and will be carried on continuously over the whole year. In addition to the ionospheric and terrestrial-magnetic investigations, observations of meteorological conditions, atmospheric electricity, and auroras will be made regularly. It is hoped that wireless communication with England—the transmission being across the auroral belt—may be maintained. The Expedition is expected to return to England in September 1936.

12. *Personalia*—Lieut. Commander Alfred L. Giacomini, United States Coast and Geodetic Survey, died February 7, 1935. As a ship's officer during the period when the Coast and Geodetic Survey carried on considerable magnetic work at sea he made many observations, especially on the Pacific Coast of the continental United States and Alaska. In recent years, he studied and made contributions to the subject of better use of the magnetic compass in air-planes.

Dr. *Axel Wallén*, director of the State Meteorological and Hydrographical Service of Sweden and president of the Association of Meteorology of the International Union of Geodesy and Geophysics, died at Stockholm on February 24, 1935, aged 58 years.

LIST OF RECENT PUBLICATIONS

By H. D. Harradon

A—Terrestrial and Cosmical Magnetism

Agostinho, J. Estudos geofísicos nas colónias dois problemas importantes. Separata, "A Terra," Coimbra, No. 14, 1934, 4 pp. [In this paper attention is called to the necessity of improving the investigations in the colonial dominions of Portugal, namely in Moçambique, where a magnetic observatory is to be established near Lourenço Marques, and also in Angola and the Cape Verde Islands where work of this kind is urgently required. The importance of the aerological observations in the Portuguese Colonies some of which lie along the most important aerial routes of the Atlantic Ocean, is also emphasized.]

Azores, Service Météorologique. Résumé d'observations de 1931. Lisbonne, Imprimerie Nationale, 1934 (22). [Contains annual values of the magnetic elements and monthly mean values of the declination at the S. Miguel Observatory for 1931.]

Bangkok, Royal Survey Department. Report on the operations of the Royal Survey Department, Ministry of Defense, for the year 1932-33. Bangkok, Printing School, Wat Sangvej (39 with 7 index maps). 29 cm. [Pp. 33-36 contain results of magnetic observations at various points in Siam.]

Batavia. Observations made at the Royal Magnetical and Meteorological Observatory at Batavia, v. 53, 1930. Published by order of the government of the Netherlands Indies, by J. Boerema, Director. Batavia, 1934 (viii+86). 36 cm. [Contains results of magnetic records, Batavia-Kuyper, 1930.]

De Beck, H. O. An accurate simplified magnetometer field method. New York, N. Y., Trans. Amer. Inst. Min. Metall. Eng., v. 110, Geophysical Prospecting, 1934 (326-333).

De Bilt, Institut Météorologique des Pays-Bas. Caractère magnétique numérique des jours. Tome XI. Avril-juin 1934. De Bilt, 1934, 20 pp. [Published under the auspices of the Association of Terrestrial Magnetism and Electricity of the International Union of Geodesy and Geophysics.]

Der magnetische Charakter des Jahres 1933. Met. Zs., Braunschweig, Bd. 51, Heft 9, 1934 (354).

Dimmler. Die wochentlichen magnetischen Beobachtungen in Jinsen, Taihoku, Otmari, und Palau 1932. Ann. Hydrogr., Berlin, Jahrg. 62, Heft 11, 1934 (464-465).

Hatakeyama, H., and M. Hirayama. On the phase-difference between the pulsation of terrestrial magnetism and of earth-current. J. Met. Soc. Japan, Tokyo, v. 12, No. 9, 1934 (449-459). [Japanese text with English abstract.]

Hazard, D. L. Results of observations made at the United States Coast and Geodetic Magnetic Observatory near Honolulu, Hawaii, in 1927 and 1928. Washington, D. C., U. S. Dept. Comm., Coast Geod. Surv., 1934, 112 pp. 27 cm.

Heiland, C. A., and W. E. Pugh. Theory and experiments concerning a new compensated magnetometer system. New York, N. Y., Trans. Amer. Inst. Min. Metall. Eng., v. 110, Geophysical Prospecting, 1934 (334-372).

Hess, V. F., and W. Illing. Terrestrial magnetism and cosmic rays. Nature, London, v 135, Jan. 19, 1935 (97-98).

India, Survey of. Geodetic report 1933. Published by order of Brigadier H. J. Couchman, Surveyor General of India. Dehra Dun, Geod. Branch Office, 1934 (xi+117 with 9 charts and 11 pls.). 25 cm. [Includes a brief report on magnetic observations at Dehra Dun for 1932.]

Kato, Y. Seismic and volcanic activities and changes in the Earth's magnetic field (the second paper). Jap. J. Astr. Geophys., Tokyo, v. 12, No. 1, 1934 (1-25).

Magnetic Compass. The Stella dual magnetic compass. Hydrogr. Rev., Monaco, v. 11, No. 2, 1934 (157-158).

Malamphy, M. C. Magnetic measurements of auriferous veins in Brazil. New York, N. Y., Trans. Amer. Inst. Min. Metall. Eng., v. 110, Geophysical Prospecting, 1934 (313-325).

Mauritius, Royal Alfred Observatory. Annual report of the Royal Alfred Observatory for the year 1933 (N. R. McCurdy, Director). Mauritius, 1934, 7 pp. 25 cm. [Contains brief report of the magnetic work for 1933.]

N., H. W. Magnetic storms and solar activity during 1934. Observatory, London, v. 58, No. 729, 1935 (49-51).

Nippoldt, A. Karte der magnetischen Störungen in Britisch-Indien. Beitr. Geophysik, Leipzig, Bd. 43, Heft 1/2, 1934 (185-209 mit 12 Figuren und 1 Karte).

PRAGUE, INSTITUT GÉOPHYSIQUE NATIONAL TCHÉCO-SLOVAQUE. Bulletin magnétique. No. 2, Année 1929. Praha, Imprimerie "Prometheus," 1933, 25 pp. 24 cm. Bulletin magnétique. No. 3, Année 1930-1931. Praha, 1934, 49 pp. 24 cm. [Contain values of the magnetic declination obtained at the magnetic observatory of Stará Dala (O'Gyalla).]

ROMAN, I., AND T. C. SERMON. A magnetic gradiometer. New York, N. Y., Trans. Amer. Inst. Min. Metall. Eng., v. 110, Geophysical Prospecting, 1934 (373-387).

SEIDLITZ, W. VON. Die geophysikalische Reichsaufnahme. Zs. Geophysik, Braunschweig, Jahrg. 10, Heft 8, 1934 (335-339).

STOYKO, N. De l'influence des perturbations magnétiques sur la vitesse de propagation des ondes électromagnétiques longues. Paris, C.-R. Acad. sci., T. 199, No. 18, 1934 (845-847).

SWANSON, C. O. Use of magnetic data in Michigan iron ranges. New York, N. Y., Trans. Amer. Inst. Min. Metall. Eng., v. 110, Geophysical Prospecting, 1934 (290-311).

TACUBAYA, OBSERVATORIO ASTRONOMICO NACIONAL. Anuario del Observatorio Astronomico Nacional de Tacubaya para el año de 1935. Formado bajo la dirección del Ing. Joaquin Gallo. Año 55. Tacubaya, Universidad Nacional de Mexico, 1934 (324). 19 cm. [Contains mean values of the magnetic elements at the Magnetic Observatory of Teoloyucan for 1933 and the first six months of 1934.]

TROMSÖ, AURORAL OBSERVATORY. Results of magnetic observations for the year 1932. By L. Harang and E. Tönsberg. Bergen, Pub. Inst. Kosmisk Fysikk, No. 4, 1934 (31). 31 cm.
Results of magnetic observations for the year 1933. By L. Harang and E. Tönsberg. Bergen, Pub. Inst. Kosmisk Fysikk, No. 5, 1934 (31). 31 cm.

B—Terrestrial and Cosmical Electricity

ALLEN, C. W. Atmospheric potential-gradient observations at the Commonwealth Solar Observatory, Mount Stromlo, Canberra. Canberra, Mem. Solar Obs., v. 1, No. 4, 1934 (47 with 9 figs.). 30 cm.

ALOCCO, G. Absorption of cosmic particles in copper and lead. Nature, London, v. 135, Jan. 19, 1935 (96-97).

APPLETON, E. V. Recent measurements of upper-atmospheric ionization. Phys. Rev., Lancaster, Pa., v. 47, No. 1, 1935 (89).

APPLETON, E. V., AND F. W. CHAPMAN. The lightning flash as source of an atmospheric. Nature, London, v. 134, Dec. 22, 1934 (968).

AUGER, P., ET P. EHRENFEST. Corpuscules ultrapénétrants du rayonnement cosmique. Paris, C.-R. Acad. sci., T. 199, No. 26, 1934 (1609-1611).

AUGER, P., ET L. LEPRINCE-RINGUET. Analyse du rayonnement cosmique en haute altitude. Paris, C.-R. Acad. sci., T. 199, No. 17, 1934 (785-787).

BROWN, G. H. The phase and magnitude of earth-currents near radio transmitting antennas. New York, N. Y., Proc. Inst. Radio Eng., v. 23, No. 2, 1935 (168-182).

CAIRNS, J. E. I. Production of induced radioactivity by the cosmic radiation. Phys. Rev., Lancaster, Pa., v. 47, No. 2, 1935 (194).

COMPTON, A. H. Magnitude of cosmic-ray bursts. Nature, London, v. 134, Dec. 29, 1934 (1006).

COMPTON, A. H., AND H. A. BETHE. Composition of cosmic rays. Nature, London, v. 134, Nov. 10, 1934 (734-735).

COULOMB, J., ET P. BENAC. Sur une réduction au plan des valeurs du champ électrique. Bull. Inst. Obs. Phys. Globe, Puy-de-Dôme, Clermont-Ferrand, No. 7, 1934 (16-22). [Pour obtenir un coefficient de réduction au plan défini au centième au moyen d'enregistrements simultanés avec des électromètres de Benndorf, il est nécessaire de poursuivre l'expérience pendant un semestre.]

DAUVILLIER, A. Recherches sur les rayons cosmiques effectués au Scoresby Sund pendant l'année polaire. J. Physique et Le Radium, Paris, Sér. 7, T. 5, No. 12, 1934 (640-648).

DAUZÈRE, C., ET J. BOUGET. Sur la cause des variations de la conductibilité de l'air dans les grottes. Paris, C.-R. Acad. sci., T. 199, No. 26, 1934 (1645-1646).

ENGELSTAD, R. B., AND N. H. MOXNES. Possible action of cosmic rays on living organisms. Nature, London, v. 134, Dec. 8, 1934 (898). [Tests were carried out on white mice in the Kongsberg silver mines west of Oslo over a period of a little more than ten months. No difference in the general conditions of the animals living in the mines and their controls was found, the only demonstrable difference being that the former had a greater mean weight than the latter.]

Ficker, H. v. Ueber die Entstehung lokaler Wärmegewitter. Vierte Mitteilung: Untersuchung der Vorgänge in der freien Atmosphäre an Gewittertagen. Berlin, SitzBer. Akad. Wiss., Nr. 29, 1934 (478-503).

Grenet, G. Le champ électrique au sommet du Puy de Dôme. Bull. Inst. Obs. Phys. Globe, Puy-de-Dôme, Clermont-Ferrand, No. 7, 1934 (2-8). [Dans ce travail, l'auteur a résumé toutes les observations de champ électrique faites au sommet du Puy de Dôme de 1928 à 1932.]

L'enregistrement du champ électrique terrestre et de ses variations. Bull. Inst. Obs. Phys. Globe, Puy-de-Dôme, Clermont-Ferrand, No. 7, 1934 (49-51). [Dans ce travail l'auteur indique comment on peut enregistrer sans aucun retard les variations du champ électrique terrestre. Il signale l'application qui a été faite de cette théorie et indique qu'on pourrait l'appliquer aux mesures du champ électrique pendant les orages.]

Grenet, G., J. Coulomb, et P. Bénac. La mesure du champ électrique à la Côte de Landais (Clermont-Fd.). Bull. Inst. Obs. Phys. Globe, Puy-de-Dôme, Clermont-Ferrand, No. 7, 1934 (9-15). [Ce travail comporte la description de l'installation utilisée depuis 1930 pour les mesures du champ électrique à la station de la plaine de l'Observatoire du Puy de Dôme.]

Grieger, H. Absolute Messungen des vertikalen Leitungsstromes in Potsdam. Berlin, Emil Ebering, [1934] (54 mit Abb.). 23 cm.

Gross, B. Zur Analyse der Absorptionskurve der Ultrastrahlung. Zs. Physik, Bd. 92, No. 11/12, 1934 (755-758).

Hawkins, R. H. Application of relativity methods to northern Ontario lignite deposits. New York, N. Y., Trans. Amer. Inst. Min. Metall. Eng., Geophysical Prospecting, 1934 (76-119).

Hoge, E. Nouvelle contribution à la carte magnétique de la Belgique. Etude de la répartition du magnétisme terrestre en Belgique et dans les régions limitrophes. Bruxelles, Hayez, 1934 (49 avec 1 carte). 29 cm. [Université de Liége, Institut d'Astronomie et de Géodésie, Physique du Globe, No. 2.]

Hubbert, M. K. Results of earth-resistivity survey on various geologic structures in Illinois. New York, N. Y., Trans. Amer. Inst. Min. Metall. Eng., v. 110, Geophysical Prospecting, 1934 (9-29). [Discussion (29-39).]

Hubbert, M. K., and J. M. Weller. Location of faults in Hardin County, Illinois, by earth-resistivity method. New York, N. Y., Trans. Amer. Inst. Min. Metall. Eng., v. 110, Geophysical Prospecting, 1934 (40-47).

Huxley, L. G. H. A theory of the origin of cosmic rays. Phil. Mag., London, v. 18, No. 122, 1934 (971-983).

Jarleton, Mlle A. Sur les relations du champ électrique avec la radiation solaire. Bull. Inst. Obs. Phys. Globe, Puy-de-Dôme, Clermont-Ferrand, No. 7, 1934 (52-71).

Jensen, J. C. Lightning-discharges to grounded conductors. Sci. Mon., New York, N. Y., v. 39, No. 6, 1934 (560-564 with 4 figs.).

Johnson, T. H. North-south asymmetry of the cosmic radiation in Mexico. Phys. Rev., Lancaster, Pa., v. 47, No. 1, 1935 (91-92).

Kawano, M. Preliminary report on cosmic-ray observation. J. Met. Soc. Japan, Tokyo, v. 12, No. 9, 1934 (445-448). [Japanese text with English abstract.]

Kihlstedt, F. H. Electrical methods in prospecting for gold. New York, N. Y., Trans. Amer. Inst. Min. Metall. Eng., v. 110, Geophysical Prospecting, 1934 (62-74).

Kolhörster, W. Höhenstrahlung und Nova Herculis. Zs. Physik, Berlin, Bd. 93, Heft 5/6, 1935 (429-431).

Kolhörster, W., und L. Janossy. Die Richtungsverteilung koinzidierender Höhenstrahlen in Seehöhe. Zs. Physik, Berlin, Band 93, Heft 1/2, 1934 (111-122).

Kulenkampff, H. Untersuchungen an Ultrastrahlungskorpuskeln. Physik. Zs., Leipzig, Jahrg. 35, Nr. 23, 1934 (996-997).

Kurtenaker, K. S. Some practical applications of resistivity measurements to highway problems. New York, N. Y., Trans. Amer. Inst. Min. Metall. Eng., v. 110 Geophysical Prospecting, 1934 (49-59).

Landsberg, H. Zählungen von Kondensationskernen auf dem Taunusobservatorium und auf dem Nordatlantischen Ozean. Bioklim. Beibl., Braunschweig, Bd. 1, Heft 3, 1934 (125-128). [Es werden die Ergebnisse von Kernzählungen an einer Gipfelstation besprochen und die Abhängigkeit der Kernzahlen von der Feuchtigkeit, der Sicht, der Windrichtung und -stärke, besonders aber der Tageszeit festgestellt, wobei sich der überragende Einfluss der vertikalen Durchmischung der Luft herausstellte. Im Anhang werden die Ergebnisse von Kernzählungen auf einer Ueberfahrt von Deutschland nach New York angeführt.]

LENZ, E. Elektrische Ablenkung von Ultrastrahlungsteilchen. Physik. Zs., Leipzig, Jahrg. 36, Nr. 1, 1935 (24-26).
MAASS, H. Zählrohrmessungen an Ultrastrahlungskorpuskeln. Physik. Zs., Leipzig, Jahrg. 35, Nr. 21, 1934 (858-861).
McEACHRON, K. B. Multiple lightning-strokes. Elec. Eng., New York, N. Y., v. 53, No. 12, 1934 (1633-1637).
MATHIAS, E. La notion d'impureté dans les foudres globulaires nues. Paris, C.-R. Acad. sci., T. 199, No. 21, 1934 (1083-1086).
MILLIKAN, R. A., AND H. V. NEHER. The equatorial longitude-effect in cosmic rays. Phys. Rev., Lancaster, Pa., v. 47, No. 3, 1935 (205-208).
MILNE, E. A. Origin of the cosmic rays. Nature, London, v. 135, Feb. 2, 1935 (183).
MÜLLER, M. Der geophysikalische Nachweis des Zechsteindolomits. Zs. Geophysik, Braunschweig, Jahrg. 10, Heft 7, 1934 (330-334).[Es wird das Ergebnis einer im mitteldeutschen Oelgebiet ausgeführten geophysikalischen Messung mitgeteilt, welches mit einem neuen elektrischen Aufschlussverfahren gewonnen wurde. Unabhängig voneinander wurden ermittelt die petrographische Schichtenfolge und deren Wassergehalt. Das neue Aufschlussverfahren ermöglicht den Nachweis des Zechsteindolomits auch unterhalb der Salzstöcke. Die Fehler in der Bestimmung der Tiefe der Schichtgrenzen betragen etwa 2 %.]
NOLAN, J. J., AND P. J. NOLAN. A new method for counting atmospheric ions and determining their mobilities. Dublin, Proc. R. Irish Acad., A, v. 42, No. 3, 1934 (15-19).
NORINDER, H. On the nature of lightning-discharges. Philadelphia, Pa., J. Frank. Inst., v. 218, No. 6, 1934 (717-738).
OSLO, NORWEGISCHES METEOROLOGISCHES INSTITUT. Jahrbuch des Norwegischen Meteorologischen Instituts für 1933. Oslo, Gröndahl und Sön, 1934 (iv+146). 33 cm. [Contains values of the electrical potential-gradient and conductivity at the Aas Meteorological Observatory.]
PIRSON, S. J. Interpretation of three-layer resistivity curves. New York, N. Y., Trans. Amer. Inst., Min. Metall. Eng., v. 110, Geophysical Prospecting, 1934 (148-158).
Effect of anisotropy on apparent resistivity curves. Bull. Amer. Ass. Petrol. Geol., Tulsa, Okla., v. 19, No. 1, 1935 (37-57).
RIEDER, F., AND V. F. HESS. Effects of cosmic radiation in a Wilson chamber at the Hafelekar Observatory (2,300 m.) near Innsbruck. Nature, London, v. 134, Nov. 17, 1934 (772-773).
ROMAN, I. Some interpretations of earth-resistivity data. New York, N. Y., Trans. Amer. Inst. Min. Metall. Eng., v. 110, Geophysical Prospecting, 1934 (183-197).
ROUGERIE, P. Analyse harmonique de la variation diurne des courants telluriques nord-sud enregistrés à l'Observatoire du Parc Saint-Maur. Paris, C.-R. Acad. sci., T. 199, No. 19, 1934 (964-966).
RUDOLPH, H. Die Energie der Heliokathodenstrahlen in ihrer Beziehung zur fortschreitenden Bewegung der Elektronen in den Polarlichtstrahlen. Zs. Geophysik, Braunschweig, Jahrg. 10, Heft 8, 1934 (356-359).
SCHLUMBERGER, C., M. SCHLUMBERGER, AND E. G. LEONARDON. Electrical exploration of water-covered areas. New York, N. Y., Trans. Amer. Inst. Min. Metall Eng., v. 110, Geophysical Prospecting, 1934 (122-134).
Electrical coring: A method of determining bottom-hole data by electrical measurements. New York, N. Y., Trans. Amer. Inst. Min. Metall. Eng., v. 110, Geophysical Prospecting, 1934 (237-288).
SHIRATORI, K. Ionic balance in air and nuclei over ocean. Formosa, Mem. Fac. Sci. Agric., Taihoku Univ., v. 10, No. 5, 1934 (175-201).
SMOSARSKI, W. Electricité atmosphérique près de Poznań. Poznań, Roczniki Nauk Rolniczych i Leśnych, T. 33, 1934 (6-12).
STREET, J. C., AND R. T. YOUNG, JR. Transition effects in the cosmic radiation. Phys. Rev., Lancaster, Pa., v. 46, No. 9, 1934 (823-824).
SWANN, W. F. G. The relation of the primary cosmic radiation to the phenomena observed. Phys. Rev., Lancaster, Pa., v. 46, No. 9, 1934 (828-829).
The origin of the hardening of cosmic rays in passing through matter. Phys. Rev., Lancaster, Pa., v. 47, No. 3, 1935 (250).
TAGG, G. F. Interpretation of resistivity-measurements. New York, N. Y., Trans. Amer. Inst. Min. Metall. Eng., v. 110, Geophysical Prospecting, 1934 (135-145).
TIELSCH, H. Die Absorption der Ultrastrahlung in verschiedenen Materialien. Zs. Physik, Berlin, Bd. 92, Heft 9/10, 1934 (589-614).
URONDO, F. E. Primeras medidas de radioactividad atmosférica en Santa Fe (República Argentina). An. Soc. Cient., Santa Fe, T. 5, Nos. 1/2, 1933 (30-34).
Conductibilidad, movilidad, y densidad iónicas del aire. An. Soc. Cient., Santa Fe, T. 6, Nos. 1/2, 1934 (26-31).

Urondo, F. E. Primeras medidas de electricidad del agua de lluvia. An. Soc. Cient., Santa Fe, T. 6, Nos. 1/2, 1934 (32-35).
Vegard, L., and L. Harang. The wave-length of the green auroral line determined by an interferometer-method. Geophys. Pub., Oslo, v. 11, No. 1, 1934 (17 with 3 pls.).
Vernoff, S. N. On the study of cosmic rays at great altitudes. Phys. Rev., Lancaster, Pa., v. 46, No. 9, 1934 (822).
Walke, H. J. Cosmic radiation and stellar evolution. Nature, London, v. 135, Jan. 5, 1935 (36).
Watson, R. J. A contribution to the theory of the interpretation of resistivity-measurements obtained from surface-potential observations. New York, N. Y., Trans. Amer. Inst. Min. Metall. Eng., v. 110, Geophysical Prospecting, 1934 (201-232).
Wölcken, K. Vergleich zwischen der Intensität der kosmischen Ultrastrahlung über Grönland und über Deutschland. Zs. Geophysik, Braunschweig, Jahrg. 10, Heft 7, 1934 (300-307).
Ziemecki, St. Rock salt absorption of cosmic rays. Nature, London, v. 134, Nov. 17, 1934 (773).

C—Miscellaneous

Brunt, D. Sir Arthur Schuster. Met. Mag., London, v. 69, No. 826, 1934 (229-230).
Compton, A. H., E. O. Wollan, and R. D. Bennett. A precision recording cosmic-ray meter. Rev. Sci. Instr., Lancaster, Pa., v. 5, No. 12, 1934 (415-422).
Egloff, K. Ueber das Klima im Zimmer und seine Beziehungen zum Aussenklima, mit besonderer Berücksichtigung von Feuchtigkeit, Staub- und Ionengehalt der Luft. Zürich, Druck von Hans A. Gutzwiller Aktiengesellschaft (84 mit 15 Fig.). 23 cm. [Dissertation der Eidgen. Techn. Hochschule, Zürich, Nr. 766.]
Gilliland, T. R. Ionospheric investigations. Nature, London, v. 134, Sept. 8, 1934 (379).
Hatakyama, H. On Benndorf's self-recording electrometer. J. Met. Soc. Japan, Tokyo, v. 12, No. 10, 1934 (501-510). [Japanese text with English abstract.]
Heiland, C. A. Geophysics in the nonmetallic field. New York, N. Y., Trans. Amer. Inst. Min. Metall. Eng., v. 110, Geophysical Prospecting, 1934 (546-576).
Henze, H. Häufigkeit und Dauer der Gewitterregen in Potsdam. Met. Zs., Braunschweig, Bd. 51, Heft 11, 1934 (417-424).
Hulburt, E. O. Analysis of recent measurements of the ionosphere. Phys. Rev., Lancaster, Pa., v. 46, No. 9, 1934 (822-823), and v. 47, No. 1 (89) and No. 5, 1935 (422).
Israël, H. Emanation in Boden- und Freiluft. Zs. Geophysik, Braunschweig, Jahrg. 10, Heft 8, 1934 (347-355).
Kaplan, J. Light of the night sky. Nature, London, v. 135, Feb. 9, 1935 (229).
Kelly, S. F., T. Zuschlag, and B. Low. Discovering gold-quartz veins electrically. Abstract: New York, N. Y., Trans. Amer. Inst. Min. Metall. Eng., v. 110, Geophysical Prospecting, 1934 (75). [This paper was published in full in Mining and Metallurgy, June, 1934.]
Köppen, W., and R. Geiger. Handbuch der Klimatologie. Bd. 2, Teil 1. Westindien. Berlin, Verlag von Gebrüder Borntraeger, 1934 (47). 26 cm. [Climatology of the West Indies, by R. DeC. Ward and C. F. Brooks.]
Lejay, P. Détermination de la quantité d'ozone contenue dans l'atmosphère, au voisinage de Shanghaï. Paris, C.-R. Acad. sci., T. 199, No. 18, 1934 (879-881).
Lenz, E. Electric deflection of cosmic radiation. Nature, London, v. 134, Nov. 24, 1934 (809).
Müller, E. A. W. Ionization of the Kennelly-Heaviside layer. Nature, London, v. 135, Feb. 2, 1935 (187-188).
Pulley, O. O. A self-synchronized system for ionospheric investigation by the pulse-method. London, Proc. Phys. Soc., v. 46, No. 257, 1934 (853-871).
Skellett, A. M. The ionizing effects of meteors. New York, N. Y., Proc. Inst. Radio Eng., v. 23, No. 2, 1935 (132-149).
Smith, F. W. Zodiacal light notes. Pop. Astr., Northfield, Minn., v. 42, 1934 (545, 607).
Steinmaurer, R. Ueber eine Vorrichtung zur selbsttätigen Veränderung der Empfindlichkeit des Benndorfschen Registrierelektrometers. Physik. Zs., Leipzig, Jahrg. 36, Nr. 3, 1935 (106-107).
Störmer, C. Luminous night clouds over Norway in 1933 and 1934. Nature, London, v. 135, Jan. 19, 1935 (103-104).
White, F. W. G. On the automatic registration of the amplitude of downcoming wireless waves: Part I. London, Proc. Phys. Soc., v. 46, No. 257, 1934 (805-817).
Witty, R. Some experimental investigations of the polarization of downcoming wireless waves. Cambridge, Proc. Phil. Soc., v. 30, Pt. 4, 1934 (540-542).

Terrestrial Magnetism and Atmospheric Electricity

VOLUME 40 JUNE, 1935 No. 2

DIE MAGNETISIERUNG DER ERDE UNTER DEN OZEANEN

VON A. NIPPOLDT

Abstract—As many years must pass before the large amount of ocean data collected by the Carnegie Institution of Washington can be analyzed by the tedious method of Gauss, it seems suitable, in the meantime, to seek some other method capable of answering, to a certain extent, some of the principal geophysical questions. One such question is that pertaining to the distribution of the non uniform magnetization of the Earth. In a previous paper the author described a new rapid method based on the computation of Bauer's "local magnetic constant" $G=H\sqrt{1+(\tan^2 I)/4}$, for a number of points distributed over the Earth's surface and investigated the law of distribution. For this investigation the magnetic results obtained at the various observatories were used. In the present paper the study is extended to the ocean areas and is based on the ocean magnetic observations obtained by the vessels operated by the Department of Terrestrial Magnetism. The method of computation is briefly described and the results of the study exhibited in a map for the epoch 1922 showing the lines of equal inhomogeneous magnetization of the Earth at intervals of 1 milligauss or 100γ. From an inspection of the map it is concluded that the distribution of the inhomogeneous magnetization over the oceans is quite systematic and not merely fortuitous and that it is not caused by the three upper kilometers (mean depth of the oceans) of the Earth's crust, having its seat principally at depths which have little to do with the surface-distribution of land and water. The details of the magnetic situation are discussed especially in comparison with the previous chart (based on observatory data). The effect of polar movement is also considered but little weight is given it for the period in question. In connection with the question of constancy of magnetization of masses at great depths in the Earth's crust, comparisons with several recent studies of anomalies are made.

Durch die emsige Tätigkeit der Magnetischen Abteilung der Carnegie Institution besitzen wir eine gleichmässige und gute Vermessung der Weltmeere. Mit Spannung wartet die Geophysik auf die Hebung der Schätze, die in diesem wissenschaftlich einzigartigen Material noch verborgen sind.

Der hierzu bisher *allein* benutzte Weg war der von C. F. Gauss vor nunmehr genau 100 Jahren eröffnete: die Darstellung aller Beobachtungen durch Karten, Entnahme von äquidistanten Werten aus diesen Karten, Berechnung numerischer Koeffizienten aus diesen Zahlen, sodann Rückberechnung der "normalen" Werte für die äquidistanten Punkte, und jetzt endlich Bildung der Differenzen aus den beobachteten und den berechneten Werten.

Erst nach dieser Rechnung war es möglich, etwas Geophysikalisches über die Verteilung der regelmässigen und unregelmässigen Magnetisierung der Erde auszusagen.

Jede einzelne dieser Rechenstufen erfordert langwierige Vor- und Zwischenstudien, und es werden noch Jahrzehnte vergehen, bis die Magnetische Weltaufnahme nach dem Verfahren von Gauss analysiert sein wird.

Natürlich muss diese Arbeit einmal gemacht werden; aber allein

schon der menschlich begreifbare Wunsch, die geophysikalischen Hauptfragen, wenigstens bis zu einem gewissen Grade *jetzt* schon beantwortet zu sehen, lässt uns nach anderen Methoden umschauen, die rascher vorzugehen vermögen.

Eine dieser Hauptfragen ist die nach der Verteilung der *unregelmässigen Magnetisierung der Erde.* Wir dürfen erwarten, aus ihr zu erfahren, wie wenigstens *dieser* Teil der Erdmagnetisierung zu stande kommt. Aber, ehe wir sie *erklären* können, müssen wir sie erst einmal *kennen* lernen. Die erste Stufe dazu ist, ihre geographische Verteilung festzulegen.

In einer Vorstudie in dieser Zeitschrift[1] ist das neue, schnelle Rechenverfahren schon bekannt gegeben worden. Es beruht darauf, dass die von L. A. Bauer eingeführte *lokale magnetische Konstante G* für ausreichend viele über die Erde verteilte Orte berechnet und das Gesetz der Verteilung über die Erde untersucht wird. Jeder einzelne Ort liefert ein G. Seine physikalische Bedeutung ist die, dass es multipliziert mit R^3 das magnetische Moment der Erde ergäbe, wenn der ganze Erdkörper so magnetisiert wäre, wie der Erdpunkt, auf welchem der Ort liegt. In anderer Weise gibt das G durch die Beziehung

$$3G/4\pi=\rho$$

die lokale räumliche Magnetisierung. Demnach verwandelt der Faktor 0.2387 alle unsere G-Zahlen in Magnetisierungen auf die Volumeinheit.

In Wahrheit sind die magnetischen Elemente oder Komponenten, aus denen G berechnet wird, an den meisten Orten lokal gestört. Aber bei einer genügend grossen Anzahl von Orten und gleichmässiger Verteilung über die Erde muss das mittlere G dem tatsächlichen quasi-homogenen nahe kommen, und die Unterschiede der einzelnen G gegen den Mittelwert G_{δ} der Erde geben dann die wahre Verteilung der unregelmässigen Magnetisierung. Ein äusseres Kennzeichen dafür, dass man sie richtig getroffen hat, ist die Abwesenheit von unsystematischen, vereinzelt herausfallenden Werten.

Die oben zitierte vorangegangene Arbeit stützte sich nur auf die Observatorien und hat schon aus diesen wenigen Punkten ergeben, dass ihr Mittelwert dem aus den Gauss'schen Koeffizienten errechneten gleich war, und ferner fand sich auch eine durchaus systematische Verteilung. Nun heisst es, das gewonnene Bild über die ganze Fläche der Erde auszudehnen. Es schien mir nur aus mancherlei Erfahrungen heraus wünschenswert, zunächst einmal die Verteilung der Magnetisierung *nur unter den Weltmeeren, den Ozeanen*, zu bestimmen. Magnetische Beobachtungen auf den Meeren haben den besonderen Vorzug, keine gänzlich lokalen Störungen besitzen zu können, die aus dem allernächsten Untergrund stammen, denn der ist hier das praktisch unmagnetische Wasser. Von einer Landstation kann man nie wissen, ob nicht die ganze Anomalie nur von einem kleinen Gestein getragen wird, das zufällig in geringer Tiefe liegt. Das ist auf dem Meere ausgeschlossen.

Das Material bilden ausschliesslich nur die Seebeobachtungen der Carnegie Institution, wie sie die Researches of the Department of Terrestrial Magnetism, "Ocean Magnetic Observations" enthalten. Die Zahlen sind so entnommen, wie sie hier zu finden sind, d. h. ohne

[1]Terr. Mag. 37, 279-286 (19`2).

jede Reduktion auf zeitliche Variationen oder eine Epoche. Die geographische Verbreitung der Magnetisierung prägt sich so deutlich schon in den ersten Dezimalen aus, dass die täglichen Variationen sie kaum noch beeinflussen. Und dieser kleine Rest wird durch die Zusammenfassung von mehreren Einzelwerten meist auch noch ausgeschieden. Über die spätere Ausschaltung der Säkularvariation ist weiter unten Näheres gesagt.

Die Erdoberfläche wurde in Felder von 10° Breiten- und 10° Längenausdehnung zerlegt. Für jedes so entstandene Feld wurden die in es fallenden Beobachtungen in Inklination und Horizontalintensität eingetragen, und das Beobachtungsjahr vermerkt. I wurde auf 1′, und H auf 0.001 Γ abgerundet. Für jedes einzelne Paar von I and H wurde der zugehörige Wert von G berechnet, wobei eine der beiden Formeln in Anwendung kam:

$$G=(H/2)\sqrt{\sec^2 I+3}=H\sqrt{1+(\mathrm{tg}^2\, I)/4}$$

Es kamen derart rund 2500 G-Werte zusammen. Die Einzelwerte wurden auf die Epoche 1922 reduziert, weil für sie die Koeffizienten des quasi-homogenen Felds von L. A. Bauer abgeleitet vorlagen. Um den säkularen Gang von G zu bekommen, sind die Berechnungen von Gauss für 1830, Adams, und Fritsche für 1842, Ad. Schmidt für 1885 und L. A. Bauer für 1922 benutzt worden. Es ergeben sich dann nachstehende Werte für G: 1830, 0.3311Γ; 1842, 0.3303Γ; 1885, 0.3254Γ; 1922, 0.3160Γ (dieser Bauer'sche Wert wird mit G_B bezeichnet). Zwischen den vier Fixpunkten ist linear interpoliert worden. Es findet sich so folgende Reduktionstabelle auf die Epoche 1922:

1905	−0.0043	1910	−0.0031	1915	−0.0018	1920	−0.0005	1925	+0.0007
1906	−0.0041	1911	−0.0028	1916	−0.0016	1921	−0.0003	1926	+0.0010
1907	−0.0038	1912	−0.0026	1917	−0.0013	1922	0.0000	1927	+0.0012
1908	−0.0036	1913	−0.0023	1918	−0.0010	1923	+0.0002	1928	+0.0015
1909	−0.0033	1914	−0.0020	1919	−0.0007	1924	+0.0005	1929	+0.0018

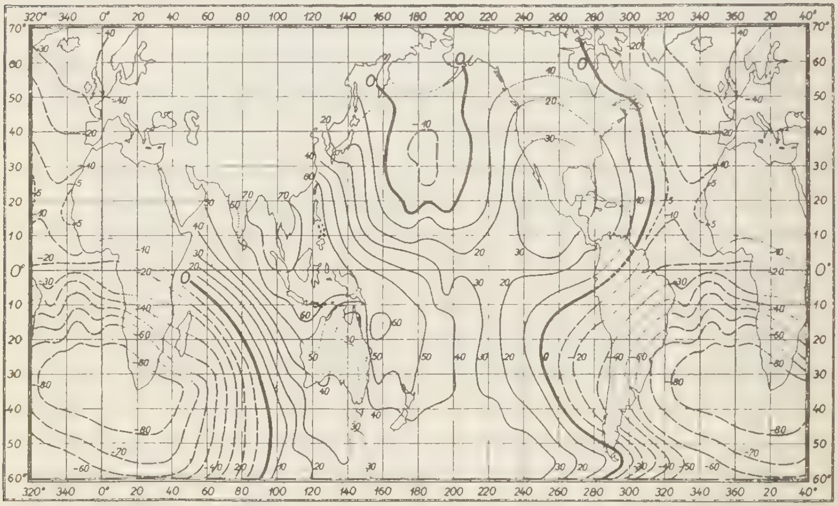
Rechenbeispiel: Gradfeld 20° bis 30° N; 210° bis 230° E. Gr.

Jahr	H	I		G	Red. 1922	G (1922)	$\triangle G=$ [G(1922)− G_B(1922)]
	Γ	°	′	Γ	Γ	Γ	Γ
1915	0.3000	39	01	0.3237	−0.0018	0.3219	+0.0059
1921	0.2896	46	01	0.3262	−0.0003	0.3259	+0.0099
1921	0.2884	45	11	0.3229	−0.0003	0.3226	+0.0066
1921	0.2900	44	30	0.3231	−0.0003	0.3228	+0.0068
1921	0.2913	42	40	0.3206	−0.0003	0.3203	+0.0043
1905	0.2868	47	52	0.3277	−0.0043	0.3234	+0.0074
1905	0.2924	45	28	0.3280	−0.0043	0.3237	+0.0077
1905	0.2927	43	35	0.3243	−0.0043	0.3200	+0.0040
1907	0.2798	48	46	0.3221	−0.0038	0.3183	+0.0023
1929	0.2866	48	10	0.3283	+0.0018	0.3301	+0.0141
					Mittel	0.3229	+0.0069

Der für dies Gradfeld ermittelte Wert 0.3229 kommt nach Ausweis dieser Zahlen auf ±0.00105 oder 0.33% seines Wertes genau heraus. Dies soll nicht mehr sein als ein roher Anhalt für die Bewertung der

Mittelwerte. Ein mittlerer Fehler im strengen Sinn des Begriffes ist es nicht.

Die hier benutzte Art, die Säkularvariation zu eliminieren, wird zukünftig verfeinert werden müssen. Es hat sich bei anderen Studien über die lokale magnetische Konstante gezeigt, dass sie nur den Wert einer Instantan-Konstanten besitzt und im Laufe der Jahre doch recht veränderlich ist. Es handelt sich da nicht um kleine Beträge, sondern an einzelnen Orten um sehr erhebliche, und vor allem variieren die einzelnen Örtlichkeiten ganz verschieden und durchaus nicht im gleichen Betrag mit der säkularen Variation des Durchschnitts aller G-Werte der Erde, also dem magnetischen Moment. Da jedoch im Augenblick noch nicht genügend viel Berechnungen vorliegen, mag es bei dieser einfachen Reduktion auf die Epoche verbleiben.



Die beiliegende Weltkarte enthält in 1/10 Milligauss den Unterschied der G-Werte gegen den für 1922 gültigen Erd-Mittelwert 0.3160 und die durch flächenhafte Interpolation nach diesen Zahlen gezeichneten Linien gleicher inhomogener Magnetisierung der Erde in Stufen 1 Milligauss oder 100γ. Wir schliessen aus ihrer Betrachtung: *Die Verteilung der inhomogenen Magnetisierung über die Ozeane ist eine durchweg systematische und nicht eine rein zufällige.*

Wäre die lokale magnetische Konstante G in engem Sinne lokal, also von den obersten Gesteinen abhängig, so könnten nicht derart weit erstreckte Gleichmässigkeiten existieren. Allerdings handelt es sich ja um Beobachtungen auf See, und die oberen Gesteine können nur in Küstennähe wirksam sein, oder in der Nähe von Inseln; aber auch hier ist nichts Störendes zu erkennen.

Vergleichen wir diese neue, für 1922 gültige Karte der Ozeane mit der früher gebrachten, überwiegend für das Festland für 1915 gegebenen,

so finden wir ein ganz klares Zusammenfallen beider Isomagnetiks, nur dass die neue Karte noch mehr gibt.

Wir ziehen aus dieser Tatsache den Schluss, dass die *inhomogene Magnetisierung der Erde nicht durch die obersten 3 km—das ist die mittlere Meerestiefe—der Erdkruste verursacht werden.*

Damit scheidet alle Klein-Geologie für diese Magnetisierungen aus. Wir müssen den Sitz der krustalen Massen tiefer suchen als je ein Bohrloch dringen wird. Die obersten Schichten der Festländer sind zwar reich an magnetischen Anomalien, aber offenbar spielen sie keine massgebende Rolle mehr für unser hier dargestelltes Feld. Dagegen zeigen sich Gross-Anomalien mitten im Ozean, obwohl dort dicke Schichten unmagnetischen Wassers dazwischen liegen.

Gegenüber der alten, nur auf die Observatorien gegründeten Karte, fehlt zwar der grosse Pol *N1*, da er überwiegend auf Land fällt, jedoch gibt die neue Karte den Ausläufer nach *SE* schärfer und zeigt die Verbindung mit den Meeren im Süden von Australien. Die in diesen Meeren gelegenen Anomalien setzen jene des Festlandes ohne Schwierigkeit fort. Ist aber magnetisch eine einheitliche Anomalie vorhanden, so wird man schliessen dürfen, dass hier auch eine geognostisch einheitliche Provinz vorliegt. Dasselbe gilt für die Umgebung des Pols *N3*, der von Nordamerika her in den nördlichen Grossen Ozean vorstösst. Dazwischen legt sich das Gebiet des Bauer'schen Pols *S3*, das von den Aleuten ausgeht und etwa bis in die Gegend der Hawaischen Inseln reicht. Diese drei Gross-Anomalien bedingen die Verhältnisse im nördlichen Grossen Ozean.

Der südliche Grosse Ozean zeigt sehr ruhige magnetische Verhältnisse, vor allem treten hier keine weiteren Polflächen auf, und nur geringe Gradienten. Daraus vermuten wir, dass sein Untergrund geognostisch gleichmässiger als der des nördlichen Stillen Weltmeers ist und beide jedenfalls von verschiedener Natur. Vielleicht macht sich das, wenn man es nur untersucht, auch noch in anderer Weise geophysikalisch bemerkbar, ich denke dabei an die Fortpflanzungsgeschwindigkeit der Erdbebenwellen.

Ganz anders liegen die Dinge im Atlantischen Ozean. Er muss geophysikalisch von ganz anderer Natur sein als der Pazifik, worüber man sich ja auch in vielen anderen Hinsichten klar geworden ist. Hier dominiert der dem Bauer'schen Pol *N2* der Lage nach entsprechende. Sein Einfluss reicht nach Westen über Südamerika hinaus bis in den Grossen Ozean hinein, und nach Osten bis tief in den Indischen Ozean hinüber, bis unmittelbar an die indisch-australische Sonderanomalie. Auffällig sind die Krümmungen aller isomagnetischen Linien nach Norden hin. Es liegt nahe, dies mit dem Aufquellen des Sima in der mittelatlantischen Schwelle in Verbindung zu bringen. Im *SW* dieser regionalen Störung auf der Höhe der Südspitze von Südamerika finden wir ein nach Osten konvexes Ausschweifen aller Isanomalen, das ebenfalls einer bekannten Bildung des Bodenreliefs entspricht, der Bodenschwelle nach Grahamland zu. Der Nordatlantik hat kein besonders charakteristisches magnetisches Verhalten, doch setzt sich in ihm die mittelatlantische Schwelle fort.

Der Indische Ozean hat magnetisch gar keine selbstständige Stellung; nirgends verrät sich eine unterseeisches Lemurien.

Das Auffallendste an dem ganzen Bilde ist, dass die regionalen

Gross-Anomalien nur aus einigen wenigen Gebieten bestehen, von denen jedes in seinem magnetischen Verhalten jedem anderen unähnlich ist; also müssen auch die geologisch-physikalischen Verhältnisse in jedem Gebiet andere sein. Diese verschiedene Natur des Untergrundes setzt sich beim ersten Blick fast rücksichtslos gegen die Gestaltung der Kontinente und Meere nach den Seiten fort. Man betrachte daraufhin namentlich die südatlantische Anomalie und die nordamerikanische. Erst in den Feinheiten kommt die mittelatlantische Schwelle und jene, welche Südamerika mit der Antarktis verbindet zur Geltung.

Die unterirdischen magnetischen Massen sind offenbar etwas ganz Anderes als die oberflächlichen Kontinente. Unter der früheren Vorstellung, dass die Festländer allein unregelmässig magnetisierte Gesteine besitzen können, und unterhalb der Ozeane petrographische Verschiedenheiten schon allein wegen der grossen Entfernung von dem messenden Instrument nicht mehr wirksam sein könnten, erwartete man, dass die Ozeane so gut wie keine Anomalien mehr aufweisen könnten. Statt dessen setzen sich nach unserer Karte nicht nur die regionalen Grossanomalien des Festlandes in das Meer hin fort, sondern es treten neue Gross-Anomalien auf, die ganz oder zum grössten Teil sich über den Ozeanen bekunden.

Und in der Tat kann man sich dem Schluss nicht entziehen, dass die anomalistische Magnetisierung der Erdkruste in der Hauptsache ihren Sitz in Tiefen hat, die mit der oberflächlich erkennbaren Verteilung von Land und Meer wenig zu tun haben. Einzig allein die südöstliche Fortsetzung der ostasiatischen Gross-Anomalie nach Australien hin und die verhältnismässig kleinen Unregelmässigkeiten in den Isanomalen des Atlantik über den bekannten Bodenschwellen deuten an, dass auch oberflächen-nähere geographische Umstände mitspielen können.

Wir haben in der früheren Arbeit gezeigt, dass das G-Feld nach den Observatorien berechnet, dem "Residual Field" L. A. Bauer's für 1885 bezüglich der Pollagen im Grossen und Ganzen entspricht. Ohne weiteres vergleichbar sind diese beiden Karten nicht, weil Bauer mit Z arbeitet, und G ja alle Komponenten umfasst. Es wurden daher die G-Werte über den Ozeanen benutzt, um aus ihnen die Verteilung der Z-Werte zu berechnen. Es lässt sich leicht ableiten, dass.

$$Z = 2\sqrt{(G-H)\,(G+H)}$$

Zieht man davon die aus dem ersten Kugelfunktionsglied berechneten Werte des quasi-homogenen Feldes ab, so erhält man die gesuchten $\triangle Z$, und damit das Überbleibende Feld in der Definition von Bauer.

Da die Ozeane allein noch nicht ausreichen, eine der Karte von Bauer gleichwertige zu liefern, verzichten wir auf Wiedergabe der so erhaltenen neuen für 1922; es besteht ohnehin der Plan, auch die Festlande in der selben Weise zu bearbeiten, wie hier die Ozeane und dann beides zu vereinen. Dann wird es auch Zeit sein, das Bauer'sche Residual Field neu zu zeichnen. Es genüge, hier zu bemerken, dass sich für die vom Ozean bedeckte Erdoberfläche jetzt ein viel komplizierteres Feld ergibt, das allerdings in den grossen Zügen noch die Gestaltung des Bauer'schen Feldes besitzt, insbesondere alle Grosspole enthält; es treten nur noch weniger weit erstreckte Nebenpole auf.

Die wichtigste Frage ist, ob die Pole in der Zwischenzeit von 1885

bis 1922 ihre Lage wesentlich verändert haben, denn offenbar verlieren unsere Betrachtungen über eine Beziehung zwischen anomaler Magnetisierung und Kontinental-Verteilung ihre geographische Unterlage, wenn das gestörte Feld über die Erde wandern sollte.

Es findet sich hier folgende Übersicht:

Pol	Ozean 1922		Bauer 1885		Verschiebung	
	Br.	Lg.	Br.	Lg.	Br.	Lg.
	°	°	°	°	°	°
N_1	+13	125	+35	110	−22	+15
N_2	−50	325	−50	325	0	0
N_3	+40	276	+42	268	− 2	+ 8
S_1'	+ 5	5	0	20	+ 5	−15
S_1'''	− 5	355	−20	340	+15	+15
S_1''	+58	355	+60	0	− 2	− 5
S_2	−45	155	−45	135	0	+20

Die Verschiebungen in Breite heben sich demnach fast im Mittel auf, und in der Länge findet sich für die 37 Jahre eine Verschiebung nach Ost von i. M. 5°. Gerade in Länge wäre eine Wanderung des Systems am ehesten geophysikalisch zu verstehen. Trotzdem wollen wir den Zahlen kein grosses Gewicht beilegen, und zwar erstens, weil die neue Karte, wie gesagt, doch komplizierter ist, als die alte, vornehmlich aber, weil die zahlenmässigen Ausgangswerte bei beiden so sehr verschieden sind. Bei Bauer waren es die durch viel graphische Extrapolationen verderbten offiziellen nautischen Karten und bei uns wirkliche Beobachtungen. Der grösste Teil der Änderungen in den Z-Werten kann man wohl auf diesen Umstand setzen, sodass die scheinbaren Wanderungen nichts Reelles bedeuten.

Wenn wir die Gestalt des Feldes der anomalistischen Magnetisierung suchen und wollen dafür eine geographisch-geologische Erklärung finden, so ist offenbar eine notwendige Voraussetzung, dass dies Feld unverändert an die Gestaltung der Erdoberfläche gebunden ist. Es muss einst hier die Entscheidung getroffen werden. Das kann nur so erzielt werden, dass man das anomalistische Feld für zwei Epochen aus ganz gleichwertigem Material berechnet. Man wird daher aus der späteren Epoche nur Beobachtungen von solchen Orten verwenden, wo auch zu den Zeiten der ersten Epoche gemessen worden ist. Das gilt natürlich nicht nur für die gerade hier behandelten Fragen, sondern ganz allgemein für alle Fälle, wo man frühere mit späteren Zuständen vergleichen will, also besonders für Studien über die Säkularvariation des beharrlichen Magnetismus der Erde.

Haben wir in den grösseren Tiefen der Erdkruste einige wenige gleichsinnig magnetisierte Massen — und das ist ja das Hauptergebnis unserer Rechnungen — so fragt es sich auch, ob die Stärke ihrer Magnetisierung konstant bleibt. In diesem Falle hätten wir eine verhältnismässig einfache räumliche Verteilung der Säkularvariation zu erwarten. Eine Durchrechnung an Hand der langen Beobachtungsreihen von Greenwich und Göttingen, die beide unter dem Einfluss des Bauer'schen

Pols S_1'' stehen, haben die sich ergebenden theoretischen Bedingungen für einen solchen Fall nur schlecht erfüllt.

Nach den soeben erschienenen Untersuchungen von N. N. Trubjatschinski[2] sind die von H. W. Fisk[3] gefundenen Centra der Säkular-Variation, anmentlich in Z an die alpinischen Synklinalen gebunden, also an Gebilde der Erdkruste, die sich an der Verteilung der Gross-Anomalien nicht beteiligen. Daraus darf man fast schliessen, dass die magnetischen tiefen Massen unserer Karte schon jenseits der säkularen Ummagnetisierungen liegen. Im augenblicklichen Stand der Forschung gibt das nichts weiter ab, als eine Richtlinie für ihre Fortsetzung. Jedenfalls haben die Fisk'schen Centra keinen erkennbaren Zusammenhang mit der Verteilung der Grösse G über die Erde.

Viel eher ist ein Zusammenhang mit der Lage der präcambrischen Massen zu finden, hier decken sich weitgehend magnetische Gross-Anomalien und Geologie[4]. Diese Bemerkung steht in einem gewissen Gegensatz gegen Jenny, der einen Parallelismus der Gross-Anomalien zu den alpinen Faltungen findet, doch gelingt ihm das erst durch Einführung einer Hilfshypothese, dass der jetzige Zustand noch remanent den vor der inzwischen eingetretenen Kontinental-Verschiebung bestandenen wiedergibt. Seine Betrachtungen stützen sich zudem auf ein von Anfang an anders definiertes normales Feld, nämlich auf das der Rotationsachse der Erde parallele.

In der Tat: sobald man davon ausgeht, dass das Hauptfeld der Erde durch ihre Rotation verursacht wird, muss man die äquatorielle Komponente des ersten Glieds irgendwie gesondert erklären, und das Naheliegende ist, dass man auch *sie* als eine Wirkung der Krustenmagnetisierung nimmt. Rechnet man derart, d. h., bildet man die Differenzen zwischen den Beobachtungen und dem Gliede $Z=2g_0^1 \sin\phi$, so erhält man ein vollkommen anderes Bild der Verteilung über die Erdoberfläche. Ich habe eine solche Karte einmal für $g_0^1=0.3254$ und für $g_0^1=0.250$ gezeichnet, die erste Zahl ist der Koeffizient des ersten Gliedes für 1885 nach Ad. Schmidt, die zweite der von mir früher einmal[5] abgeschätzte Betrag für diesen Koeffizienten, wenn man den Einfluss der Rinde abzieht. Zufällig kommt Jenny bei ganz anderer Arbeitsweise auf die ähnliche Grosse 0.240 und legt sie seiner Karte zu grunde. Auch mein Potsdamer Kollege H. Haalck[6] hat eine solche Karte, bezogen auf den ersten Koeffizienten des ersten Glieds bekannt gegeben. Auch auf eine neue, mehr skizzenhafte ähnliche Darstellung von S. W. Visser[7] sei hingewiesen.

Alle diese Karten geben sehr viel einfachere Verhältnisse als bei Benutzung des ganzen quasi-homogenen Felds. Es treten einfach zwei Gebiete auf: ein in Z positives längs des amerikanischen Kontinents, das über den Nordpol hinüber noch mit dem Pol über China zusammenhängt, und einen räumlich bedeutend grösseren negativen Teil für alles Übrige. Centra giebt es nur auf den Festländern, die Ozeane erscheinen nur mit geringen Gradienten belegt. Es ist anzunehmen,

[2]Geotektonik und Geomagnetismus, Verh. 7. Tagung Baltisch. Geod. Komm. (1934).

[3]Unsymmetrical distribution of magnetic secular variation, Terr. Mag., **37**, 235-240 (1932).

[4]vgl. auch W. P. Jenny, Problems in the geologic interpretation of the Earth's major magnetic anomalies, Terr. Mag., **38**, 97-105 (1933).

[5]A. Nippoldt, Terr. Mag., **26**, 101 (1921).

[6]Zs. Geophysik, **8**, 154-163 (1932).

[7]Amsterdam, Proc. Akad. Wet. 3776, 37-81 (1933).

dass auch diese Art von Karten mehr Einzelheiten geben wird, wenn man nicht mehr die vielfach ausgeglichenen und entstellten magnetischen Weltkarten zu grunde legt, sondern neue, unmittelbar auf Beobachtungen gestützte Karten verwendet.

Die grösste sachliche Schwierigkeit liegt darin, die Krustenmagnetisierung rein für sich abzusondern.

Wenn wir aus den Beobachtungen über die ganze Erdoberfläche das Potential ableiten, so hat das quasi-homogene Feld die Gestalt:

$$V/R = g_0^1 \cos u + g_1^1 \sin u \cos \lambda + h_1^1 \sin u \sin \lambda$$

Diese Berechnung erfasst an jedem Orte den Teil der dort herrschenden Magnetisierung, der in die durch

$$\operatorname{tg} u_0 = \sqrt{[(g_1^1)^2 + (h_1^1)^2]}/g_0^1, \qquad \operatorname{tg} \lambda_0 = h_1^1/g_1^1$$

definierte Richtung fällt, d. h., die Komponente parallel der magnetischen Achse der Erde. Das Überbleibende Feld wird rechnerisch durch die Summe aller übrigen der theoretisch unendlich vielen Glieder der Kugelfunktionsreihe dargestellt. Setzt man die Krustenmagnetisierung damit gleich, und analysiert ihr Feld auf die gleiche Weise, so kann danach in ihm kein erstes Glied auftreten; sie besitzt also weder ein magnetisches Moment, noch hat sie eine magnetische Achse.

Solche Felder sind physikalisch durchaus möglich, jeder homogen magnetisierte Ring ist ein Beispiel dafür. Arbeiten wir beim Erdmagnetismus so, so heisst das, wir haben die Hoffnung, dass die Erdrinde eine derartige ringförmige Magnetisierung trage. Die Schiefe des Normalfeldes gegen die Rotationsachse wird —wenn sie überhaupt erklärt wird —als eine Art Überbleibsel aus Zeiten angesehen, wo die Erdachse der magnetischen noch näher lag und schliesslich einmal mit ihr zusammenfiel. Aus dieser Vorstellung heraus erwuchs die Meinung, dass ein schiefes Feld für die ganze Erde normal sein könne.

Es ist jedoch klar, dass in dem ersten Glied, wie eben gesagt, alle Komponenten enthalten sind, die parallel der Gesamtachse liegen, also auch jene, die dem Krustenfeld zukommen. Nennen wir *gh* jetzt die Koeffizienten des wirklich normalen Erdfelds, und $\mathfrak{g}\mathfrak{h}$ jene der Krustenmagnetisierung allein, so wäre das gesamte Feld darzustellen durch:

$$V/R = (g_0^1 + \mathfrak{g}_0^1) \cos u + (g_1^1 + \mathfrak{g}_1^1) \sin u \cos \lambda + (h_1^1 + \mathfrak{h}_1^1) \sin u \sin \lambda$$

Wegen der Ein-Mehrdeutigkeit der Beziehung zwischen Feld und verursachender Verteilung der magnetisierten Massen ist eine Trennung aus den Zahlwerten allein unmöglich. Man muss schon von aussen neue physikalische Vorstellungen einführen.

Einen gewissen Anhalt zu neuen physikalischen Gedanken liefern Untersuchungen über die zeitlichen (Säkularvariation) und örtlichen Veränderungen der Koeffizienten und hier besonders die[5] sogenannte "Torsion" der magnetischen Achse der Erde. Aber das Beste wäre es, wenn es gelänge, das mit cos *u* proportionale Teilfeld des ersten Gliedes aus der Rotation der Erde heraus zu erklären und zu berechnen. In diesem Falle würde sich g_0^1 als einziger Koeffizient des Normalfelds getrennt von $\mathfrak{g}_0^1$ ergeben, und die Komponenten der äquatoriellen

[5] L. A. Bauer, Terr. Mag., **17**, 79 u. ff. (1912) und A. Nippoldt, **26**, 101 (1921).

Quermagnetisierung fielen in ihrem Gesamtbetrag auf die Kruste. Versuche, dies Ziel zu erreichen, sind z. Z. in Gang.

In der Zwischenzeit helfen uns nur tastende Versuche, die alle darauf hinauskommen, das Verhältnis $g_0^1 : \mathfrak{g}_0^1$ abzuschätzen. Der vom Verf. und von S. W. Visser benutze Wert $=0.24$ ist ein erster Versuch in dieser Richtung gewesen. Trotzdem ist diese Unterlage noch zu schwach, um danach das Feld der Krustenmagnetisierung ableiten zu dürfen. Das war der Grund, warum wir noch bei der alten üblichen Definition verblieben sind.

Bei weiteren Schlüssen aus unserer Ozeankarte ist zu bedenken, dass sie die Ausdehnung der in der Tiefe lagernden magnetisierten Massen erkennen lässt, dass sie aber nichts darüber aussagt, warum jene Massen verschieden magnetisiert sind. Natürlich ist die Magnetisierungsfähigkeit entscheidend, aber diese kann sowohl durch das petrographische Material, als auch durch seine Zustandsgrössen gegeben sein also durch Krystallisationszustand, Temperatur, u. a. m.

In diesem Sinne betrachtet besagt unsere Karte, dass die Wasserdeckung durch die Weltmeere keine wesentlichen Wirkungen auf die Zustandsphasen der tieferen magnetisierten Schichten äussert.

Magnetisches Observatorium,
Potsdam, Deutschland

DETERMINATIONS OF THE MAGNETIC DECLINATION AT SEA ON A MOTOR-BOAT

By Gustaf S. Ljungdahl

Abstract—An account is given of the method used to determine the magnetic declination at sea on an ordinary motor-boat. The reliability of the results is ascertained by repeat-measurements. Dynamic deviations of the liquid-compass are demonstrated. Results in a rough sea determined by observations on a single heading may be less trustworthy than a mean obtained on various headings.

During the summers of 1925 and 1926 the Estonian non-magnetic yacht *Cecilie* was employed by the Hydrographic Service (Kungl. Sjökarteverket) Stockholm, in collaboration with the Meteorological Central Office, Helsingfors, for the measurement of the magnetic declination, horizontal force, and vertical force at 117 sea-stations in the Baltic[1]. These measurements embraced mainly an area of one to one and one-half degrees of latitude and about one degree of longitude, situated in the narrow waters between Sweden and the Finnish Åland Islands northeast of Stockholm. During these observations the *Cecilie* was towed by a survey-steamer.

During the years 1932-34, the above-mentioned measurements were

[1]J. Keränen and H. Odelsiö, Magnetic measurements in the Baltic Sea, South Quarken, and Northern coast of the Baltic Sea, Stockholm, Kungl. Sjökarteverket, Jordmag. Pub., Nr. 5-6; Helsingfors, Suomen Valtion Met. Keskuslaitos, Maamag. Tutk., Nr. 14, 16 (1928 and 1927).

Fig. 1—The motor-boat showing compass-stand before the mast

increased by 160 determinations of the magnetic declination on board an ordinary motor-boat (Fig. 1) belonging to the Hydrographic Service. An account of the results, obtained in 1932, has been published[2] but a short recapitulation of the method used may be of some interest, since the measurements were made from a relatively small and cheap vessel. The craft was only about ten meters in length, driven by a crude-oil engine of 15 horse-power. During the first experimental period the crew consisted of two observers and one "all-round man," but for the major part of the expeditions during the three summers only one observer and the "all-round man" were engaged. The measurements were always made during swings and twice on each helm.

The instrument used was liquid-compass No. 15802, manufactured by Lyth of Stockholm. In 1932, the card was an ordinary linen one, graduated from 0° to 360°. Since that card was affected by some errors of graduation, it was replaced on the subsequent expeditions by one of mica with engine-divided graduations. As on the *Cecelie*, the observations were made with the simple device known as the shadow-pin. The errors due to eccentricity or to inclination of the shadow-pin were eliminated by turning the compass-bowl through 180°. The diameter of the shadow-pin was equal to the distance between two adjacent lines of the graduation on the card—this greatly facilitated the readings.

The compass was placed on a wooden stand before the mast. This place was chosen, because a location farther abaft the motor and other large masses of iron would have caused rather large deviations. In all other respects this location may be considered a bad one. If the compass were placed amidships and lower, its behavior would have been much less affected by the motion of the sea.

The magnetic deviations of the compass, as obtained from the eight main headings, were

Deviation in $1932 = A + 73'.4 \sin \psi + 29'.0 \cos \psi + 34'.2 \sin 2\psi - 5'.4 \cos 2\psi$; probable error $= \pm 1'.7$

Deviation in $1933 = A + 81'.2 \sin \psi + 37'.5 \cos \psi + 40'.5 \sin 2\psi + 1'.5 \cos 2\psi$; probable error $= \pm 2'.3$

Deviation in $1934 = A + 93'.1 \sin \psi + 32'.5 \cos \psi + 38'.7 \sin 2\psi + 0'.1 \cos 2\psi$; probable error $= \pm 2'.4$

The constant correction A refers not only to the collimating error, that is, the angle between the magnetic axis of the magnetic-system and the north line of the card, but also, perhaps, to the dynamic deviation. In my opinion, there may be a theoretical possibility that the regular vibrations of the single-cylinder motor might produce periodical fore-and-aft and athwartship oscillations of the fore part of the boat, which, in turn, might cause the compass to swing in small circles or ellipses.

In 1932 the constant A was determined by comparison with values at stations on shore. During the years 1933 and 1934 the expeditions were begun and finished by swingings on a control-station in Lake Mälaren, in the immediate vicinity of the magnetic observatory at Lovö (Stockholm). At this control-station the magnetic declination was determined previously (during the spring) on the ice with instru-

[2]G. S. Ljungdahl, An attempt to determine the magnetic declination at sea on board an ordinary motor-boat. Hydrogr. Rev., **10**, No. 1, 52-60 (1933).

ments of the land survey. The values of the constant A, thus obtained, may be regarded as quite reliable.

From later repeat-measurements at a number of the sea-stations, made in 1932, it appeared that a correction of about $+0°.3$ should be added to the published values[2], that they might agree with the results of measurements in 1933 and 1934. The positions of these stations could all be found with sufficient accuracy, for example, by using two terrestrial angles.

All the observations were made as determinations of deviation at the eight main headings north, northeast, east, etc., by swinging clockwise as well as in the opposite direction. At each heading the boat was steered with as little yawing as possible for at least one minute before the reading was taken. Between the first and the second swings the compass-bowl was turned through 180°, as well as between the third and the fourth swings. Thus each series of obse vations consisted of at least 32 readings -four on each heading. The effect of the errors in reading was considered to be sufficiently minimized by the large number of observations taken. Thus, an error of $\pm 1°$ in a single reading affects the average by $\pm 1/32°$ or $\pm 2'$. All the readings were corrected for magnetic deviation in order to estimate the relative accuracy of the results, as expressed by mean errors.

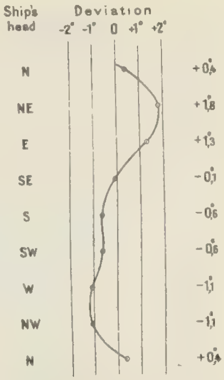


FIG. 2—Magnetic deviation of motor-boat in 1932

During each expedition several of the earlier stations were reoccupied, thus making possible judgment of the reliability of the results. Observations made during different years and reduced to the common epoch 1929.5, show in only a few cases discrepancies greater than 0°.2 (see Table 1).

Thus it appears the accuracy of the determinations of the magnetic declination at sea-stations may be considered quite satisfactory, especially

TABLE 1—*Repeat-determinations of magnetic declination at sea-stations*

Station No.	Latitude, north		Longitude, east		$D_{1929.5}$ according to measurements in years				Adopted value	Mean error $= \triangle$
					1925*	1932	1933	1934	1929 5	
	°	′	°	′	°	°	°	°	°	°
14	60	28.9	19	02.7	−0.8	−0.6	.....	.....	−0.7	±0.1
27	59	49.8	19	05.2	−2.4	−2.2	.....	−2.3	−2.3	±0.1
137	59	44.9	19	16.0	−1.7?	.....	.....	−2.2	−2.2	(0.5)
136	59	43.7	19	09.6	−2.3	.....	.....	−2.3	−2.3	±0.0
64	58	58.5	18	40.0	−2.9	.....	−2.7	.....	−2.8	±0.1
0	58	52.0	18	01.0	.....	−3.2	.....	−3.3	−3.2	±0.1
32	58	48.4	17	57.8	.....	−4.4	−4.3	−4.3	−4.3	±0.1
28	58	43.9	18	04.4	.....	−3.1	−2.7	−3.0	−2.9	±0.2
30	58	39.9	18	09.6	.....	−2.0	−1.8	.....	−1.9	±0.1
36	58	36.1	19	08.6	.....	−2.8	−2.5	.....	−2.6	±0.2
37	58	24.1	19	12.2	.....	−2.0	−1.9	.....	−2.0	±0.1
91	57	44.8	16	59.7	.....	.....	−3.0	−3.2	−3.1	±0.1

* Results obtained on the *Cecilie*.

in view of the great magnetic irregularities prevailing in the Baltic regions. While the accuracy of the results obtained at stations on shore undoubtedly is greater than at the sea-stations, the former, as a rule, may be more affected by local influences of the underlying rock, since at land-stations the instruments are nearer the rock. Therefore, it may be a question whether the sea-stations are not in general the more representative.

It may often happen that with a rough sea, the observations made on certain headings are less trustworthy than those made on the other headings. In such cases the mean was computed from the remaining headings only. Such a procedure may be considered arbitrary but as a matter of fact it corresponds to that used on board the *Cecelie*. The principal difference is that the observations on the *Cecelie* were taken only on that heading which was considered the best with regard to sea and swell, whereas observations on the motor-boat were taken on all the eight main headings, the selection being made during the subsequent computations.

The greatest difficulty inherent in this kind of work lies in deciding when it will be possible to get trustworthy observations on a rough sea, that is, to determine whether or not conditions are favorable for observation. This difficulty will always cause much trouble. As a rule, it was impossible to work with greater winds than 2 to 3 Beaufort and with corresponding sea-conditions. If often happened too that with lesser winds or even with no wind at all, the motion of the sea prevented observations.

All movements produced by the rolling, pitching, yawing, rising, and falling of the vessel probably produce dynamic deviations of a complex nature, on the assumption that the center of gravity of the magnetic needle, or system of magnetic needles, does not coincide with the point of suspension[3], and that the card-system is immersed in a damping fluid[4]. Every displacement of the card-system, obviously, may impart to the fluid some rotational motion. The rolling and pitching may sometimes combine with the rotational movements, especially when the vessel is running at an oblique angle to the waves. The compass, behaving as a conic pendulum, causes the fluid to rotate (wherefore the shape and dimensions of the card-system must be carefully designed) and the rotation may continue to deviate the card-system owing to the inertia of the fluid.

I have tried to verify the actual existence of such deviations by statistical treatment of the observed values from stations occupied during a somewhat regular sea. Unfortunately, the records from the stations do not always contain sufficiently detailed information regarding sea and swell; however, at a number of stations during 1933 and 1934, the annotations indicate that the movements of the sea were probably rather regular. The regular sea may, as a rule, have been produced from an almost calm surface through the effect of increasing wind. These cases are treated statistically. (Results obtained when the wind was coming from one direction and the sea from another or when the sea apparently was moving in more than one direction were not included.)

[3]Fr. Bidlingmaier, Deutsche Südpolar Expedition, 1901-1903, **5**; Erdmagnetismus, **1**, Heft 2, Teil I, 273-309 (1909); W. J. Peters, Tilting deviations in magnetic declinations, Terr. Mag., **34**, 93-115 (1929); W. J. Peters, Present status of the investigation on dynamic and tilting deviations in the Department of Terrestrial Magnetism, Carnegie Institution of Washington, Terr. Mag., **39**, 203-207 (1934).

[4]I have given the expression "dynamic deviations" a somewhat broader sense than usual, including the deviations, produced by the rotation of the fluid of a liquid-compass.

The material treated was divided into two groups: (1) One group included values obtained at stations when the sea was almost calm and (2) the other included values obtained when the wind was from 2 to 3 Beaufort with corresponding sea.

Most of the 24 stations forming the first group were located in the Stockholm Archipelago, where the waves were too small to produce any noticeable rolling or pitching. The difference between a single observed value, corrected for magnetic deviation, and the adopted mean of the station is considered as a *residual consisting of a systematic dynamic deviation as well as of accidental errors*. The average residuals from the 24 stations of the first group, arranged according to the eight main magnetic headings, are shown in Figure 3. The dotted lines signify the limits of the mean errors of the averages. (The mean error of the value of a single station will be $\sqrt{24}$ or almost five times greater.) The waves were in one case coming from north, in five cases from northeast, in two from east, in four from southeast, in five from south, in five from southwest, in one from west, and in one case from northwest. The average residuals of the same group, arranged according to the direction of wind and waves, are shown in Figure 4. A small dynamic deviation is indicated.

The second group includes values from 22 stations in the open sea. Here also all observed values are corrected for magnetic deviation. The average residuals, arranged according to the direction of wind and sea, are shown in Figure 5. The dotted lines signify the limits of the mean

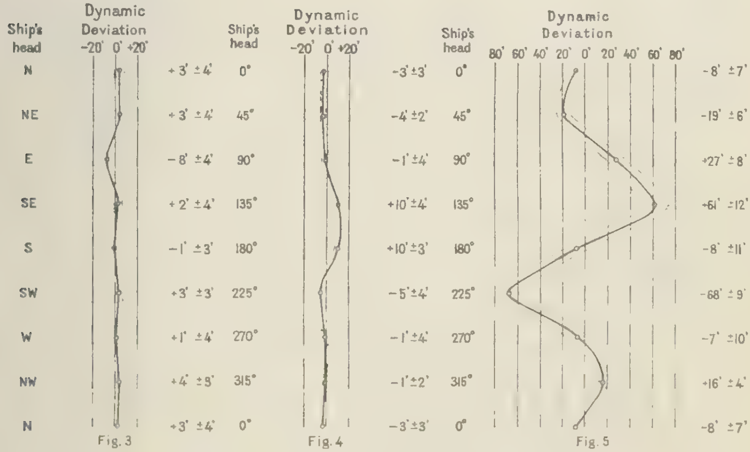


FIG. 3—Average residuals, corrected for magnetic deviation, from 24 stations, for wind corresponding to 1 Beaufort and for almost calm sea with very small waves

FIG. 4—Average residuals of Figure 3 arranged according to direction of wind (0° signifies boat running with wind and other figures signify helms clockwise from that course)—a small dynamic deviation is indicated

FIG. 5—Average residuals, corrected for magnetic deviation, from 22 stations in the open sea, for wind of 2 to 3 Beaufort and corresponding fairly regular sea, arranged according to direction of wind and waves—an influence of dynamic deviation is clearly indicated

errors of the averages. (The mean error of the value from a single station will be $\sqrt{22}$ or not quite five times greater.) The waves were in three cases coming from north, in nine cases from northeast, in one case from southeast, in four cases from south, in three from southwest, and in two from northwest. The influence of dynamic deviation is clearly seen. The average dynamic deviation, as shown in Figure 5, is rather regular. Running with the waves (0°), and against the waves (180°), the dynamic deviation is, on an average, almost nil. With the waves abeam (90° and 270°), the same condition would have been expected but a tendency of dynamic deviations is indicated, with opposite sign for waves abeam on port or on starboard. With the waves on the bow (135° and 225°), the dynamic deviation reaches its greatest value—about 1°—with opposite sign for waves on the port bow or on the starboard bow. With the waves on the quarter (45° and 315°), the dynamic deviation is about 0°.3, with opposite sign for waves on the starboard or on the port quarter. For waves on the quarter, the mean error is only about half as large as for waves on the bow.

As may be readily understood from the smoother movements of the vessel for waves directly aft or on the quarter, the best observations ought to be expected from these courses and, moreover, the mean errors are also shown to be least. The qualifications for dynamic deviations remain, however, and with a disturbed sea the observations on a single heading (for example, while under tow) may be less trustworthy than a mean obtained from various headings suitably symmetrical as to sea and swell.

Notwithstanding the *average regularity* of the dynamic deviation of a given compass, on a given boat, and for a certain degree of disturbed sea, observations on a single heading may always be somewhat doubtful when the craft is rolling and pitching.

During the three expeditions in question, the surveys embraced mainly an area north of the Stockholm Archipelago and another area between Gotland and the mainland of Sweden as shown by the chart of Figure 6. This chart shows clearly local as well as pronounced regional anomalies of the magnetic declination. The existence of a number of very prominent local anomalies, previously determined, nevertheless, is omitted on the chart. In the western part, the magnetic conditions generally are irregular; more to the east they are more homogeneous over greater areas but are by no means normal. The stations on shore in northern Gotland, for example, show the same irregularities as the sea-stations. It must be supposed that the causes of these irregularities are to be found at very great depths. With the exception of certain regions with heterogeneous material at the surface, Gotland consists mainly of non-magnetic Cambro-Silurian strata. Regarding the Archaean beneath Gotland nothing is known but the fact that its upper level at Visby lies about 388 meters below sea-level and that the rock there is a gneiss containing garnet. It is probable therefore that the regional anomalies in question may arise from causes at *much* greater depths.

The results of magnetic measurements at sea-stations are, as a matter of fact, the only basis for drawing conclusions regarding the conditions at great depths beneath the Baltic Sea. (Gravity-measurements probably can not be relied on, at any rate in the immediate future.) From a geophysical point of view it appears extremely desirable that magnetic

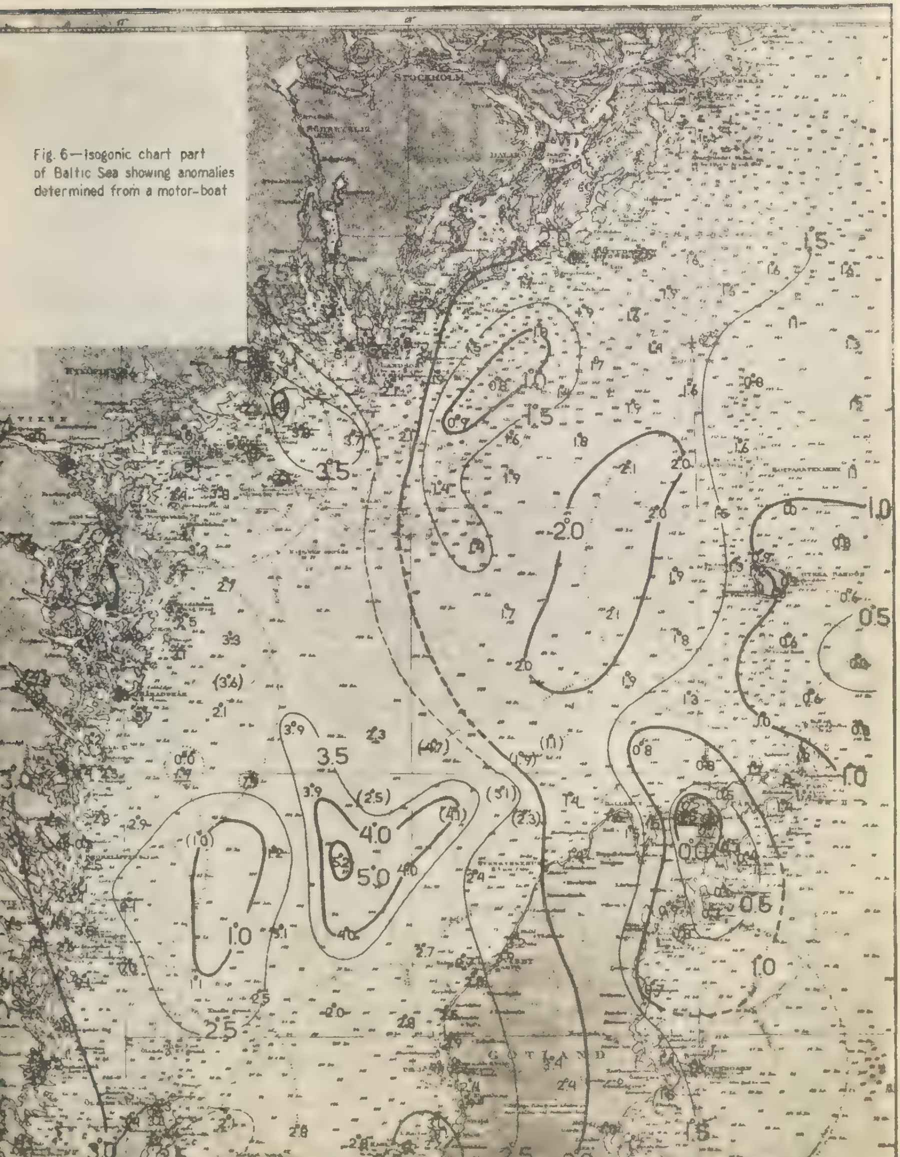


FIG. 6—Magnetic declination, epoch 1935, westerly declination prevails (underlined values signify easterly declination; parentheses signify doubtful values; dotted rings signify values in locally perturbed regions)

surveys at sea be continued and that they include magnetic elements other than the declination. From the navigational viewpoint, the need of additional measurements appears beyond question.

The experiences from the three expeditions seem to warrant the following conclusions:

(*a*) From a motor-boat of the size used it was possible to obtain fairly reliable determinations of the magnetic declination at sea-stations with an accuracy of $\pm 0°.2$, but only under especially good conditions. For a force of wind greater than 2 to 3 Beaufort and a corresponding sea, it was found impossible to secure trustworthy results.

(*b*) The movements of the vessel owing to perturbed sea, always caused dynamic deviations of the card-system of the liquid-compass used (dynamic deviations also embracing the effect of rotation of the fluid). Dynamic deviations were indicated when the sea was almost calm with very small waves and even without any noticeable rolling or pitching (see Fig. 4). With increasing movements of the vessel the dynamic deviations increased (see Fig. 5). The observational data were not sufficient to define the relation between the dynamic deviations and the state of the perturbed sea.

(*c*) By swinging the vessel on two helms—port and starboard—and by steadying on the eight cardinal and intercardinal points for each helm, the magnetic deviation was sufficiently eliminated and also, to a certain degree, the effect of dynamic deviations.

(*d*) The motor-boat used, however, was not suitable for magnetic measurements. It was too easily moved, even by a slightly perturbed sea. The magnetic deviation was changing too rapidly (see Fig. 2), especially on headings northwest, north, and northeast. The impossibility of obtaining determinations of magnetic elements other than those of the declination was a serious deficiency.

(*e*) In my personal opinion, a high-sea motor-boat about the size of those used by Swedish fishermen would be very appropriate and would not be unreasonably expensive. Such a vessel, about 18 meters in length over all and 5.5 meters in breadth, built of oak, with all iron mainly concentrated in the motor, anchors, chains, and winches, would allow distances of 5 to 6 meters from the compass to all the larger or movable masses of iron, and 3 to 4 meters apart for the magnetic instruments. By suitable ballasting it should be possible to obtain the requisite steadiness. The cost may be estimated at $10,000 including crude-oil motor, etc., but excluding suitably arranged living quarters below and the instrumental outfit.

(*f*) Considering that the Swedish waters are among the most magnetically disturbed regions (see, for example, Fig. 6), it seems extremely desirable, from a geophysical as well as from a navigational point of view, that the magnetic measurements in these waters be continued—including determinations of all three magnetic elements.

Magnetic Section,
Hydrographic Service,
Stockholm, Sweden

CONTINUATION OF THE OCEANIC MAGNETIC SURVEY OF THE CARNEGIE INSTITUTION OF WASHINGTON BY THE BRITISH ADMIRALTY

By J. A. Fleming

One of the major objectives of the Department of Terrestrial Magnetism of the Carnegie Institution of Washington has been the accomplishment of the magnetic and electric survey of the Earth both on land and on sea. The Institution's earliest work at sea was done with the chartered vessel *Galilee* during 1905-08. The experience gained during her three cruises proved conclusively that oceanic observations of the magnetic elements sufficiently precise for practical and scientific needs could be assured only by a vessel designed specially for such work. The *Carnegie* was accordingly designed in 1908 primarily for magnetic and electric surveys and investigations and her construction and equipment were completed in 1909. Through the seven cruises of this unique vessel during 1909-29 in all oceans, the international character of the activities of the Carnegie Institution of Washington is probably as widely recognized as through any other of its agencies. The theoretical and practical values of the knowledge acquired and of the resulting discussions of the Earth's magnetic and electrical fields are attested by many expressions of appreciation made by the leading hydrographical establishments and by investigators of geophysics in all countries.

The vast amount of observational work accomplished before the destruction of the *Carnegie* by explosion and fire in the harbor of Apia, Western Samoa, November 29, 1929, was obtained during seven cruises aggregating 297,579 nautical miles, or 342,681 statute miles. The combined data resulting from these cruises and the three previously made by the *Galilee*, include values of declination at 3844 points, of inclination and of horizontal intensity at 2321, and 2322 points, respectively, and of the atmospheric-electric elements on 1913 days.

While more information on secular-variation changes in the Earth's magnetism is required for navigation, yet future magnetic and electric data over the vast extent of the oceans are far more necessary for the advancement of theoretical studies. The full value of magnetic results of the few earlier expeditions under government auspices has never been attained because of the shortness of the cruises. A point of first importance in considering the continuation of the work of the *Carnegie* is that of the enhanced theoretical value of the work already done which would result from further surveys of like accuracy. As an example of the theoretical application, attention may be called to the apparent diminution of the intensity of the Earth's magnetic field discovered by the Department's investigations of the data thus far obtained, this diminution being practically zero over continental areas and of an appreciable amount over oceanic areas. The confirmation and interpretation of this deduction doubtless will be important in geophysical and geological research to advance understanding and interpretation of Earth phenomena. On the side of practical application the increasing use of the oceans in the commerce of nations makes the continuation of the survey a matter of international concern and benefit.

Those theoretical investigations demanding continuation of the oceanic survey in terrestrial magnetism include, among others, the following:

(*a*) Determination of secular-variation of progressive changes of the

Earth's magnetic field involving particularly their accelerations which the data accumulated so far indicate can not be extrapolated reliably over periods as long as five years. A definite control is necessary for a number of epochs to facilitate the investigation of causes producing and governing these progressive changes which, it appears, would be favored by accurate knowledge of their accelerations and distribution.

(*b*) The study of regions of local disturbance and particularly of those indicated by the work of the *Carnegie* over "deep-sea" areas including accompanying determination of oceanic depths by sonic-sounding devices and of gravity.

(*c*) The determination of additional distribution-data in a few large areas not already covered.

As regards the domain of terrestrial electricity continuation of the survey of the oceans initiated during the cruises of the *Carnegie* is desirable in several directions. Among these are the following:

(*a*) Additional determinations to establish changes in the values of the atmospheric-electric elements with geographic position. Such distribution-data are needed in the further investigations of the origin and maintenance of the Earth's electric charge and of the relations to its magnetic condition.

(*b*) More and widely distributed determinations of the diurnal variations in atmospheric electricity particularly to confirm the discovery that such variations in the potential gradient progress with universal time, a deduction first indicated from results obtained on the *Carnegie*. Conditions at sea for such work are superior to those on land where variable meteorological disturbances and topography mask the true characteristics of the phenomena.

(*c*) Determinations and investigations of earth-currents—a field not yet touched at sea. Two outstanding characteristics of the water-area of the globe are (1) its extent and (2) its far greater homogeneity as compared to the land-area.

The question arises whether the theoretical requirements might not be met in a less expensive way than through construction and maintenance of a vessel similar to the *Carnegie*. A careful study was made by the Department subsequent to the loss of the *Carnegie* to determine what might be done in an attempt to control magnetic secular-variation data through observations on land only over the oceans between 60° north and 60° south latitude[1]. The maximum control so effected would result from 150 secular-variation stations along the coasts of the continents and on islands; about 90 of these have been occupied by the Carnegie Institution of Washington one or more times during 1905 to 1935, but the remainder include the more inaccessible islands of the oceans and are subject, generally, to magnetic local disturbance. Such disturbance introduces uncertainties both in the effects upon secular-variation changes and in the relation between the normal and the island value, even though the inaccessibility of stations insures possibility of exact reoccupations. The reduction to common epoch would be more difficult because of the length of intervals between reoccupations and of the lack of the better distribution of data which would result from observations at sea. The study shows that the regions for which the necessary data for the con-

[1]In any case requisite additional data on land- and ocean-areas in the polar regions beyond the parallels of 60°—less than one-seventh of the surface of the globe—can be secured only, as in the past, through or in cooperation with special expeditions.

tinued theoretical investigations would be lacking are very large even if the complete scheme for control by observations on land could be carried out as based on the assumption that the distribution of secular-variation stations need not be greater than one every 800 miles. These areas (see Fig. 1) approximate 3400 by 800 miles in the north Pacific,

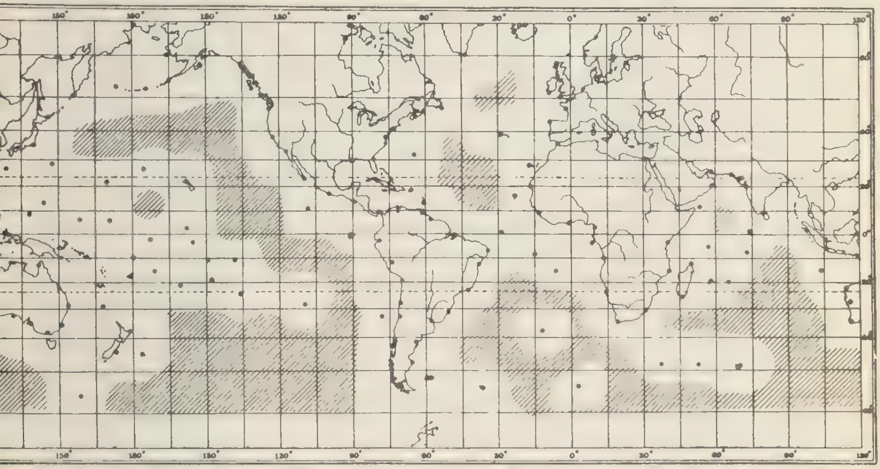


FIG. 1—SHOWING OCEANIC AREAS (SHADED) BETWEEN PARALLELS OF 60° NORTH AND SOUTH LATITUDE FOR WHICH SECULAR VARIATION OF MAGNETIC ELEMENTS COULD NOT BE CONTROLLED BY LAND STATIONS ON CONTINENTS AND ISLANDS

3600 by 1500 miles in the east central Pacific, 3600 by 1800 miles in the south Pacific, 600 by 600 miles in the north Atlantic, 2400 by 800 miles in the middle north Atlantic, 1900 by 900 miles in the west south Atlantic, 1500 by 700 miles in the east Indian, 3600 by 750 miles in the central Indian, and 2400 by 900 miles in the southeast Indian to the south of Australia. [In view of the local disturbances existing at many of the possible island-stations, which doubtless would make data from a majority of them unsuitable for discussion, these areas are actually much greater than indicated in Figure 1.] The great desirability of continued work at sea is emphasized by the fact that these areas involve portions of the Earth's surface where there are at present the greatest irregularities in the progressive character of the secular variation, namely, in the central and south Atlantic, Indian, north Pacific, east central Pacific, and south Pacific oceans.

On the other hand, the lack of a survey-vessel would mean that future observations for the distribution of the absolute values of the atmospheric-electric elements could be made at relatively few stations and at relatively great expense since, to eliminate for short series of observations the topographic and meteorological conditions at stations on land, only selected points in wide bays or estuaries could be used where it would be possible to observe on floats. Atmospheric-electric observations could be obtained on board ordinary vessels, and doubtless some of the maritime companies could be interested to the extent of permitting the installation of the special equipment at reasonable cost, but it is not feasible to obtain on such vessels the calibration-observations re-

quired for the determination of the necessary reduction-factors nor, despite the most earnest desire to cooperate, is it possible to control the deck-space and eliminate vitiating effects of smoke and exhaust gases. Furthermore, it would be necessary to repeat such work and control of such conditions on many vessels in order to accomplish the requisite distribution of observations over the oceans. Despite the considerable expense that would be incurred, the accumulated data would be subject to many uncertainties and would involve an expenditure of time for reductions in the office out of all proportion to that required were there a survey-vessel available.

Because of the great desirability of continuing the operations conducted for a quarter-century by the vessels of the Carnegie Institution of Washington, it therefore is especially gratifying to learn, in view of the Institution's decision not to replace the *Carnegie* by a similar vessel, that the British Admiralty has decided to build one. The chief reason for this action on the part of Great Britain is to be found in her world-wide maritime interests. Magnetic charts published for the last two decades by the American, British, French, German, and other governments for use at sea have been based in an increasingly large degree upon data obtained by the *Carnegie*. There are now serious gaps in the present data which would have been filled had the *Carnegie* completed her last cruise and had the rapid change in the secular variation in certain regions been determined. One of the first tasks, therefore, of the new vessel would appear to be the repetition of the observations of the *Carnegie* in these regions to determine the secular change so that the isogonic charts may be corrected to date and prepared for succeeding epochs.

It is to be hoped also that the Admiralty, as was done on the last cruise of the *Carnegie*, may find it possible to have the program include, besides terrestrial magnetism and electricity, also marine meteorology, physical and chemical oceanography, marine biology, topographical survey of ocean beds, and electrical phenomena of the atmospheric ocean over the sea.

The details of the design of the new vessel have not yet been made public although Rear-Admiral Edgell, Hydrographer, advises it probably will be somewhat larger than the *Carnegie*. The proposed instrumental equipment will parallel closely that used on the *Carnegie* as it has not appeared advisable to depart from designs gradually evolved from the experience of many years of observational work at sea. It is expected that the vessel will be ready for commissioning some time in 1936.

The building of this new vessel to continue the oceanic survey is an outstanding event in the advance of geophysical research. Geophysicists everywhere will welcome this action and will extend cordial cooperation and collaboration to achieve its aims. Not only will the resulting additional observations increase the opportunities of geophysical investigation but they will enhance the value of the earlier data. In view of the long record of valuable scientific work which the British Admiralty has to its credit, students of terrestrial physics may look forward with confidence to further oceanic data of the highest quality and to the further enrichment of our knowledge of Earth science.

Department of Terrestrial Magnetism,
Carnegie Institution of Washington.
Washington, D. C.

SECULAR CHANGE IN THE MAGNETIC SOLAR-DIURNAL VARIATIONS AT THE HUANCAYO MAGNETIC OBSERVATORY

By A. G. McNish

Abstract—A complete change of the character of the solar-diurnal variations of vertical magnetic intensity observed at the Huancayo Magnetic Observatory during the past sunspot-cycle is described. Characteristics of the southern type of variation, present during the northern summer solstice during 1922, give place to a form of variation characteristic of the Northern Hemisphere in 1932. This change is associated with the secular change of the Earth's general magnetic field which consists partly of a southern shift of the magnetic equator in the region around Huancayo.

Detailed features of this change are discussed and found to be qualitatively in agreement with expected changes. The large magnitude of the changes indicates that the solar-diurnal variation is a complex function of the Earth's general field such as is called for by the atmospheric-dynamo theory of Balfour Stewart. Statistical considerations lead to the conclusion that the reality of the change is established to an extraordinarily high degree of probability.

It is quite generally accepted that the magnetic solar-diurnal variations are due to an effect of the Sun acting in conjunction with the Earth's general magnetic field. A few theories, proposed many years ago, which assumed that the variation-field was superposed upon but independent of the Earth's general field met with insuperable difficulties. Distinctly different variations recorded at stations in approximately the same geographic latitude have been explained as due to irregularities in the Earth's general field.

If the Earth's general field does play an essential part in causing the variation-field then there should be a secular change in the form of the variations at every station following the secular change in the Earth's general field. Owing to fluctuations in the variation-field caused by seasonal effects and by sunspot-effects the observation of such secular changes is extremely unlikely. Even the lengthy series of data from Bombay and Greenwich show no distinct change in the form of the diurnal variations from 1879 to 1923 and from 1889 to 1923, respectively. Such changes, however, should not be expected at those stations during a few decades for the grosser features of secular variation are consummated but gradually. On the other hand, a magnetic observatory located in one of the transition-regions where the form of the diurnal variation changes rapidly with location might be expected to show a definite trend in the form of the variations within an even shorter time.

The observatory of the Carnegie Institution of Washington at Huancayo, Peru, is advantageously located for the discovery of such an effect. Shortly before the Observatory was established in 1922 the magnetic equator was slightly north of the site of the Observatory. When magnetograph-observations were begun the magnetic equator had shifted to a position south of the Observatory, which trend has continued up to the present time at a fairly constant rate. The value of the vertical intensity and inclination in 1932 were 1021γ and $1°\ 58'.5$, respectively, both characteristic of the Northern Hemisphere. This shift in the magnetic equator is illustrated in Figure 1.

The effect of this shift of the magnetic equator on the magnetic diurnal-variations is illustrated in Figure 2, in which are plotted the variations in magnetic vertical intensity for the five international quiet days of May, June, July, and August from 1922 to 1932. The international quiet days were selected for this study to minimize any effect due to increased magnetic activity during years of high sunspot-numbers.

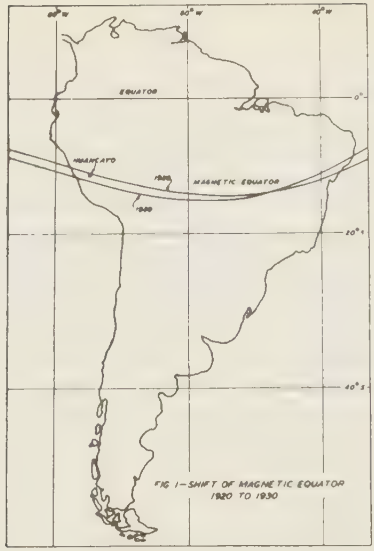


FIG 1—SHIFT OF MAGNETIC EQUATOR 1920 TO 1930

A distinct change in the character of the variations is evident. During the earlier years when the magnetic equator was quite close to Huancayo the variations exhibited two maxima, the second of which, occurring about noon, was characteristic of variations in vertical intensity in the Southern Hemisphere. By gradual changes this second maximum disappears, giving place to a midday minimum, characteristic of the vertical-intensity variations in the Northern Hemisphere.

Similar but less marked changes are exhibited by the curves in Figure 2 for the months of January, February, November, and December, The variations during the earlier years are of a distinct southern type in which a midday maximum of vertical intensity is manifest. During the later years the amplitude of the variation is less and its symmetry is greatly impaired. In this series of curves the effect of increased magnetic activity on the variations is revealed by the large amplitudes from 1926 to 1929.

Less striking but perhaps more convincing evidence that a change has occurred in the form of the daily variations in vertical intensity at Huancayo is supplied by the curves in Figure 2, based upon the yearly means of the quiet days. While the variations during the earlier years are distinctly characteristic of the Southern Hemisphere, the variations during the later years are not characteristic of either hemisphere. A continual shift in phase of the variations is evident from the gradual movement of the maximum from noon to about 8 hours.

A simple explanation of these changes may be offered. Seasonal changes in magnetic diurnal-variations are due to the expansion and intensification of the area covered by the northern or southern variation-system around the June or December solstices, respectively. During the June solstice the northern variation-system had less effect on Huancayo

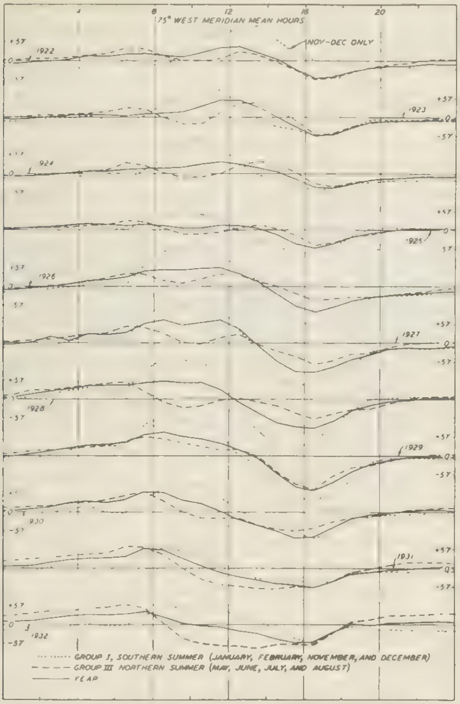


FIG. 2—DIURNAL VARIATION IN VERTICAL INTENSITY, HUANCAYO MAGNETIC OBSERVATORY, 5 INTERNATIONAL QUIET DAYS, 1922-1932

in the earlier years, but, with the southern shift of the magnetic equator, the northern variation-system increased its domains and became dominant in determining the variations at Huancayo. Throughout the entire period the southern variation-system has dominated the variations during the December solstice, although its effect has decreased in later years.

It is supposed, and a preliminary examination of current records sustains this supposition, that during the next few years the variations at Huancayo will more strongly take on the characteristics of the Northern Hemisphere, a shift back toward the southern type occurring only after a very long time, perhaps a century. Similar changes in the character of the variations in magnetic declination have also been noted but since they are less pronounced they are not presented.

A more comprehensive insight into the nature of these changes is derived from a study of the Fourier components of the variations during the season of northern summer for the eleven years considered. The amplitudes in gammas and the phase-angles in hours, reckoned from midnight 75° west meridian mean time as origin or 0^h, are represented in Table 1. The definitions of the terms are given by the expression

$$\triangle Z_h = \Sigma c_n \sin (2\pi n/24) (h+\phi_n)$$

in which $\triangle Z_h$ is the departure of Z from the mean of the day h hours after local midnight. Increase of vertical intensity, considered positive in the Northern Hemisphere, is reckoned positive.

TABLE 1—*Fourier analyses of mean diurnal variations in magnetic vertical intensity, Huancayo Magnetic Observatory, for five international quiet days in May, June, July, August, 1922-1932*

Year	Amplitudes				Phase-angles			
	c_1	c_2	c_3	c_4	ϕ_1	ϕ_2	ϕ_3	ϕ_4
	γ	γ	γ	γ	*hours*	*hours*	*hours*	*hours*
1922	2.37	0.83	1.34	0.95	23.8	3.6	4.5	0.3
1923	1.56	1.11	1.82	1.35	22.5	1.9	4.6	0.2
1924	1.49	0.76	1.52	0.97	21.5	1.2	4.1	0.0
1925	1.50	0.38	1.22	0.79	1.2	2.9	4.4	0.1
1926	2.78	0.69	0.87	0.68	22.1	3.3	4.7	0.9
1927	3.16	0.87	1.17	0.88	0.1	4.7	4.6	0.5
1928	3.24	0.45	1.43	0.76	1.8	0.4	4.7	0.9
1929	4.27	2.10	0.88	0.95	0.1	5.0	5.7	0.3
1930	3.56	1.44	0.38	0.73	1.9	7.1	3.6	0.5
1931	4.66	2.06	0.59	0.72	2.8	7.5	3.1	0.8
1932	5.28	2.03	0.76	0.89	4.0	8.1	4.0	0.3

These data are represented graphically in Figure 3, the components of the single years being grouped to form triennial means for smoothing. During the earlier years the 24-hour wave culminated around 7^h and had an amplitude of less than 2γ. By progressive changes the amplitude increased to 4.4γ for the last triennial mean while the time of culmination shifted to about 3^h. The total change, vectorially, amounted to about 4γ. The 12-hour wave experienced a change in amplitude from 0.8γ to 1.8γ with a nearly complete reversal of phase, the time of culmination being changed from 1^h and 13^h to 7^h and 19^h. The vectorial change for this wave amounted to 2.6γ. The 8-hour wave suffered a reduction in amplitude from 1.5γ to 0.5γ with a small phase-change, while the

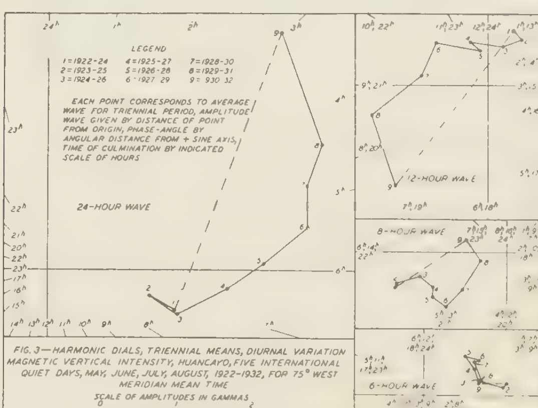


FIG. 3—HARMONIC DIALS, TRIENNIAL MEANS, DIURNAL VARIATION MAGNETIC VERTICAL INTENSITY, HUANCAYO, FIVE INTERNATIONAL QUIET DAYS, MAY, JUNE, JULY, AUGUST, 1922-1932, FOR 75° WEST MERIDIAN MEAN TIME

6-hour wave remained fairly constant throughout the period. It must be borne in mind that the actual changes from 1922 to 1932 were in general greater than this, the changes being somewhat diminished by taking triennial means.

The revelation of these changes in the Fourier components of the magnetic variation is astounding, but in view of the changes in the 24-, 12-, and 8-hour waves, the relative constancy of the 6-hour wave, which at many stations is most variable, stands out as a paradox. Such an effect would be observed if the 24-, 12-, and 8-hour waves have different phases north and south of the equator while the 6-hour wave is symmetrical with respect to the equator, or in other words, if spherical harmonics of degree $m=n+1, 3, 5, \ldots$ contribute most to the 24-, 12-, and 8-hour components while spherical harmonics of degree $m=n+0, 2, 4, \ldots$ contribute most to the 6-hour component.

To test this possibility, the first four components of the diurnal variation in vertical force at 10° north, 0°, and 10° south were computed from the coefficients obtained by Chapman[1] from an analysis of the vertical-intensity variations at 19 widely-distributed observatories during the months of January 1903, and February, May, June, July, and December, 1902. The coefficients given by Chapman apply for the solstices on the assumption that the variations are symmetrical with respect to the equator, an assumption which, as he himself pointed out, is not in accord with the actual conditions.

The results of this calculation are shown in Figure 4. The trends

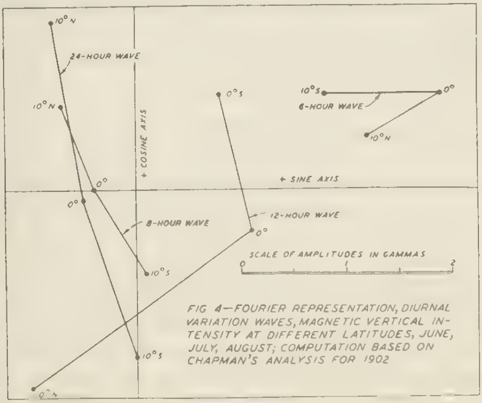


FIG 4—FOURIER REPRESENTATION, DIURNAL VARIATION WAVES, MAGNETIC VERTICAL INTENSITY AT DIFFERENT LATITUDES, JUNE, JULY, AUGUST; COMPUTATION BASED ON CHAPMAN'S ANALYSIS FOR 1902

of the various components as observed at Huancayo are roughly in agreement with the trends shown in the Figure, considering that Huancayo has apparently "shifted" from a southern latitude to a northern one. The comparative constancy of the 6-hour wave is conspicuous. Strict agreement of the magnitude and direction of the observed and predicted changes is not to be expected because the variations at Huancayo are very anomalous[2], being only roughly predictable from Chapman's

[1]See p. 24 of S. Chapman, Trans. R. Soc. A., **218**, 1-118 (1919).

[2]H. F. Johnston and A. G. McNish, C.-R., Cong. Intern. Electricité, Paris, **12**, 41-52 (1932).

coefficients. From the calculated values it would appear that Huancayo had shifted about 20° in magnetic latitude while the actual shift is less than 1° as deduced from the change in inclination by the relationship $\tan I = 2 \tan \Phi$, Φ being the geomagnetic latitude. In fact, from so small a change in geomagnetic latitude a change in the character of the diurnal variations was not to be expected and its appearance can be explained only by the hypothesis that the transition from northern to southern variations occurs over a very narrow region in the Western Hemisphere. Further evidence upon this hypothesis will be presented subsequently.

The reality of the phenomena which have been described is not questionable. The continual and gradual trend of the variations from one year to another consummating a total change which considerably exceeds the variations themselves in magnitude is most convincing. However, as a matter of interest, a meager attempt has been made to determine the statistical certainty of the change. For this purpose analyses of the first two components of the variations for the individual days of May, June, July, and August, 1922, were computed. The results of these analyses are shown in Figure 5. The days are numbered in their sequence, there being twenty days in all.

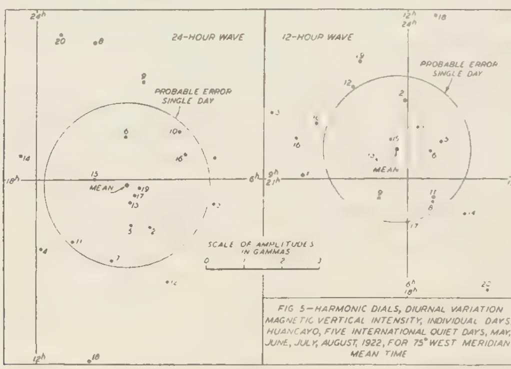


FIG 5—HARMONIC DIALS, DIURNAL VARIATION MAGNETIC VERTICAL INTENSITY, INDIVIDUAL DAYS, HUANCAYO, FIVE INTERNATIONAL QUIET DAYS, MAY, JUNE, JULY, AUGUST, 1922, FOR 75° WEST MERIDIAN MEAN TIME

The root-mean-square or standard deviation of the 24-hour component of a single day for this interval was found to be 2.6γ. Strong interdependence of days which are closely associated in time warns against too free a use of conventional statistical methods in calculating the probable error of a mean. Sixteen of these 20 days were found to be successive, that is, May 2, 3, May 30, 31, June 2, 3, etc. The means of each of these pairs were formed and treated as single days together with the unpaired days, thus making 12 "individual" days. The standard deviation of these 12 "days" was found to be 2.4γ. Had the original days been strictly independent the new standard deviation should have been

$$\sigma\sqrt{(N-n+n/4)/(N-n+n/2-1)} = 0.85\ \sigma$$

in which σ is the standard deviation of a single day, and N and n are the total number of days and the number of paired days, respectively. The computed standard deviation is nearly 1.15 times as large as was to be

expected, indicating that the accuracy of the mean is not much improved by the paired days. When monthly means are formed their standard deviation is found to be 1.77γ. The expected value of the standard deviation of monthly means is $\sigma\sqrt{4}/(5\times 3)$ or 1.33γ. The ratio of the expected to the observed standard deviation is 0.75, indicating a lesser interdependence. Though the number of data used is insufficient to give an accurate measure of precision these considerations clearly invalidate the variability of a mean given as the quotient of the variability of a single day divided by the square root of the number of days included in that mean.[3]

Considering the theory of Lexis, the days within any one season exhibit the tendency of a Lexis-series, that is, days within any one month are more alike than those in different months. For this reason the variability of the mean of many days in one season will be *greater* than the quotient of the variability of single days included in that mean divided by the square root of the total number of days. On the other hand, when days for the same season are taken from several different years, as was done in the formation of triennial means plotted in Figure 5, the condition of the Poisson-series arises. For illustration, a day in May 1922 will probably resemble a day in May 1923 more than it will a day in August 1922. In a Poisson-series the variability of the mean is less than the variability of the single individuals included in that mean divided by the square root of the number of such individuals. Thus the effects of the Lexis-series and the Poisson-series tend to compensate so that one might be justified in giving the variability of one of the triennial means, which include 60 days each, as the variability of a single day divided by $\sqrt{60}$. However, since the pairing of days indicates that this figure should be reduced by a factor of about 0.75, the conservative quantity $0.7\sqrt{60}$ will be used in calculating the precision of the triennial means.

Assuming that the standard deviation obtained for 1922 applies to all the years, the standard deviation of the difference of the triennial means 1922-24 and 1930-32 is $1.4\times 2.6\gamma/0.7\sqrt{60}=0.7$. The actual difference is 4.0γ, over 5 times this standard deviation. The chance that this difference is due to statistical fluctuations is only e^{-25}, or roughly 1 in 8×10^{10}. Allowing that the limits of precision are only roughly calculated one may wipe off several orders of magnitude in this probability and still feel that the reality of the change in the 24-hour component is established beyond all reasonable doubt. Comparable reliability is found for the 12-hour wave, the standard deviation of which is 2.4γ for single days in 1922. No attempt was made to determine the reliability of the 8- and 6-hour waves.

An interesting feature brought out by the Fourier-representation of the individual days is the association of large departures from the means for the first two harmonics. Thus of the nine days with departures of the 24-hour wave from the mean greater than the probable error, seven exhibit departures greater than the probable error in the 12-hour wave also. It may be seen that, even in 1922, days occurred in which the 24-hour component closely approached the mean 24-hour component for the last triennial group. On no day, however, does the 12-hour component

[3] J. Bartels in Terr. Mag., **40**, 1-60 (1935), has given an adequate discussion of the effect of interdependence of data and methods of arriving at correct conclusions when such effects are present.

closely approach the mean of the 12-hour component for the last triennial group.

Thus observations during the last decade at the Huancayo Magnatic Observatory clearly demonstrate the effect of secular change of the Earth's general magnetic field on the magnetic solar-diurnal variations. Consideration of the individual Fourier-components of the variation indicates that the changes are qualitatively in accord with what was to be expected although the greater magnitude of the observed changes demonstrates that the change from northern- to southern-type variations occurs over a very limited region. The phenomenon is practically conclusive proof that the daily variations depend upon the Earth's general field and that the relationship must be a complicated one, mathematically, such as is involved by the atmospheric-dynamo theory of Balfour Stewart.

Department of Terrestrial Magnetism,
Carnegie Institution of Washington,
Washington, D. C.

BERICHT ÜBER DIE ERDMAGNETISCHE NEUAUFNAHME OESTERREICHS 1930.0

VON M. TOPERCZER

In den Jahren 1928, 1929, und teilweise auch noch 1930 wurde eine erdmagnetische Aufnahme für das Gebiet des heutigen Oesterreich durchgeführt. Veranlasst wurde sie teilweise auch durch Bedürfnisse der Praxis, da ja seit der letzten Landesaufnahme durch J. Liznar,[1] die sich auf die Epoche 1890.0 bezog, fast 40 Jahre vergangen waren. Es wurde daher auch für die Neuaufnahme als Reduktionsepoche 1930.0 gewählt, sodass nunmehr zwischen den einzelnen Landesaufnahmen auf österreichischem Gebiet (Kreil 1850.0, Liznar 1890.0) ein Zeitraum von je 40 Jahren liegt. Erhöhte Bedeutung gewinnt die Neuaufnahme Oesterreichs dadurch, dass annähernd gleichzeitig auch analoge Arbeiten in den benachbarten Ländern (Schweiz, Bayern, Polen, und der Tschechoslovakei) stattfanden, deren Ergebnisse teilweise auch schon veröffentlicht sind. Das schweizerische und das bayerische Netz hängen durch Kontrollmessungen an gemeinsamen Stationen mit dem österreichischen zusammen.

Da der grösste Teil Oesterreichs von Gebirgen bedeckt ist, wurde ein möglichst dichtmaschiges Stationsnetz angelegt, das 112 gleichmässig verteilte Stationen umfasst, die auf ein Gebiet von rund 80,000 km^2 entfallen. Die Dichte des Stationsnetzes ist also grösser, als sie im allgemeinen für magnetische Aufnahmen 1. Ordnung geforderet wird. Vor Beginn der Feldarbeiten wurde in der Nähe von Wien eine erdmagnetische Basisstation errichtet, die seither dauernd in Betrieb geblieben ist. Damit befindet sich nun in der Nähe vom Wien wieder eine registrierende magnetische Beobachtungsstation, nachdem im Jahre 1898 infolge überhandnehmender Stromstörungen die magnetischen Beobachtungen in Wien-Hohe Warte aufgegeben worden waren. Diese Magnetwarte stellt derzeit das südöstlichste Observatorium Mitteleuropas dar und hat infolge ihrer vorgeschobenen Lage erhöhte Bedeutung für die Ueberwachung der magnetischen Säkularvariation. Als Basisstation für die Landesaufnahme ist ihre Lage nicht sehr günstig, da sie fast am Ostrand des Vermessungsgebietes liegt. Doch konnte nur bei Errichtung in der Umgebung Wiens daran gedacht werden, sie später dauernd in Betrieb zu erhalten. Die Basisstation wurde mit photographisch registrierenden Variometern von Eschenhagen-Typ ausgestattet. Deklination und Horizontalintensität werden mit einem Theodoliten von G. Schulze, Potsdam, die Inklination mit einem Erdinduktor aus der gleichen Werkstätte gemessen. Aehnliche Instrumente wurden bei der Feldmessung verwendet. Vor Beginn (1928) und nach Abschluss der Landesaufnahme (1930) wurden die Absolutinstrumente der Basisstation in Potsdam bezw. Seddin angeschlossen. Indirekte Vergleiche ergeben sich aus den gemeinsamen Punkten mit der Schweiz und mit Bayern, da auch die dort verwendeten Instrumente an Potsdam angeschlossen wurden.

Bei den Feldmessungen wurden die Arbeiten so geteilt, dass durch einen Beobachter der Zeitdienst und die astronomisch-geodätischen Arbeiten durchgeführt wurden, während dem anderen Beobachter nur die rein magnetischen Messungen oblagen. Den astronomischen Teil der Feldarbeiten besorgte F. Siegl vom Bundesamt für Eich- und Ver-

[1] Wien, Denkschr. Ak. Wiss., **62** (1895); **67** (1898).

messungswesen in Wien, die magnetischen Feldmessungen A. Schedler Die Arbeiten an der Basisstation wurden vom Verfasser durchgeführt. Ein jeder Feldmesspunkt wurde unter- und oberirdisch analog wie ein geodätischer Punkt vermarkt, wobei selbstverständlich nur unmagnetische Materialien verwendet wurden. Durch Nachtragsvermessungen wurde dann jeder magnetische Messpunkt an das geodätische Netz angeschlossen und ist so jederzeit wieder genau feststellbar. Bei der Auswahl der Punkte wurde auch darauf Rücksicht genommen, dieselben für möglichst lange Zeit ausser Bereich von Störungen durch Stromquellen, Siedlungen usw. zu halten. Nach Tunlichkeit wurden die Stationen der Aufnahme von 1890, von denen 35 auf das jetzt vermessene Gebiet entfielen, wieder aufgesucht. Doch konnte nur an ganz wenigen Stellen an genau dem gleichen Ort, wie damals gemessen werden. In den meisten Fällen waren die damals aufgesuchten Plätze schon verbaut oder doch im Bereiche von Störungsquellen, sodass die Beobachtungsstationen verlegt werden mussten.

Bei den Feldmessungen wurde die Deklination mittelst Pinnenmagnet gemessen. Die Messung der Horizontalintensität erfolgte an den meisten Stationen durch Ablenkungs- und Schwingungssätze mit zwei verschiedenen Magnetstäben, nur wenn es die Witterungsverhältnisse nicht zuliessen, wurden Ablenkungsmessungen allein vorgenommen. Einige entsprechend über das Gebiet verteilte Stationen sind als Säkularstationen ausersehen; an ihnen soll alle fünf Jahre eine vollständige Bestimmung der erdmagnetischen Elemente durchgeführt werden. Diese Säkularstationen wurden sowohl 1928 als auch 1929 gemessen. Selbstverständlich wurden während der Feldarbeiten die Feldinstrumente auch zeitweise an die der Basisstation angeschlossen. Die Genauigkeit der Feldmessungen war hinreichend. Es ergab sich teils aus den Doppelmessungen an gleicher Station zu verschiedener Zeit, teils aus den Messungen an den Anschlusstationen mit den Nachbarländern folgende mittlere Fehler: in $D \pm 0'8$, in $H \pm 8^{\gamma}$, und in $I \pm 0'3$. Darin sind übrigens auch die Fehler herrührend von der Reduktion auf die Epoche enthalten, sofern man die Säkularvariation ermittelt nach der Basisstation Wien-Auhof zugrundelegt. Es ist freilich, wie schon oben erwähnt, noch nicht untersucht, wie weit die Angaben dieser Station repräsentativ für das ganze Vermessungsgebiet sind, da sie ja im Osten liegt. Immerhin aber werden ihre Angaben noch immer besser den Verlauf der Säkularvariation darstellen, als die Werte des zur Zeit der Aufnahme nächst benachbarten Observatoriums von Seddin.

In Tabelle 1 sind die Werte von 29 Stationen, an denen schon 1890 beobachtet wurde, mitgeteilt.

In der Tabelle bedeuten ϕ_N die nördliche Breite, λ_E die östliche Länge gezählt vom Meridian von Greenwich. Die Bedeutung der verwendeten Symbole ist die übliche. Das ganze Vermessungsgebiet liegt im Bereich westlicher Deklination und nördlicher Inklination. Von den angegebenen Stationen ist Klagenfurt lokal gestört.

Um die Grösse der Störungen festzulegen, wurden aus den gemessenen Werten der magnetischen Elemente an den einzelnen Stationen Normalwerte abgeleitet, indem jedes magnetische Element E nach den Methoden der Ausgleichsrechnung in der Form

$$E = E_0 + a \cdot \triangle \phi + b \cdot \triangle \lambda$$

TABELLE 1

Station	ϕ_N	λ_E	D	I	H	Z
	° ′	° ′	° ′	° ′	γ	γ
Admont	47 35	14 28	−5 00.9	+62 57.9	20728	40621
Aflenz	47 33	15 14	4 39.0	62 51.3	20782	40535
Bleiberg	46 37	13 41	5 18.0	62 07.2	21159	39997
Bludenz	47 10	9 50	6 59.7	62 46.6	20734	40305
Bregenz	47 30	9 43	7 05.9	63 06.0	20564	40534
Bruck	47 24	15 16	4 38.1	62 45.1	20845	40478
Eisenerz	47 33	14 54	4 48.2	62 53.9	20765	40576
Golling	47 36	13 11	5 33.0	63 04.6	20665	40693
Graz	47 02	15 24	4 33.6	62 23.6	21053	40260
Hofgastein	47 11	13 06	5 30.1	62 35.6	20899	40306
Horn	48 39	15 40	4 15.4	63 51.4	20237	41228
Imst	47 13	10 45	6 40.5	62 48.9	20752	40407
Innsbruck	47 16	11 23	6 25.6	62 46.3	20713	40352
Ischl	47 42	13 37	5 28.2	63 10.7	20607	40758
Klagenfurt	46 38	14 17	5 13.4	62 07.3	21066	39825
Kremsmünster	48 04	14 08	5 08.7	63 26.9	20390	40805
Lienz	46 49	12 46	5 42.4	62 17.9	21057	40106
Liezen	47 34	14 14	5 00.3	62 56.0	20734	40578
Linz	48 18	14 16	5 03.4	63 36.8	20334	40986
Melk	48 14	15 20	4 35.8	63 29.6	20441	40985
Radstadt	47 23	13 28	5 24.1	62 48.4	20827	40537
Rattenberg	47 27	11 53	6 01.8	62 53.6	20687	40413
Salzburg	47 50	13 06	5 38.2	63 20.2	20474	40774
St. Anton a. Arlberg	47 08	10 17	6 55.4	62 46.1	20777	40375
St. Johann i.T.	47 31	12 26	5 42.8	62 56.9	20690	40514
Schärding	48 27	13 27	5 29.8	63 44.3	20271	41083
Strasswalchen	47 59	13 16	5 34.0	63 23.8	20426	40784
Vöcklabruck	48 00	13 39	5 21.5	63 23.0	20435	40778
Wr. Neustadt	47 48	16 16	4 10.2	63 04.1	20692	40730

dargestellt wurde. Dabei bedeutet E_0 den ausgeglichenen Wert dieses Elementes für den Punkt mit den Koordinaten $\phi_0 = 47°.5$ und $\lambda_0 = 13°.5$, ferner ist $\triangle\phi = \phi - \phi_0$, $\triangle\lambda = \lambda - \lambda_0$. Selbstverständlich ist diese Darstellung des Verlaufes der magnetischen Elemente nur sinnvoll in dem Gebiete, für das sie abgeleitet wurde. Es ergeben sich auf diese Art folgende Gleichungen für die Normalwerte der magnetischen Elemente D, H, und Z.

$$-D_n = 5°\ 23'.10 - 0'.0\ 17748\ \triangle\phi - 0'.451352\ \triangle\lambda$$
$$H_n = 20721^{\gamma}.48 - 7^{\gamma}.961878\ \triangle\phi + 0^{\gamma}.644221\ \triangle\lambda$$
$$Z_n = 40530^{\gamma}.11 + 10^{\gamma}.170713\ \triangle\phi + 0^{\gamma}.116013\ \triangle\lambda$$

Aus den abgeleiteten Normalwerten kann nun noch die Verteilung der Komponenten X (positiv gegen Norden) und Y (positiv gegen Osten) berechnet werden. Sie sind dann gegeben durch die folgenden Beziehungen, die nun von 2. Ordnung in den Koordinatendifferenzen sind, da ja durch Wahl der normalen D- und H-Werte schon die Werte von X und Y festgelegt sind und nunmehr eine Form der Darstellung zu suchen ist, die ihre Verteilung genügend genau wiedergibt.

$$X_n = 20630\ .05 - 7.918702\ \triangle\phi + 0.897717\ \triangle\lambda + 0.0_5 95293 \triangle\phi^2 - 0.0_4 950137\ \triangle\phi\ \triangle\lambda - 0.0_3 1766157\ \triangle\lambda^2$$

$$Y_n = -1944^{\gamma}.66 + 0.853639\ \triangle\phi + 2.647524\ \triangle\lambda - 0.0_4357936\ \triangle\phi^2 - 0.0_210388485\ \triangle\phi\ \triangle\lambda + 0.0_31017386\ \triangle\lambda^2$$

Natürlich ist dabei zu beachten, dass die hier gegebene Definition einer "normalen" Verteilung vom physikalischen Standpunkt aus ziemlich willkürlich ist, da ja eine mathematische Darstellung der oben angegebenen Form für das darzustellende Objekt keinen entsprechenden Ausdruck bildet. Doch ist sie für die Beantwortung verschiedener Fragen auch von physikalischem Standpunkt als Näherungslösung hinreichend geeignet. Auf die oben angegebenen Normalwerte wurde die Darstellung der Störungsgebiete bezogen. Es zeigt sich, dass Oesterreich trotz seines vorwiegenden Gebirgscharakters viel weniger gestört ist, als ursprünglich angenommen wurde. Die hier mitgeteilte Landesaufnahme ist übrigens die erste, deren Stationszahl für das Gebiet hinreichend gross ist, um Aufschlüsse über regionale Störungen geben zu können. Trotzdem ist auch hier der Stationsabstand noch zu gross, sodass unter Umständen eine lokal gestörte Station bei ungünstiger Gruppierung der Werte eine regionale Anomalie vortäuschen könnte. Es wird daher zu den jetzt in Angriff zu nehmenden Aufgaben gehören, die durch die Landesaufnahme gelieferten Störungsgebiete durch entsprechend gewählte weitere Stationen zu kontrollieren und genauer abzugrenzen.

Da bei der Landesaufnahme von 1890.0 durch J. Liznar ebenfalls Normalwerte der oben dargestellten Form abgeleitet worden waren, so bieten sie ein bequemes Mittel, um den Verlauf der säkularen Aenderung in den einzelnen magnetischen Elementen für den 40-jährigen Zeitraum, der zwischen den beiden Vermessungen liegt, zu untersuchen. Dabei ist allerdings zu bedenken, dass der Ausgleich Liznars über das ganze Gebiet von Oesterreich-Ungarn vorgenommen wurde, daher die individuellen Züge im mittleren Verlauf der einzelnen magnetischen Elemente im Gebiet der Ostalpen, auf das sich die Messung 1930.0 bezieht, wohl durch den Ausgleich über ein grosses Gebiet teilweise verwischt sein werden. In Tabelle 2 sind die säkularen Aenderungen der drei Elemente D, H, und Z von 1890.0 auf 1930.0 für die Gradschnittpunkte des Beobachtungsgebietes angegeben. In Deklination zeigt sich

TABELLE 2—*Säkulare Änderung 1930.0 — 1890.0*

ϕ	E	λ							
		10°	11°	12°	13°	14°	15°	16°	17°
°									
49	D	....	....	....	....	5° 14.′7	5° 12.′0	5° 09.′7	5° 07.′9
	H	....	....	....	....	−112γ	−146γ	−182γ	−219γ
	Z	....	....	....	....	−117γ	− 71γ	− 32γ	− 2γ
48	D	....	....	....	5° 11.′3	5° 09.′6	5° 08.′2	5° 07.′5	5° 07.′2
	H	....	....	....	− 35γ	− 69γ	−103γ	−138γ	−175γ
	Z	....	....	....	−242γ	−182γ	−131γ	− 87γ	− 51γ
47	D	5° 12.′1	5° 10.′4	5° 09.′1	5° 08.′4	5° 08.′1	5° 08.′3	5° 09.′0	5° 10.′2
	H	96γ	67γ	35γ	3γ	− 30γ	− 64γ	− 99γ	−135γ
	Z	−531γ	−445γ	−368γ	−296γ	−233γ	−177γ	−127γ	− 85γ

ebenso wie beim Element selbst auch bei der säkularen Aenderung hauptsächlich eine Abhängigkeit von λ, bei den anderen Elementen ist eine gemischte Abhängigkeit vorhanden. In Tabelle 3 sind für D, H, und Z die auf den Jahresbeginn bezogenen Jahresmittel nach den Registrierungen der Basisstation Wien-Auhof mitgeteilt. Die Angaben

TABELLE 3

Epoche	D	H	Z
	° ′	γ	γ
1929.0	−4 16.95	20525.9	40971.2
1930.0	−4 07.42	20514.7	41018.3
1931.0	−3 58.31	20509.3	41068.6
1932.0	−3 49.15	20502.7	41125.9
1933.0	−3 39.93	20508.0	41180.5

dieser Basisstation wurden ja für die Reduktion der Feldmessungen auf 1930.0 verwendet. Die mitgeteilten Werte geben auch einen Ueberblick über den Verlauf der Säkularvariation in dem Gebiet. Die geographischen Koordinaten der Basisstation Wien-Auhof sind: $\phi = 48°\ 12'.2$, $\lambda = 16°\ 14'.2$. Die bisherigen Ergebnisse der Landesaufnahme, im wesentlichen eine Uebersicht über die gewonnenen Beobachtungsergebnisse sind in zwei Publikationen[2,3] niedergelegt.

[2] A. Schedler und M. Toperczer, Die Verteilung der erdmagnetischen Deklination in Oesterreich zur Epoche 1930.0. Wien, Jahrb. ZentrAnst. Met. Geodyn., Beiheft Jahrg. 1929 (1932). [Pub. No. 138.]

[3] A. Schedler und M. Toperczer, Die Verteilung der erdmagnetischen Kraft in Oesterreich zur Epoche 1930.0. Wien, Jahrb. ZentrAnst. Met. Geodyn., Beiheft Jahrg. 1929 (1932). [Pub. No. 138.]

ZENTRALANSTALT FÜR METEOROLOGIE UND GEODYNAMIK,
Wien

REVIEWS AND ABSTRACTS

(See also page 171)

STAGG, J. M.: *The diurnal variation of magnetic disturbance in high latitudes.* London, Proc. R. Soc., A, v. 149, No. 867 (298-311).

In this very interesting paper, Stagg gives the results of his investigations of irregular magnetic disturbances at ten high-latitude stations. The stations chosen are widely distributed in longitude and lie between magnetic latitudes (ϕ_m) 54° and 88°. Three of the stations are in Antarctica. He shows that a diurnal variation in irregular magnetic disturbance (D) exists, controlled by local time throughout the region within at least 35° of the magnetic axis-pole. (D) is not dependent upon the regular diurnal variation (S_d) associated with disturbed days which is also a local-time phenomenon. A marked decrease in the scale of (D) occurs below $\phi_m = 70°$; a single evening maximum is typical of (D) in all seasons of the year. The time of this maximum becomes later by one hour for approximately each 5° of increase in ϕ_m until at $\phi_m = 70°$ the maximum is at midnight. Above $\phi_m = 78°$ the dominant maximum is in the forenoon. The scale of (D) is much reduced in winter in this zone. In the transition-zone, $\phi_m = 70°$ to 78°, a double maximum in (D) exists: One in the forenoon is prominent throughout the year; the other, in late evening, is much more pronounced in winter than in summer. The change with season is such as would occur if the conditions normally obtaining below $\phi_m = 70°$ advanced right up to the magnetic-axis pole in winter, the scale of (D) being very much damped above $\phi_m = 80°$ in winter. Stagg suggests that it is more than a coincidence that the zone within 12° of the magnetic axis where such marked seasonal change occurs in (D), is also the zone of continuous direct solar radiation in summer and of none in winter.

It is interesting to note that Stagg found that even for periods as short as one month, character-figures for hours gave the form of (D) quite as well as other estimates of (D) such as hourly values of ($Hr_H + Zr_Z$). He also points out that for stations where H is low and r_H, r_Z are of similar magnitude, the expression ($Hr_H + Zr_Z$) is well expressed by Zr_Z alone.

The maximum-disturbance region at about $\phi_m = 70°$ is probably closely related to the maximum auroral-frequency zone. Data from three high-latitude stations are discussed in relation to the results of Stagg's investigation elsewhere in this JOURNAL. Two of the three stations, Chesterfield, Canada, and Point Barrow, Alaska, for 1932-33, fit in very well with the results of this investigation. The data for Little America, in Antarctica, for 1929-30, do not agree so well.

F. T. DAVIES

RÖSTAD, A.: *Beziehung der Nordlichterscheinungen zu den weltweiten magnetischen Störungen.* Geofys. Pub., Oslo, v. 10, No. 10, 1935 (10 with 33 figs.).

This paper is a continuation of two previous ones, by the same author, dealing with the relations between the auroral and magnetic disturbances. The method was to compare the magnetic pole-distances, θ, of the brightest parts of auroral displays, as derived from Störmer's photographs, with simultaneous horizontal-intensity departures from normal, P, as determined at different observatories during world-wide magnetic storms. A statistical treatment of 121 cases shows, as was to be expected, that the angle θ increases with the disturbance-intensity P. For northern stations, such as Potsdam and Rude Skov, the curves obtained by plotting θ against P are straight lines, while for more southerly stations, as Tucson and Batavia, the curves assume parabolic forms. No conclusions are drawn by the author in explanation of these results. He suggests that the world-wide magnetic storm may be caused by the Earth's entrance into a cosmic-electric stream, giving rise primarily to current-systems at a considerable distance from the Earth which cause the initial positive phase of the magnetic storm. Then follows the negative phase accompanied by strong polar disturbances, the transition from positive to negative intensities being conceived as a veiling of the positive intensities by current-systems nearer to the Earth.

A number of diagrams are produced to show the changes in the horizontal-intensity disturbing forces and the intensities of the accompanying auroras.

W. F. WALLIS

SUR LA VARIATION DIURNE DES ELÉMENTS MAGNÉTIQUES AU SCORESBY SUND (EST-GROENLAND)

Par J. P. Rothé

En examinant rapidement les courbes enregistrées par les magnétographes La Cour à la station française de l'Année Polaire au Scoresby Sund (longitude, 21° 57′.7 W; latitude, 70° 29′.1 N), on est frappé par la régularité des variations diurnes des éléments magnétiques. Les résultats détaillés de nos observations sont publiés, sous la direction du Professeur Maurain dans les Annales de l'Institut de Physique du Globe de Paris[1] et l'auteur résumera ici quelques observations inspirées par ce dépouillement.

Variation diurne de la déclinaison

Le régime diurne de la déclinaison est particulièrement net, avec un maximum vers l'est, vers 6 à 7 heures du matin (T. M. G.) et un maximum plus étalé vers l'ouest le soir vers 20 à 22 heures. Le graphique (Fig. 1) représente l'allure des différentes courbes mensuelles, toutes très concordantes; il montre aussi que l'amplitude diurne moyenne suit une marche annuelle très accentuée passant de 16 minutes en janvier à 38 minutes en avril. La seule série d'observations comparables directement à nos mesures est constituée par des lectures horaires de déclinaison faites en 1891-1892 à l'île de Danemark, dans le Scoresby Sund, au cours de l'hivernage de l'expédition danoise Ryder.[2] La variation diurne de la déclinaison y présentait une marche tout à fait analogue à celle que nous avons constatée; toutefois l'amplitude diurne moyenne était plus forte au printemps, atteignant 61 minutes en mars, parce que 1891-1892, à l'encontre de 1932-1933, correspondait à une période de maximum d'activité solaire.

Variation diurne des jours calmes et des jours agités

Les courbes de déclinaison des jours dits "agités" présentent un aspect très perturbé, à première vue désordonné; cependant la variation diurne, analogue à celle qui ressort des moyennes mensuelles, mais avec une amplitude beaucoup plus grande, *apparaît encore très nettement*. Cette variation diurne à une seule onde journalière est celle de la plus grande partie des jours du mois et par suite c'est celle qui se traduit dans les moyennes mensuelles.

De même que l'amplitude de la variation diurne suit la périodicité undécennale du soleil, de même au cours de chaque cycle de rotation solaire, la variation diurne s'amplifie périodiquement, en même temps que les variations brusques et les "perturbations" sont plus nombreuses, surtout aux heures du maximum et du minimum de la variation diurne. Ceci nous indique que *"l'agitation magnétique" et la variation diurne procèdent de la même cause* que nous préciserons plus loin. A l'examen des courbes du Scoresby Sund, il m'a semblé que *caractériser l'état magnétique d'un jour par l'amplitude de la déclinaison* en minutes, amplitude calculée par les moyennes horaires graphiques afin d'éliminer les "pointes," était un moyen satisfaisant. Appliqué à nos observations,[1] il fait bien ressortir les remarquables périodicités tant de jours calmes,

[1] J. P. Rothé, Observations magnétiques au Scoresby Sund pendant l'Année Polaire avec tableaux horaires des éléments *D*, *H*. et *Z*. Ann. Inst. Physique du Globe, Paris, **13** (sous presse).

[2] H. Vedel, Observations météorologiques, magnétiques, etc. . . . faites dans l'île de Danemark en 1891-1892, Copenhague, Dansk Met. Inst. (1895).

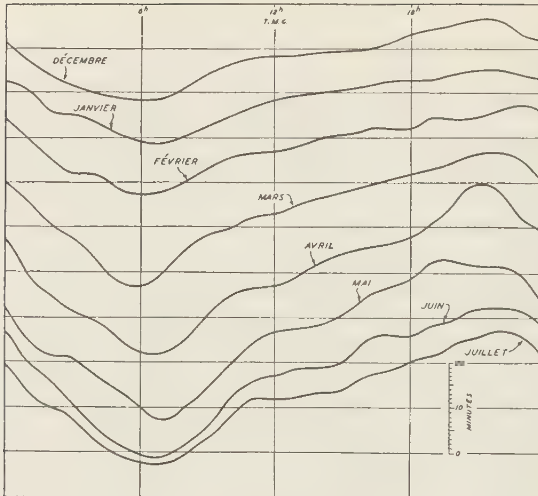


FIG. 1—VARIATION DIURNE DE LA DÉCLINAISON AU SCORESBY SUND, DÉCEMBRE 1932–JUILLET 1933

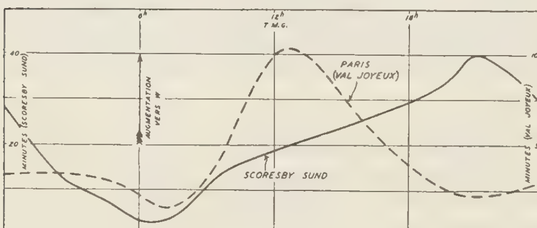


FIG. 2—VARIATION DIURNE DE LA DÉCLINAISON À PARIS ET AU SCORESBY SUND, AVRIL 1933 (L'ÉCHELLE EST QUATRE FOIS PLUS PETITE POUR LE SCORESBY SUND)

à faible variation, que de jours agités à forte amplitude diurne, périodicités signalées récemment par ailleurs par M. Bartels.[3]

En examinant, d'après le tableau suivant, la marche diurne de la déclinaison les jours "calmes" choisis par la commission internationale on constate une *marche différente* de celle des jours agités.

Si le minimum trés prononcé du matin persiste, le maximum de 22 heures est moins net et un autre maximum apparaît vers 13 à 15 heures (T. M. G.), c'est-à-dire $11^h\ 30^m$ à $13^h\ 30^m$, heure locale. Pour certains mois le maximum de midi est prépondérant; en janvier le maximum du soir est à peine visible, en mai il a complètement disparu. La marche diurne qui résulte de ce tableau est une combinaison de celle—à onde simple—trouvée pour les jours agités et d'une autre—également simple—caractérisant quelques jours exceptionnels chaque mois. *Le fait d'associer des éléments non comparables montre le caractère arbitraire de l'étude de la variation diurne par les jours "calmes,"* particulièrement pour les régions polaires.

[3]Terr. Mag., **39**, 201-202 (1934).

Variation diurne de la déclinaison, les jours "calmes" internationaux

Jours internationaux	0–1	1–2	2–3	3–4	4–5	5–6	6–7	7–8	8–9	9–10	10–11	11–12
, 7, 12, 21, 23 déc	38.4	34.7	33.1	32.9	32.1	30.0	31.4	35.1	36.0	38.0	39.1	39.0
, 5, 10, 11, 21 janv	37.4	36.2	34.7	35.0	35.0	34.5	35.3	36.1	35.8	37.4	38.8	37.8
, 11, 13, 16, 17 févr	38.4	33.7	32.4	33.2	32.7	34.4	32.6	34.0	35.0	35.7	35.5	34.9
, 6, 7, 9 mars	35.3	30.6	31.3	30.0	29.9	30.3	29.8	31.1	32.0	33.4	35.7	38.6
1, 12, 13, 28, 29 avr	27.6	25.6	22.6	22.9	19.8	17.4	18.0	20.7	23.7	27.7	28.4	28.3
, 10, 12, 24, 26 mai	26.6	25.9	25.1	20.7	16.9	17.4	19.3	17.1	18.5	24.4	28.4	30.1
, 6, 16, 18, 24 juin	31.5	28.5	24.7	23.7	21.2	17.2	13.1	14.8	19.2	23.9	27.6	30.4
3, 15, 21, 30 juil	35.8	33.6	31.1	29.1	24.4	25.6	25.6	27.0	27.8	29.5	32.7	36.0

Jours internationaux	12–13	13–14	14–15	15–16	16–17	17–18	18–19	19–20	20–21	21–22	22–23	23–24
, 7, 12, 21, 23 déc	39.4	38.0	38.2	37.8	36.7	37.4	37.2	37.7	38.2	40.0	41.3	39.9
, 5, 10, 11, 21 janv	39.4	40.3	39.0	38.9	38.4	37.3	37.3	37.3	37.9	39.9	38.9	38.0
, 11, 13, 16, 17 févr	36.6	36.8	37.5	36.7	35.5	35.6	35.8	36.3	38.0	37.2	39.4	38.7
, 6, 7, 9 mars	39.5	40.6	38.5	38.2	36.8	35.1	35.0	36.2	37.5	38.8	40.4	36.7
1, 12, 13, 28, 29 avr	32.0	34.7	35.2	35.4	34.2	32.1	34.6	34.8	37.0	36.6	33.3	30.5
, 10, 12, 24, 26 mai	32.9	35.9	34.8	33.7	31.1	29.9	28.9	30.1	31.2	31.6	31.9	30.1
, 6, 16, 18, 24 juin	32.2	33.0	34.8	34.0	34.0	32.2	32.8	30.2	34.9	34.5	38.0	36.0
3, 15, 21, 30 juil	38.2	39.4	41.4	41.0	36.5	37.6	37.2	39.2	44.8	49.4	39.8	45.0

Comparaison de la variation diurne de la déclinaison au Scoresby Sund et en France

Au Val Joyeux (Paris) le maximum vers l'ouest de la déclinaison se produit vers 12 heures et un minimum vers l'est correspond au contraire au maximum du soir, vers l'ouest, au Scoresby Sund (Fig. 2). La marche diurne au Val Joyeux est encore celle des stations de moyenne latitude, comme Sodankylä, Lerwick, Pavlovsk en Europe, Meanook et Sitka en Amérique. Au contraire la marche diurne moyenne du Scoresby Sund se retrouve, en attendant les résultats des stations de la récente année polaire, dans les observations des stations de haute latitude comme celle du Spitzberg en 1882-1883.

Mais l'étude de la variation diurne des jours calmes au Scoresby Sund nous a montré que *certains jours exceptionnels présentaient une marche diurne tout à fait analogue à celle de Paris*, avec un maximum absolu vers midi (heure locale). Je crois intéressant d'en donner ici la liste—une partie seulement correspond aux jours calmes internationaux—pour la période qui va du 24 novembre 1932 au 14 août 1933: 24 novembre, 4, 12, 20, et 21 décembre, 6, 11, 16 et 17 janvier, 5, 6, 10, 12, 16, et 17 février, 5, 9, 13, 15, et 31 mars, 12 et 13 avril, 10, 11, 24, et 26 mai, 22 juin, 5 et 10 juillet, 1, 9, et 10 août. Ce sont, en général, les jours, où, *sans être insensible*, la variation diurne est la plus faible: le 24 novembre 2'.7 seulement; les 10 et 16 février 8'.3 et 7'.5.

On sait, depuis longtemps les liens qui unissent l'activité magnétique et les aurores. Les jours à forte variation diurne de la déclinaison sont au Scoresby Sund ceux où se produisent les aurores les plus intenses.

Lemström[4] signalait en 1886, sans préciser, le changement de caractère

[4]L'aurore boréale, 73 (1886).

de la variation diurne de la déclinaison dans les régions arctiques et la marche inverse des perturbations magnétiques des deux côtés de la zone aurorale maximum. Le déplacement de cette zone aurorale, lié à la rotation de la Terre[5], se traduisait au Scoresby Sund par une recrudescence des aurores vers 7 heures et 21 heures, c'est-à-dire à peu près aux heures des maxima et minima de la déclinaison.

De plus l'étude de certains phénomènes auroraux comparés aux enregistrements magnétiques nous a montré qu'une aurore au sud augmentait la déclinaison, et qu'une aurore au nord au contraire la diminuait. A l'aide de ces diverses considérations, nous pouvons donc préciser: *la variation diurne de la déclinaison, qui, en une station, dépend de la position de cette station par rapport à la zone électrisée de la haute atmosphère, zone rendue apparente par les phénomènes auroraux:*

(1°) Au Scoresby Sund la déclinaison augmente vers l'ouest tant que la zone aurorale maximum (dans le sens défini ci-dessus) se trouve au sud. Elle diminue vers l'est tant que cette zone est au nord de la station.

(2°) A Paris la déclinaison passe par son maximum vers l'ouest lorsque la zone aurorale est la plus proche de Paris, c'est-à-dire, occupe la position la plus méridionale. Elle passe par son minimum lorsque la position de la zone aurorale est la plus nord.

(3°) Au Scoresby Sund certains jours très calmes présentent une variation diurne analogue à celle de Paris, parce que la zone aurorale occupe ces jours-là la même position relative par rapport aux deux stations, c'est-à-dire qu'elle se trouve au nord du Scoresby Sund réduite à la région du pôle de Gauss. Cette situation correspond aux minima d'activité solaire. On n'observe alors au Scoresby Sund que des aurores peu intenses ou même pas d'aurores.

L'étude d'une "perturbation" comme celle des 5-6 août 1933 confirme les faits précédents.

Malgré les variations accidentelles de détail, qui troublent l'allure régulière de la courbe, *la variation diurne habituelle des régions polaires apparaît nettement.* La déclinaison est maximum vers l'ouest au Scoresby Sund, le 5 vers 19 heures (T. M. G.), tandis qu'elle est mimimum vers l'est au même moment à Paris. Il n'y a plus de décalage correspondant au déplacement de la zone aurorale parce que la quantité d'énergie électrique mise en jeu dans la haute atmosphère est à ce moment suffisamment grande pour provoquer la variation de la déclinaison magnétique en même temps à Paris et au Scoresby Sund, *mais en sens inverse parce que cette source d'énergie se trouve située entre ces deux stations.*

Variation diurne des composantes

L'étude des variations de l'intensité du champ magnétique par ses composantes H et Z est plus malaisée que l'étude des variations de sa direction. Déjà difficile dans les régions tempérées, elle se complique dans les régions polaires par suite de la proximité de la source perturbatrice. La variation diurne de la composante verticale est comparable et de même sens à celle de Paris, mais le minimum localisé vers 12 heures à Paris s'étend beaucoup plus largement, de 8 à 20 heures, au Scoresby Sund et progressivement se scinde en deux minima vers 9 et 18 heures.

[5]A. Dauvillier, J. Physique et Le Radium, 5, 405 (1934).

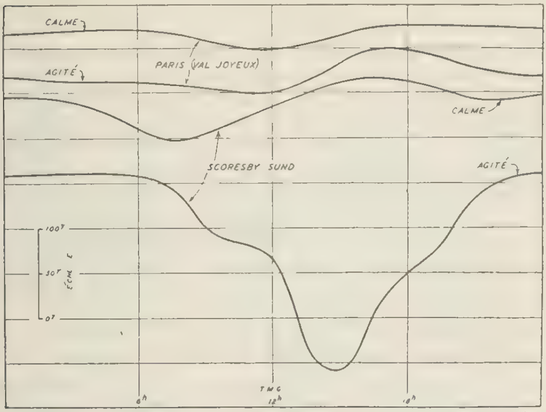


FIG. 3—VARIATION DIURNE DE LA COMPOSANTE VERTICALE A PARIS (VAL JOYEUX) ET AU SCORESBY SUND POUR LES 5 JOURS CALMES ET LES 5 JOURS AGITÉS DE MAI 1933

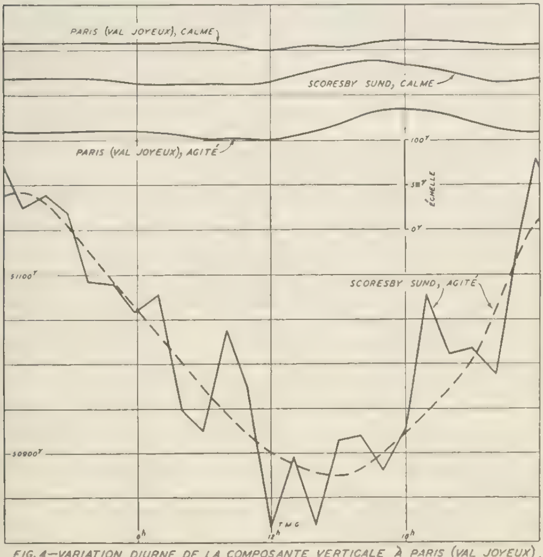


FIG. 4—VARIATION DIURNE DE LA COMPOSANTE VERTICALE À PARIS (VAL JOYEUX), ET AU SCORESBY SUND UN JOUR CALME (17 FÉVRIER 1933) ET UN JOUR AGITÉ (24 FÉVRIER 1933); LES QUATRE COURBES SONT À LA MÊME ÉCHELLE

séparés par un maximum secondaire vers midi. Ici encore les courbes mensuelles sont des courbes moyennes résultant de l'addition de courbes qui ne sont pas directement comparables mais l'étude de cas particuliers confirme ce que nous avons déjà observé pour la déclinaison. En effet les courbes correspondant aux jours calmes et aux jours agités sont très différentes (Fig. 3 pour mai 1933). Les jours calmes ou du moins certains d'entre eux présentent une marche diurne tout à fait analogue à celle

de Paris avec un minimum vers midi et un maximum vers 18 heures. Fait remarquable, que montrent bien les figures 3 et 4 (les courbes y sont toutes à la même échelle), *l'amplitude de la variation diurne des jours calmes au Scoresby Sund est tout à fait du même ordre de grandeur que celle des jours agités à Paris. Ainsi le Scoresby Sund occupe ces jours-là par rapport à la zone électrisée de la haute atmosphère une situation analogue à celle de Paris les jours agités.* La source d'énergie dont l'action combinée à la rotation de la terre est la cause de la variation diurne est, les jours calmes, réduite aux régions polaires voisines du pôle de Gauss.

Notons encore qu'à Paris le minimum très régulier de Z correspond au minimum vers l'est de la déclinaison, c'est-à-dire au moment où la zone aurorale est la plus proche de Paris. Ceci confirme les conclusions auxquelles nous avait conduits l'étude de la variation diurne de la déclinaison.

En ce qui concerne les variations de la composante horizontale, l'observation aurorale montre que de fortes aurores correspondent toujours à une diminution de l'intensité de cette composante. Aussi pendant les mois d'hiver *retrouve-t-on nettement les doubles passages journaliers de la zone aurorale* sur les courbes de la variation diurne de H au Scoresby Sund (Fig. 5), marqués par deux minima, le soir vers

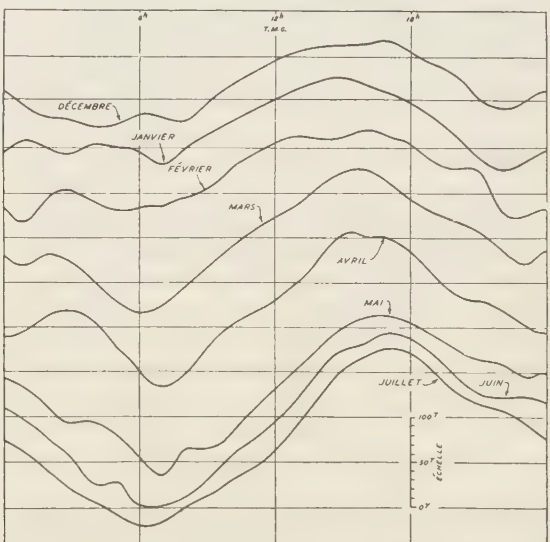


FIG. 5—VARIATION DIURNE DE LA COMPOSANTE HORIZONTALE AU SCORESBY SUND, DÉCEMBRE 1932—JUILLET 1933

22 heures et le matin vers 6 heures. Au fur et à mesure qu'on avance vers le solstice d'été, les courbes se déforment et le minimum du matin devient de plus en plus prépondérant de même que le maximum de 18 heures.

Pour conclure, l'étude de la variation diurne des éléments magné-

tiques telle qu'on est tenté de la faire en considérant les moyennes soit des mois entiers soit seulement des jours calmes est arbitraire et masque le phénomène essentiel: la variation diurne est d'autant plus nette que l'activité solaire est plus grande, par suite la situation magnétique plus agitée, elle s'explique, en examinant des cas particuliers et en les confrontant avec les observations aurorales, par le déplacement journalier de la zone électrisée de la haute atmosphère. Cette explication rendue plus claire dans les régions polaires, s'applique aux régions tempérées. Limitée comme elle l'a presque toujours été à ces dernières, l'étude des phénomènes magnétiques ne saurait être que peu fructueuse.

INSTITUT DE PHYSIQUE DU GLOBE,
Paris, Décembre 1934

REVIEWS AND ABSTRACTS

(See also page 164)

WIECHERT, ERWIN: *Untersuchungen an Baystörungen unter besonderer Berücksichtigung der Störmerschen Theorie.* Mit. Geophysik. Warte Gr. Raum, Univ. Königsberg, Nr. 22, 1934 (26 mit 37 Fig.). 30 cm.

A refreshing investigation of possible physical explanations of a terrestrial-magnetic phenomenon is presented in this paper. The author follows Steiner in the belief that bay-disturbances are due to electric currents consisting of streams of electrified corpuscles such as were hypothesized by Birkeland to explain elementary polar storms. Various conditions which this theory imposes are formulated by the author and the observed phenomena are investigated to ascertain the extent to which they satisfy the required conditions.

Application of the mathematical investigations of Störmer and the results of the terrella-experiments of Birkeland lead to the conclusion that bays should occur at certain regions, called ϕ-centers by the author, which are determined by the asymmetry of the Earth's magnetic field, the hour-angle of the Sun, and the declination of the Sun for given rigidities of the corpuscular streams. These ϕ-centers move about the Earth with the Sun's apparent motion and should result in certain seasonal and diurnal distributions of bay-frequency. On the assumption that bays are due to Birkeland-currents, the form of the perturbation associated with a bay and the rate of progression of its maximum from station to station may be predicted.

For the investigation of the diurnal and seasonal distributions the author uses the results of Steiner's investigation for O'Gyalla, Lubiger's investigation for Samoa, and his own for Grosser Raum, the terrestrial-magnetic station of the Geophysical Observatory of the University of Königsberg, Prussia. Unfortunately the series from the latter station is rather short and incomplete. In general the distributions at the three stations are in agreement.

Both O'Gyalla and Samoa exhibit maxima at the equinoxes as called for by the theory. Grosser Raum exhibits a single maximum in winter, in disagreement with the theory. The author attempts to explain this as due to obliteration of bays during the equinoxes by storms which occur most frequently then (during which bays were not counted). Although the author has calculated probable errors for his variation-curves he has done so without due regard to interdependence manifested by geophysical phenomena so that the absence of the equinoctial maxima may be accidental, due to fewness of data.

All three series of data exhibit similar diurnal-distribution curves in which are conspicuous maxima of positive bays around midnight and negative bays around midday. The author has not been content with so rough an approximation, however, and has predicted several maxima for each station from theoretical considerations, defining the time of each within fractions of hours. Such high degrees of exactness cannot be verified with the limited data available for the investigation. The observed and predicted maxima are not in very good agreement, in general. Evidence for fine-structure in the spectrum of bay-frequency throughout the day, which the author computes and believes he finds, in certain cases is not very convincing.

Assuming that the part of the Birkeland-current effective in producing those magnetic phenomena is an element parallel to the Earth's surface about one kilometer in length, the author proceeds to compute the form of the perturbation produced and from the gradual increase and decrease of the current as it slowly moves into different longitudes, the relative time of maximum perturbation at different stations. The element of current is assumed to exist at heights of from 400 to 1200 km above the Earth's surface. The rate of progression of the maximum is computed to be of the order of 10 to 20 minutes later for 15° of west longitude.

Measurements of individual perturbations at several European stations to test this requirement of the theory are also not as convincing as might be desired, although, in consideration of the complications involved, no better agreement is to be expected. A significant fact is that the progression in most cases indicates that the maximum perturbation occurs later at more westerly stations. In some cases the progression appears to be eastward, which fact is accounted for by the assumption that the bay is due to a current which occurs over a region where there are no observatories, the recorded perturbation being due to distance-effect.

Selecting two disturbances which exhibited very similar characteristics the author notes that they occurred 55 days apart. Since similar bays should have similar sources he concludes that these two are of common origin which he associates with some region on the Sun owing to their separation being twice 27.5 days. In view of the close dependence of terrestrial-magnetic phenomena on solar conditions such a conclusion seems reasonable but the proof offered for it is weak.

Although the theory and the proofs presented are not sufficient to establish the contentions of the author they furnish perhaps as reasonable a physical picture of the origin of bays as has been presented. The author's presentation has been weakened by attempting to give too detailed an explanation of the phenomena when a grosser explanation would suffice. The physical possibility of the Birkeland-currents because of electrostatic instability of the stream of corpuscles naturally is a primary consideration for the theory and that has not been discussed. The paper represents a type of investigation of great value to terrestrial-magnetic research—while presenting a theory which points out favorable lines of investigation it leaves open large regions for further speculation.

A. G. McNish

THE DIURNAL VARIATION IN MAGNETIC AND AURORAL ACTIVITY AT THREE HIGH-LATITUDE STATIONS

By Frank T. Davies

The data used in this article were prepared for the purpose of comparing magnetic and auroral diurnal-activity. Hourly magnetic character-figures on a scale of 0, 1, and 2 are used to express magnetic character. As J. M. Stagg[1] has found that such figures express satisfactorily the diurnal variation in irregular magnetic disturbance in high latitudes, the magnetic character-graphs for the three high-latitude stations considered in this article serve also to compare with the results of Stagg's investigation for ten other high-latitude stations. The data used are for the following stations: The Byrd Antarctic Expedition Station at Little America, Antarctica, 1929-30; The United States Polar-Year Station at Barrow, Alaska, 1932-33; and the Canadian Polar-Year Station at Chesterfield, Northwest Territory, Canada, 1932-33.

The months of January and June 1933 used by Stagg as typical winter and summer months in his comparison of Fort Rae and Godhavn Polar-Year stations, have also been chosen for Barrow and Chesterfield. The locations of Little America, Barrow and Chesterfield are given in Table 1 and illustrated in Figure 1.

Table 1—*Geographic positions and magnetic latitudes*

Station	Geographic Latitude ϕ	Geographic Longitude λ	Magnetic latitude ϕ_m
	°	°	°
Little America	78.4 S	164 W	74.4 S
Barrow	71.4 N	156 W	68.6 N
Chesterfield	63.3 N	91 W	74.0 N

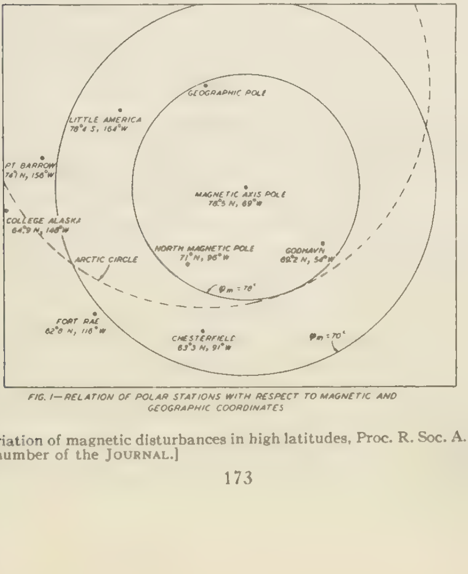


FIG. 1—RELATION OF POLAR STATIONS WITH RESPECT TO MAGNETIC AND GEOGRAPHIC COORDINATES

[1]The diurnal variation of magnetic disturbances in high latitudes, Proc. R. Soc. A., **149**, 298-311 (1935). [See review in this number of the Journal.]

Diurnal-variation graphs of magnetic character for typical summer, winter, and equinoctial months are shown in Figure 2. For Barrow and Chesterfield the months January, March, and June, 1933, are used as typical of winter, equinox, and summer. For Little America July and September, 1929, and January, 1930, are chosen; July rather than June was chosen as the winter month because many more auroral data were available for comparison in July. Local mean time for each station is used throughout the discussion. The zones between ϕ_m 70° to 78°, 78° to 90°, 55° to 70° are referred to in this article as the transition, inner, and outer zones, respectively.

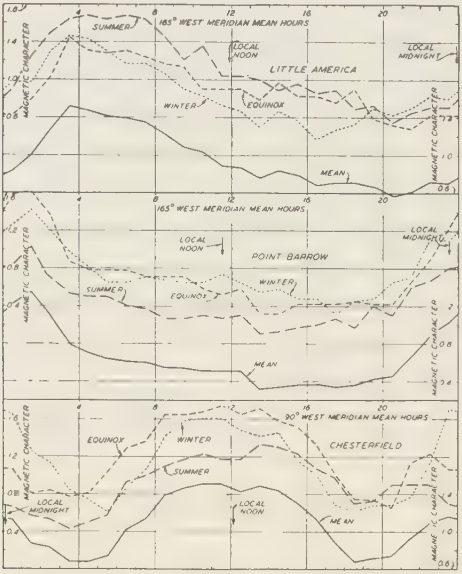


FIG. 2—DIURNAL VARIATION IN MAGNETIC CHARACTER FOR DIFFERENT SEASONS AT POLAR STATIONS

Figure 2 shows that magnetic activity at Barrow in all seasons exhibits the single wave with night maximum, typical of high-latitude stations below $\phi_m = 70°$. Although the data for Barrow fit in with the outer-zone type the time of maximum activity is at 2^h in all seasons. From its magnetic latitude, which is very nearly the same as for Fort Rae, the maximum might be expected to occur just before midnight. Winter and equinox are about equally disturbed; summer is the quietest season.

Chesterfield fits in well with its position in the transition-zone. The graphs for all seasons show a strong midday maximum. A night maximum is very prominent in winter but less so in spring and still less in summer. The midday maximum is centered at noon all through the year and is more intense in winter and at equinox than in summer. The night maximum at $23^h.5$ in winter and spring is much diminished in summer and occurs two hours earlier. The minima occur throughout

the year at approximately 4^h and $19^h.5$. As at Barrow, summer is the quietest season.

The graphs for Little America at nearly the same magnetic latitude as Chesterfield, although in the opposite hemisphere, might be expected to show the transition-zone two-maxima type. Actually the single-maximum type is evidenced for all seasons with a maximum at $3^h.5$. The graphs for Little America are similar to those for Barrow with a less sharply defined maximum occurring 1½ hours later. Summer activity is greater than for winter or equinox.

The data of Antarctic stations examined by Stagg fitted in well with data from seven northern stations as to type of variation and times of maximum disturbance. The controlling factors were found to be local time and magnetic latitude. It is thus somewhat surprising to find the data from Little America exhibiting the outer-zone type.

The times of the night maxima in magnetic character-graphs are given in Table 2 together with geographic and magnetic latitude, time of magnetic midnight—referred to the magnetic poles—and times of auroral maxima.

TABLE 2—*Magnetic and auroral particulars*

Station	Latitude		Approximate midnight		Maximum character	
	Geog. ϕ	Mag'c ϕ_m	Mag'c axis	Mag'c pole	Mag'c	Auroral
	°	°	h	h	h	h
Chesterfield	63.3 N	74.0 N	1	23.5	23.5	1
Barrow	71.4 N	68.6 N	2	4	2	23.5
Little America	78.4 S	74.4 S	20	16	3.5	3

From Table 2 it appears that the time of magnetic character-maximum is more closely related to geographic latitude than to any other factor in the table. A retardation of the night maximum in magnetic character by about one hour local time for each increase of 4° in geographic latitude is indicated. Only three stations are here considered. Stagg has definitely shown that data on magnetic character for ten well-distributed stations is related not to geographic but to magnetic latitude. The relationship of night maxima in diurnal magnetic-character to geographic latitude indicated in the above table may therefore be considered

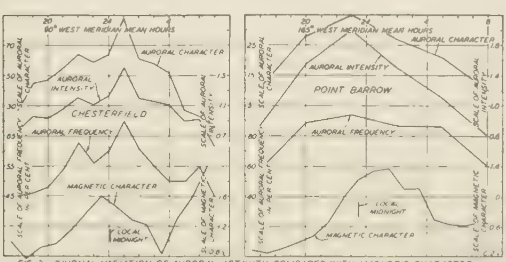


FIG. 3—DIURNAL VARIATION OF AURORAL ACTIVITY COMPARED WITH MAGNETIC CHARACTER, 1932-33, CHESTERFIELD AND POINT BARROW

doubtful, but some explanation of the apparent contradiction is necessary.

The diurnal activity of auroral disturbance at Chesterfield, Barrow, and Little America is shown in Figures 3, 4, and 5. Three different measures of auroral activity are plotted, namely, auroral character-number, auroral intensity, and auroral frequency. The first is an estimate of the total amount of aurora seen without regard to conditions of cloud or daylight. Auroral intensity is the average brightness of the displays seen on a scale of 0 for no aurora to 4 for a brilliant display. Auroral frequency is the percentage of times when aurora was seen at any given hour to the number of times when conditions of cloud and light allowed observation. These graphs are plotted from the results of visual observations (*a*) at three-hour intervals for Barrow, (*b*) at one-hour intervals for Chesterfield and (*c*) at half-hour intervals for Little America. Because of the higher geographic latitude of Little America, the day hours were sufficiently dark to allow of many more auroral observations during the day than at the other two stations. Consequently, whereas auroral graphs are drawn for the night hours for Chesterfield and Barrow, they are drawn for the whole day for Little America.

The auroral graphs for Barrow in Figure 3*b* are for the winter of 1932-33. The magnetic-character graph is the mean for the winter months November, December, January, and February. A maximum in auroral activity is shown at $23^h.5$ by all three estimates while the maximum of magnetic character is at $1^h.5$. The latter is an half-hour earlier than in the magnetic-character graphs of Figure 2. The maximum in auroral activity may be considered approximate because the observations are for three-hour intervals. The maximum in magnetic character at Barrow is about two hours later than the maximum in auroral activity. The center of auroral activity was reported by C. J. McGregor[2], who was in charge of the Barrow Station, to be in a northeasterly direction, that is, the band of maximum auroral activity in this locality is nearer the pole of the magnetic axis than the magnetic latitude (68°.6) of Barrow.

The graphs of auroral conditions and of magnetic character for Chesterfield in Figure 3*a* are for 56 winter nights during which the sky was clear. All three estimates of auroral activity show a maximum at 1^h with a secondary maximum at 22^h. Magnetic character has a maximum at $23^h.5$, or 1.5 hours earlier than the auroral maximum. At Barrow aurora preceded magnetic maximum by a similar period. The subsequent rise in magnetic character at Chesterfield after a minimum at $3^h.5$ shows the influence of the intense day-time magnetic disturbance. Except for the difference in times of maxima the auroral semi-diurnal graphs for Barrow and Chesterfield, including all three estimates of auroral activity, are similar.

Figures 4 and 5 show auroral diurnal activity for Little America during the winter of 1929. The graph of auroral character shows a single maximum at 3^h, and a sharp rise between 17^h and 18^h. The curve of magnetic character for July 1929, which was a very clear month with the greatest number of displays for any month, is plotted in Figure 4. It is a single wave with maximum at 3^h and minimum at $16^h.5$. The times of maximum of magnetic and auroral character agree quite closely. The

[2]In paper, Auroral observations at Point Barrow, Alaska, during the international Polar Year, presented April 26, 1935, before the sixteenth annual meeting of the American Geophysical Union and to be published in the Transactions of that meeting.

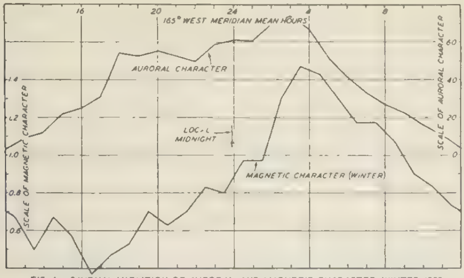


FIG. 4—DIURNAL VARIATION OF AURORAL AND MAGNETIC CHARACTER, WINTER, 1929, LITTLE AMERICA

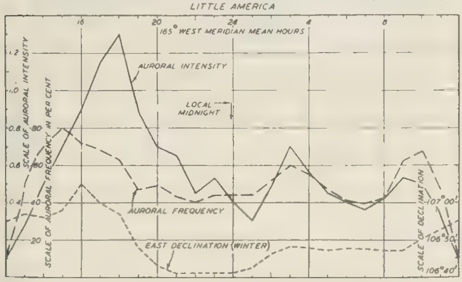


FIG. 5—DIURNAL VARIATION OF AURORAL INTENSITY, OF AURORAL FREQUENCY, AND OF MEAN HOURLY DECLINATION, WINTER, 1929, LITTLE AMERICA

magnetic maximum is an half-hour later than the auroral maximum. Chesterfield, at the same magnetic latitude, shows maximum auroral character two hours earlier and maximum magnetic character four hours earlier, local time, than Little America.

A marked difference exists between the three estimates of auroral activity at Little America. Figure 5 shows the diurnal variation of auroral intensity and frequency and also the diurnal variation of mean hourly values of declination during winter. The latter graph is from data used by C. C. Ennis[3] of the Department of Terrestrial Magnetism of the Carnegie Institution of Washington. Ennis has shown that not only mean declination at Little America but hourly variability of declination or the difference between mean values of declination from one hour to the next, shows a marked diurnal maximum in the afternoon. The graphs of auroral intensity and frequency show marked maxima also at about this time of day. This is quite in contrast with the auroral graphs for the two northern stations. Fewer auroral data are available during the day-hours than at night, but sufficient observations were made to establish the maximum of intensity at 18^h and to make the maximum of frequency at 15^h quite probable. If we consider only the night-hours, both auroral intensity and frequency show maxima at 3^h which is approximately the time of maximum of auroral character-number.

It is possible that if auroral observations could be made to a similar extent at Barrow and Chesterfield in the day-hours, similar differences in the three estimates of auroral activity might be found. The auroral

[3]In paper, Correlation of auroral and magnetic activity at Little America, first Byrd Antarctic Expedition, presented April 26, 1935, before the sixteenth annual meeting of the American Geophysical Union and to be published in the Transactions of that meeting.

data actually obtained at these two stations, however, do not suggest that a maximum occurs at any but the midnight hours. It is of interest to note that although only a single maximum at $3^h.5$ occurs in the graphs of magnetic character for all seasons at Little America, different estimates of magnetic disturbance based on mean hourly declination values show maxima at 16^h. The mean hourly value estimate of magnetic disturbance, however, is more closely related to what is termed in Stagg's paper[1] "The regular diurnal variation of magnetic disturbance" which, although a local-time phenomenon, is not the same thing as the diurnal variation of irregular magnetic disturbance, which is also dependent upon local time.

The graph of magnetic character has a minimum value at the time when the graphs of auroral intensity and frequency show maximum values. A marked difference existed at Little America between auroral conditions at the times of maxima of auroral character and of frequency-intensity. At $3^h.5$ when the largest number of displays was seen, the center of auroral activity was very near Little America. In the afternoon and early evening, displays were much lower in the sky in a direction opposite to the magnetic axis. Magnetic disturbances, if associated with aurora, might be expected to be more intense at $3^h.5$ when aurora was much nearer the station. The writer, when at Little America in 1929, had a strong impression of the existence of two separate bands of maximum auroral activity[4]. The time of maximum activity in the band furthest from the station (and from the magnetic axis) was 18^h. In the band nearer Little America, the maximum was $3^h.5$ or 9.5 hours later. Rough estimates based on visual observations placed these auroral maximum activity-bands at about 70° and 74° magnetic latitude, respectively. A marked difference between these bands was that, in general, great activity occurred in the afternoon and early evening in the 70°-band but not in the 74°-band, whereas great activity in the 74°-band at $3^h.5$ was accompanied by increased activity in the 70°-band also.

At Chesterfield the band of maximum auroral activity was in a direction away from the magnetic axis at about 70° magnetic latitude; Chesterfield and Little America are situated similarly with respect to the auroral maximum activity-bands in either hemisphere. We might, therefore, expect to see some similarity in diurnal variation of auroral activity. At Chesterfield a secondary maximum in auroral activity occurs three hours before the major maximum. The tendency for auroral displays to mount higher in the sky during the evening at Chesterfield indicates that at the time of secondary maximum, 22^h, the center of auroral activity was some distance further from the station than at the time of major maximum at 1^h. Although a relative lull in auroral activity occurred between 22^h and 1^h no such marked difference is shown as at Little America. If two maximum auroral activity-bands exist near Chesterfield then they are much closer together than in the case of Little America and the times of maximum activity are 3 hours instead of 9.5 hours apart. At both stations, however, auroral activity in the outer band precedes activity in the inner one.

Figure 6 shows for Chesterfield in March 1933, a month in which

[4] F. T. Davies, Observations of the Aurora Australis, Byrd Antarctic Expedition, 1929, Terr. Mag., **36**, 199-230 (1931).

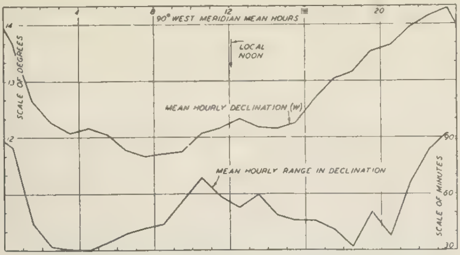


FIG 6—DIURNAL VARIATION OF MEAN HOURLY DECLINATION AND MEAN HOURLY RANGE IN DECLINATION, MARCH, 1933, CHESTERFIELD

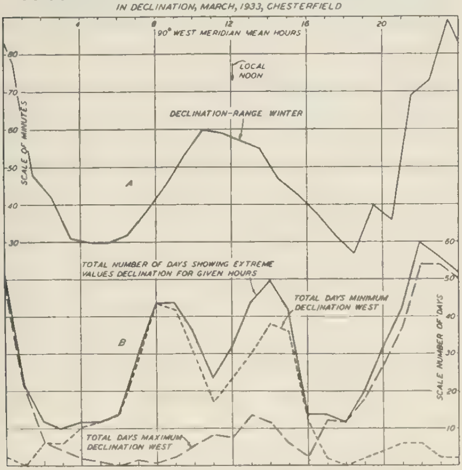


FIG. 7—(A) DIURNAL VARIATION OF MEAN HOURLY RANGE IN DECLINATION, WINTER, 1932–33, AND, (B) DIURNAL INCIDENCE OF DAILY EXTREME VALUES OF DECLINATION, 1932–33, CHESTERFIELD

aurora was quite active, graphs of hourly mean values of declination and hourly mean declination-range. The former does not show the marked midday maximum seen in graphs of magnetic character for all seasons at this station. The graph of declination-range, however, does show a midday maximum value. If declination-traces were integrated for each hour, the midday maximum would be much enhanced because of the oscillatory character of the traces at this time of day. It is evident that graphs of mean hourly values of declination at this station do not correspond very well with graphs of magnetic character. Hourly mean values of declination-range are then a better estimate of magnetic disturbance. Figure 7*A* shows the diurnal variation of mean declination-range for three active auroral months, January to March, 1933, at Chesterfield. The pronounced maximum at $23^h.5$ and the secondary maximum at noon agree closely with the variation of magnetic character for winter. It is evident then that what is termed magnetic character, which is a personal estimate of magnetic disturbance, is better represented by range in declination than by mean values. Figure 7*B* shows for Chesterfield the number of days in the year on which

daily extreme values of declination occurred at given hours. This graph also brings out the essential form of the graph of magnetic character.

It seems probable then that the graph of mean declination for Little America in Figure 5 is not as good an indicator of magnetic disturbance as the graph of magnetic character. The following marked difference exists between the two stations: At Chesterfield the times of maximum diurnal range in declination, maximum magnetic character, and maximum departure in the declination mean values, all coincide near local midnight. At Little America maximum magnetic character occurs 9½ hours later than the maximum departure in declination mean values. It appears then that during the winter at Chesterfield maxima in irregular (D) and regular (SD) magnetic disturbance occur at the same time. At Little America the times of maxima are widely separated. Further analysis of the magnetic data at these stations is necessary, however, before this point can be definitely established.

Diurnal variation of hourly range of earth-potential at Chesterfield—Figure 8 shows for typical summer and winter months (June and January 1933—the same months used in magnetic character comparisons) the diurnal variation in hourly range of earth-potential in north-south and east-west directions, respectively. The ordinates are expressed in millivolts per kilometer. The graphs are essentially the same in type and times of maximum values as the graphs of magnetic character, with wide maxima at midday in summer and winter, sharp maxima at $23^h.5$ in summer. Earth-potential ranges are much less in summer than in winter. The intensity of magnetic disturbance is also less in summer.

Figure 9 shows the diurnal variation in hourly range of earth-potential in an east-west direction for January and June 1933 at College, Alaska, on the same scale as Figure 8. The corresponding variation in north-south direction is very similar. For winter and summer the graphs show single maxima near local midnight. This is what might be expected

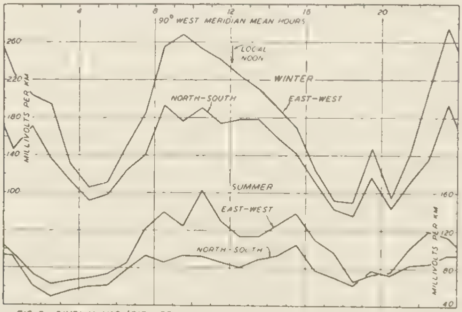


FIG. 8—DIURNAL VARIATION OF MEAN HOURLY RANGE IN EARTH POTENTIAL, SUMMER AND WINTER, 1933, CHESTERFIELD

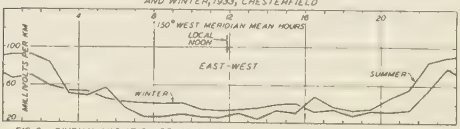


FIG. 9—DIURNAL VARIATION OF MEAN HOURLY RANGE IN EARTH POTENTIAL, SUMMER AND WINTER, 1933, COLLEGE

for diurnal variation of magnetic character at this station situated, as it is, in the outer zone.

Figures 8 and 9 indicate that the diurnal range of the earth-potential changes in a similar way to the diurnal variation of magnetic character as the ($\phi_m = 70°$)-boundary is crossed. Range in earth-potential is probably a good measure of activity. Integration of the traces for each hour would be a better one, but, owing to the very oscillatory character of the traces, this would be a difficult task. Figures 6, 7, and 8 show that for Chesterfield the hourly ranges in declination and in earth-potential are good measures of activity for magnetic or earth-potential disturbance, as they are free from any personal estimate and yet bring out very clearly the diurnal variation.

SUMMARY

(1) The boundary of the transition-zone $\phi_m = 70°$ in diurnal variation of irregular magnetic disturbance, which J. M. Stagg also shows is the region of greatest magnetic disturbance, seems to be also the region of maximum auroral activity and probably of earth-potential disturbance too.

(2) At Chesterfield diurnal variation of magnetic and earth-potential disturbance conforms to the transition-zone type.

(3) At Barrow diurnal variation of magnetic disturbance conforms to the outer-zone type except for the maximum occurring two hours later.

(4) The diurnal variation in earth-potential range at Chesterfield and College, is similar in form to the variations of magnetic character in the transition-zone and outer-zone, respectively.

(5) At Little America for 1929-30, the diurnal variation of magnetic character does not conform in type or time of maximum with the results of Stagg's investigation.

(6) The times of the maximum at night in diurnal variation of magnetic character at Chesterfield, Barrow, and Little America appear to be related to geographic latitude.

(7) Three different estimates of auroral activity agree closely for Barrow and Chesterfield, but differ considerably for Little America. At the three stations the auroral character-number shows maxima at approximately the same hours as magnetic character, which is also a measure of irregular magnetic disturbance. At Little America, auroral frequency and intensity show maxima at the same hours as maximum departure in mean declination-values, which is a measure of the regular variation in magnetic disturbance. Magnetic and auroral activity, judged by different estimates, reach their maxima at the same hours. The difference in phase of the diurnal irregular and regular magnetic disturbance at Little America paralleled by different estimates of auroral activity, may explain the apparent disagreement in the diurnal variation of magnetic character with respect to the magnetic latitude of the station. That 1929 was close to the maximum of the sunspot-cycle whereas 1932-33 was at minimum may have a bearing on this question.

The auroral and magnetic records of the second Byrd Antarctic Expedition 1934-35 will be of particular interest in clearing up this point.

Thanks and acknowledgments are due Dr. B. W. Currie of the University of Saskatchewan for the Chesterfield earth-potential data, to C. J. McGregor of the United States Weather Bureau for the Barrow auroral data, and to C. C. Ennis, W. J. Rooney and others of my colleagues in the Department of Terrestrial Magnetism for helpful comment and discussion.

DEPARTMENT OF TERRESTRIAL MAGNETISM,
CARNEGIE INSTITUTION OF WASHINGTON,
Washington, D. C.

SEASONAL VARIATION IN EARTH-CURRENTS AT TUCSON, ARIZONA

By W. J. Rooney

Abstract—The seasonal changes in earth-current flow at Tucson, Arizona, have been determined from a study of the calm day diurnal-variation records for the three-year period 1932-34 obtained through the cooperation of the Department of Terrestrial Magnetism of the Carnegie Institution of Washington, the United States Coast and Geodetic Survey, the American Telephone and Telegraph Company, and the Mountain States Telephone and Telegraph Company. The records are of unusual interest because of the location of Tucson in the transition-belt where the diurnal variation of magnetic dip and intensity change from high-latitude to equatorial type. The diurnal variation in earth-current potential-gradient, as shown by a series of monthly hodographs also changes radically with season. During the winter the hodographs are nearly circular with the current-vector proceeding in two clockwise loops. During the summer they are elongated greatly along a line which corresponds closely to the magnetic meridian and the clockwise rotation of the vector tends to disappear at midday. A decided parallelism with the changes noted in the diurnal variation of the magnetic elements as recorded at the same station adds to the evidence linking the two phenomena together.

Earth-current measurements at Tucson, Arizona, have been carried on since March, 1931, as a cooperative project of the Department of Terrestrial Magnetism of the Carnegie Institution of Washington, The United States Coast and Geodetic Survey, the American Telephone and Telegraph Company, and the Mountain States Telephone and Telegraph Company. Two components of the potential gradient are recorded continuously by two Leeds and Northrup galvanometers with current-sensitivity about 2.5×10^{-9} ampere per millimeter scale-deflection, each used in series with a resistance of 1.5 megohms. The northerly line extends from the Tucson Magnetic Observatory to an electrode located in Mammoth, a distance of 56.8 kilometers in a direction 19° 33′ east of north and the eastward line from the Observatory to Wilcox is 93.9 kilometers long, its direction being less than one-half degree north of true east. Connections of the electrodes to the recording mechanism are made with the lines of the Telephone Company. The true northward component of earth-current is calculated from the combined records obtained, while the eastward component is given directly by the Wilcox line. Details of the site and installation have been given in a previous paper.[1]

The records for the three-year period, 1932-34 have been reduced and analyzed and, pending their publication in full, there is presented here a condensed summary of the mean results by months with a brief discussion of one feature of the records—the variation of earth-current flow with season of the year. In this respect the Tucson results are of unusual interest because of the location of the Tucson Observatory. It is well known that, as one passes from the poles toward the equator there is a decided change in the type of diurnal variation in magnetic dip and intensity. This is because the center of the electric current-systems circulating in the ionized layers of the upper atmosphere, and now generally accepted as the chief cause of the diurnal variation, is always south of the first stations encountered, while at stations close to the equator, the center is always to the north. Tucson is situated in the transition-belt between the two regions, close to the mean central point of circulation, with the result that it lies north of the center during the winter when the Sun is low and to the south of it at midsummer.

[1]Carnegie Year Book No. 30, 1930-31, 347-348.

The diurnal-variation curves of the several magnetic elements obtained at Tucson consequently differ greatly at different times of the year. In view of the close tie-up between earth-currents and the Earth's magnetic field it is to be expected that the earth-current records should show a similar change in type of diurnal variation with season of the year.

Table 1—*Diurnal variation northward earth-current gradient in millivolts per kilometer, Tucson, Arizona for all days recorded during three-year period 1932-34*

(Tabular values are average departures from mean of day of 60-minute means centering on the half-hour, a positive sign indicating current flowing northward is greater than mean of day.)

105° west M.M.T. hour	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.	Mea
0– 1	−0.82	−0.54	−0.57	−0.15	−0.39	−0.24	−0.35	−0.48	−0.23	−0.31	−0.21	−0.67	−0.
1– 2	−0.72	−0.56	−0.46	−0.32	+0.10	−0.10	+0.14	−0.34	−0.21	−0.76	−0.39	−0.87	−0.
2– 3	−0.62	−0.35	−0.24	+0.09	+0.08	+0.15	−0.05	+0.20	−0.10	−0.59	−0.40	−0.33	−0.
3– 4	−0.59	−0.51	−0.09	+0.17	+0.25	+0.47	+0.61	+0.28	+0.14	−0.06	−0.26	−0.34	0.
4– 5	−0.26	−0.19	+0.12	+0.37	+0.84	+0.85	+0.67	+0.73	+0.44	−0.02	−0.21	−0.52	+0.
5– 6	−0.39	+0.16	+0.14	+0.88	+1.26	+1.22	+1.14	+1.04	+0.60	+0.20	0.00	−0.39	+0.
6– 7	−0.02	+0.19	+1.09	+1.86	+2.02	+2.21	+2.30	+2.48	+1.84	+1.19	+0.37	−0.38	+1.
7– 8	+0.51	+0.62	+1.93	+2.34	+2.48	+2.32	+2.74	+3.21	+2.34	+2.07	+1.36	−0.30	+1.
8– 9	+1.69	+0.75	+1.69	+1.15	+0.92	+0.81	+1.34	+1.46	+0.49	+1.74	+1.13	+1.11	+1.
9–10	+2.29	+0.81	+0.06	−1.27	−1.45	−1.40	−1.33	−1.76	−1.86	−0.03	+0.11	+1.19	−0
10–11	+1.34	+0.12	−1.82	−2.83	−2.98	−3.18	−3.57	−3.96	−3.15	−2.03	−0.91	+0.58	−1.
11–12	−1.47	−0.97	−2.46	−2.70	−3.50	−3.45	−3.93	−3.98	−3.14	−2.76	−1.83	−0.75	−2.
12–13	−2.44	−1.71	−2.48	−2.08	−2.43	−2.66	−3.20	−2.97	−1.95	−1.89	−1.78	−1.18	−2.
13–14	−2.00	−1.26	−1.46	−1.48	−1.34	−1.39	−1.74	−1.60	−0.58	−0.61	−0.56	−1.09	−1.
14–15	−1.12	−0.48	−0.32	−0.58	−0.33	−0.43	−0.21	+0.18	+0.68	+0.40	−0.06	−0.50	−0.
15–16	+0.16	+0.23	+0.73	+0.42	+0.69	+0.81	+0.74	+1.13	+1.47	+0.82	+0.49	+0.20	+0.
16–17	+1.08	+0.97	+1.16	+1.20	+1.08	+1.21	+1.55	+1.62	+1.00	+0.67	+0.76	+0.89	+1.
17–18	+1.15	+0.84	+1.39	+1.66	+1.56	+1.44	+1.53	+1.14	+0.59	+0.74	+0.85	+1.16	+1
18–19	+1.13	+0.91	+1.02	+0.93	+0.58	+0.60	+0.88	+0.59	+0.64	+0.86	+0.97	+1.07	+0.
19–20	+0.72	+0.83	+0.88	+0.39	+0.32	+0.18	+0.22	+0.24	+0.39	+0.72	+0.64	+0.74	+0.
20–21	+0.60	+0.51	+0.28	+0.34	+0.38	+0.24	+0.20	+0.27	+0.36	+0.55	+0.45	+0.50	+0.
21–22	+0.42	+0.06	+0.27	−0.05	+0.44	+0.16	+0.23	+0.30	+0.12	+0.08	+0.13	+0.09	+0.
22–23	−0.08	−0.02	−0.41	−0.37	−0.03	+0.13	+0.12	+0.25	+0.14	−0.11	−0.15	−0.23	−0.
23–24	−0.46	−0.41	−0.34	−0.16	−0.43	+0.18	+0.08	−0.01	−0.14	−0.76	−0.50	−0.77	−0.
No. days recorded	86	85	92	84	86	76	80	76	75	77	82	88	9

Tables 1 to 4 show the mean diurnal variation by months for all days recorded and for ten calm days per month. The values for the northward component in Tables 1 and 2 were obtained by combining the records from the northerly and the eastward lines using the formula

$$N = 1.064\ N' - 0.356E$$

The values for the eastward component are given as measured directly on the line from the Observatory to Wilcox.

Figure 1 shows the mean diurnal-variation curves for the entire three-year period, both northward and eastward components being given for all days recorded and for ten calm days per month. It has been found here, as elsewhere, that a definite and consistent difference exists between the diurnal variation on calm and disturbed days but the

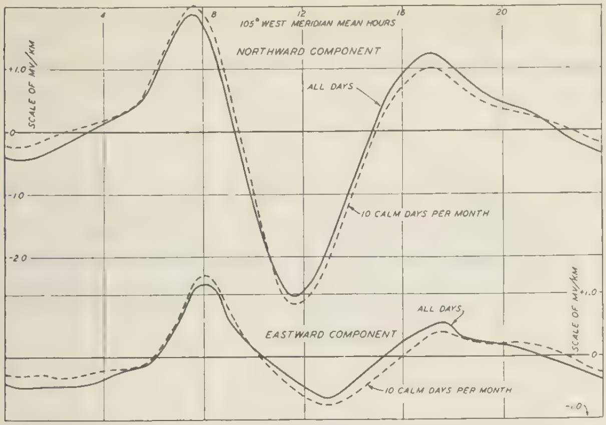


FIG. 1—DIURNAL VARIATION EARTH-CURRENT POTENTIAL-GRADIENT AT TUCSON, ARIZONA, MEAN FOR THREE-YEAR PERIOD, 1932-1934

TABLE 2—*Diurnal variation northward earth-current gradient in millivolts per kilometer, Tucson, Arizona, for ten calm days per month recorded during three-year period, 1932-34*

(Tabular values are average departures from mean of day of 60-minute means centering on the half-hour, a positive sign indicating current flowing northward is greater than mean of day.)

105° west M.M.T. hour	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.	Mean
0–1	−0.65	−0.43	−0.23	−0.08	−0.18	+0.10	−0.08	+0.05	−0.29	−0.28	−0.39	−0.52	−0.25
1–2	−0.42	−0.50	−0.11	+0.17	+0.16	−0.02	+0.30	+0.03	−0.03	−0.57	−0.14	−0.57	−0.14
2–3	−0.28	−0.36	−0.17	+0.10	+0.30	+0.28	−0.06	+0.12	+0.39	+0.04	−0.14	−0.21	0.00
3–4	−0.40	−0.20	−0.04	+0.08	+0.33	+0.49	+0.62	+0.59	+0.31	0.00	−0.10	−0.39	+0.11
4–5	−0.36	−0.20	+0.14	+0.44	+0.84	+0.57	+0.74	+0.74	+0.46	+0.21	−0.10	−0.28	+0.26
5–6	−0.16	+0.13	+0.28	+0.89	+1.13	+1.33	+0.98	+1.09	+1.08	+0.29	+0.07	−0.19	+0.57
6–7	−0.16	+0.31	+0.77	+1.90	+2.26	+2.30	+2.38	+2.41	+2.40	+1.23	+0.49	−0.09	+1.35
7–8	+0.46	+0.97	+1.71	+2.84	+2.62	+2.43	+2.84	+3.29	+2.60	+2.48	+1.44	+0.44	+2.02
8–9	+1.95	+1.36	+1.50	+1.36	+1.42	+1.03	+1.57	+1.71	+0.91	+2.15	+1.35	+1.03	+1.44
9–10	+2.74	+1.02	+0.06	−1.27	−1.60	−1.35	−0.98	−1.97	−1.71	−0.26	+0.62	+1.34	−0.19
10–11	+1.70	+0.14	−1.56	−3.35	−3.05	−3.21	−3.47	−4.09	−3.31	−2.30	−0.50	+0.80	−1.87
11–12	−1.35	−1.18	−2.39	−3.10	−3.77	−3.63	−3.90	−4.10	−3.66	−3.01	−1.64	−0.99	−2.73
12–13	−2.47	−1.87	−2.37	−2.30	−2.73	−2.80	−3.18	−3.18	−2.65	−2.32	−1.84	−1.35	−2.42
13–14	−2.10	−1.45	−1.66	−1.64	−1.57	−1.54	−1.96	−1.82	−1.05	−0.81	−0.98	−1.19	−1.48
14–15	−1.40	−0.71	−0.40	−0.63	−0.26	−0.49	−0.36	−0.03	+0.32	+0.23	−0.40	−0.50	−0.39
15–16	−0.02	−0.19	+0.53	+0.48	+0.85	+0.67	+0.38	+1.09	+1.23	+0.54	+0.05	0.00	+0.46
16–17	+0.67	+0.68	+0.97	+1.32	+1.06	+1.20	+1.20	+1.28	+0.98	+0.58	+0.31	+0.73	+0.91
17–18	+1.08	+0.74	+0.92	+1.45	+1.28	+1.23	+1.31	+0.91	+0.40	+0.37	+0.61	+1.01	+0.94
18–19	+0.73	+0.57	+0.92	+0.92	+0.59	+0.66	+0.62	+0.36	+0.48	+0.63	+0.62	+0.71	+0.64
19–20	+0.60	+0.59	+0.60	+0.29	+0.18	+0.24	+0.14	+0.31	+0.34	+0.54	+0.47	+0.57	+0.40
20–21	+0.41	+0.56	+0.43	+0.49	+0.51	−0.02	+0.07	+0.24	+0.36	+0.29	+0.29	+0.18	+0.31
21–22	+0.18	+0.08	+0.22	−0.06	+0.08	+0.20	+0.17	+0.53	+0.32	+0.11	+0.21	+0.13	+0.18
22–23	−0.10	0.00	+0.17	−0.27	−0.07	+0.13	+0.31	+0.46	+0.20	−0.10	−0.16	+0.04	+0.03
23–24	−0.39	−0.01	−0.22	−0.04	−0.28	+0.21	+0.15	−0.11	−0.07	−0.34	−0.20	−0.81	−0.14

difference is small and, for the general considerations involved at present, it is immaterial whether all days or only calm days are considered. The mean curves, which as will be seen later are fairly typical for the equinoctial periods, are very like those obtained at Watheroo except for a reversal of the sign of the northward component, due to the fact that the two stations lie on opposite sides of the equator.

In Figure 2 is a series of hodographs which bring out more clearly the variations in current-flow with season of the year than do the usual time-curves of the two components. The points mark the successive positions of the ends of vectors representing the magnitude and direction of the potential gradient, and hence of current-flow, for each hour of the day. The center figure represents the same data as those in the two calm-day curves of the previous figure. In type it is very like the graphs

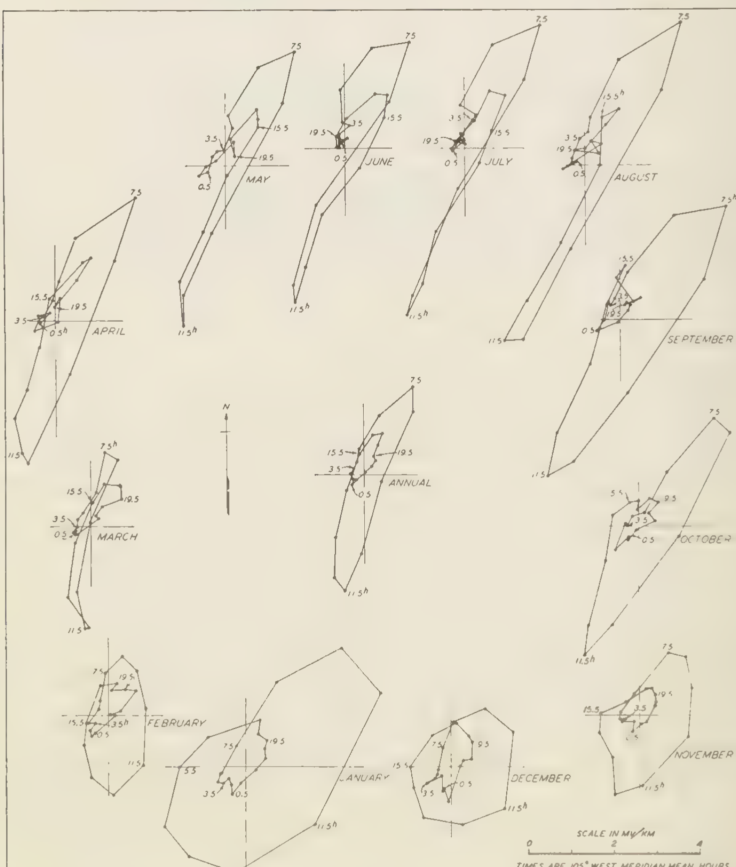


FIG. 2—SEASONAL CHANGE IN EARTH-CURRENT DIURNAL-VARIATION AT TUCSON, ARIZONA, 10 CALM DAYS PER MONTH FOR THREE-YEAR PERIOD, 1932-1934

TABLE 3—*Diurnal variation eastward earth-current gradient in millivolts per kilometer, Tucson, Arizona, for all days recorded during three-year period, 1932-34*

(Tabular values are average departures from mean of day of 60-minute means centering on the half-hour, a positive sign indicating current flowing eastward is greater than mean of day.)

105° west M.M.T. hour	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.	Mean
0–1	−0.36	−0.49	−0.77	−0.26	−0.53	−0.68	−0.64	−0.42	−0.62	−0.39	0.00	−0.46	−0.48
1–2	−0.52	−0.54	−0.58	−0.40	−0.36	−0.11	−0.14	−0.34	−0.64	−0.87	−0.61	−0.52	−0.48
2–3	−0.64	−0.46	−0.44	−0.56	−0.28	−0.35	−0.45	−0.02	−0.46	−0.74	−0.65	−0.52	−0.47
3–4	−0.79	−0.49	−0.59	−0.32	−0.17	−0.17	−0.04	−0.22	−0.47	−0.38	−0.72	−0.66	−0.42
4–5	−0.61	−0.78	−0.25	−0.02	+0.17	+0.09	+0.07	+0.08	−0.26	−0.42	−0.52	−0.60	−0.26
5–6	−0.54	−0.24	−0.14	+0.22	−0.07	−0.20	−0.03	+0.03	+0.14	+0.10	−0.44	−0.82	−0.17
6–7	−0.56	−0.26	+0.05	+0.57	+0.72	+0.55	+0.45	+0.83	+1.08	+0.55	−0.32	−0.58	+0.24
7–8	−0.27	−0.01	+0.55	+1.58	+1.37	+1.26	+1.53	+2.30	+2.46	+1.62	−0.67	−0.34	+1.04
8–9	+0.84	+0.18	+1.05	+1.23	+0.92	+0.72	+1.17	+1.76	+1.87	+1.98	+0.86	+0.04	+1.04
9–10	+1.85	+0.67	+0.38	+0.15	−0.45	−0.69	−0.46	−0.16	+0.34	+1.25	+0.89	+0.94	+0.37
10–11	+3.24	+0.89	−0.32	−0.45	−0.98	−1.22	−1.37	−1.29	−0.68	−0.28	+0.95	+1.69	+0.02
11–12	+1.53	+1.13	+0.08	−0.41	−0.90	−1.12	−1.57	−1.72	−1.43	−1.11	+0.14	+1.42	−0.30
12–13	−0.22	+0.09	−0.04	−0.78	−0.94	−0.94	−1.09	−1.36	−1.15	−0.91	−0.56	+0.65	−0.60
13–14	−1.42	−0.57	−0.37	−0.70	−0.38	−0.22	−0.49	−0.81	−0.57	−0.64	−0.11	−0.46	−0.56
14–15	−1.71	−0.28	0.00	−0.29	+0.29	+0.60	+0.27	+0.34	−0.24	−0.36	−0.46	−0.85	−0.23
15–16	−1.39	−0.25	+0.13	+0.03	+0.85	+1.10	+0.74	+0.59	+0.09	+0.11	−0.52	−0.83	+0.05
16–17	−0.32	+0.14	+0.52	+0.63	+0.91	+1.15	+1.21	+0.67	−0.19	+0.06	−0.18	−0.16	+0.37
17–18	+0.26	+0.20	+0.81	+0.98	+0.89	+1.06	+1.02	+0.39	+0.32	+0.14	+0.15	+0.43	+0.56
18–19	+0.19	+0.30	+0.62	+0.29	0.00	+0.28	+0.34	+0.14	+0.26	+0.18	+0.39	+0.53	+0.29
19–20	+0.41	+0.58	+0.31	−0.30	+0.09	−0.24	+0.10	−0.01	+0.03	+0.68	+0.53	+0.53	+0.22
20–21	+0.43	+0.37	−0.17	−0.08	−0.12	+0.01	−0.11	0.00	+0.05	+0.29	+0.46	+0.48	+0.13
21–22	+0.28	+0.10	+0.03	−0.21	−0.18	−0.17	−0.23	−0.49	+0.02	+0.45	+0.26	+0.30	+0.01
22–23	+0.29	−0.08	−0.60	−0.49	−0.33	−0.34	−0.08	+0.02	+0.12	−0.25	0.00	+0.18	−0.14
23–24	0.00	−0.21	−0.31	−0.45	−0.56	−0.33	−0.21	−0.33	−0.05	−0.72	−0.13	−0.36	−0.31
No. days recorded	83	85	93	83	85	79	78	72	77	79	81	88	983

for three of the equinoctial months (April, September, and October), but for some reason or other the graph for March is distinctly different. As far as amplitude goes the mean curve for Tucson might be rather misleading, since, because of the phase-shifts with season, neither the northward nor the eastward spread of the figure is nearly as great as the mean of the corresponding amplitudes in the individual months.

It might be emphasized here that the graphs for the individual months do not, like the mean annual graph, give merely a general average obtained from diverse figures, but are distinctly typical and reproduce themselves with remarkable closeness from year to year. For example, the graph for March, which is quite unlike those for the other three equinoctial months and bears very little resemblance to that for February or April, was not obtained by combining three individual records, one of which resembled that for February, one that for April, and a third of different appearance. On the contrary, the graphs for the three individual months of March and also that for March, 1931, are all very like each other and like the graph in Figure 2. Or if we consider January, which differs from the neighboring months chiefly in amplitude, we find that the *smallest* of the four January graphs obtained between 1932 and 1935 is larger than the *largest* of the four graphs for

TABLE 4—*Diurnal variation eastward earth-current gradient in millivolts per kilometer, Tucson, Arizona, for ten calm days per month recorded during three-year period, 1932-34*

(Tabular values are average departures from mean of day of 60-minute means centering on the half-hour, a positive sign indicating current flowing eastward is greater than mean of day.)

105° west M.M.T. hour	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.	Mean
0–1	−0.33	−0.33	−0.38	−0.40	−0.45	−0.14	−0.32	−0.21	−0.58	−0.25	−0.19	−0.19	−0.31
1–2	−0.35	−0.41	−0.38	−0.14	−0.34	−0.18	−0.09	−0.32	−0.39	−0.57	−0.15	−0.22	−0.29
2–3	−0.41	−0.46	−0.43	−0.42	−0.22	−0.17	−0.32	−0.33	−0.30	−0.32	−0.43	−0.25	−0.34
3–4	−0.55	−0.32	−0.33	−0.30	−0.06	+0.08	+0.17	−0.14	−0.26	−0.28	−0.36	−0.59	−0.25
4–5	−0.66	−0.50	−0.34	−0.09	+0.14	−0.08	+0.26	+0.05	−0.15	−0.17	−0.47	−0.40	−0.20
5–6	−0.60	−0.25	−0.21	+0.06	+0.05	−0.13	−0.13	+0.11	+0.14	+0.11	−0.46	−0.34	−0.14
6–7	−0.58	−0.24	+0.11	+0.46	+0.75	+0.60	+0.63	+0.79	+1.24	+0.64	−0.15	−0.32	+0.33
7–8	−0.24	−0.10	+0.31	+1.87	+1.61	+1.49	+1.74	+2.24	+2.46	+1.76	+0.65	−0.21	+1.13
8–9	+0.70	+0.31	+0.52	+1.38	+1.33	+1.00	+1.36	+1.77	+1.95	+2.13	+1.07	+0.03	+1.13
9–10	+2.22	+0.64	−0.04	+0.31	−0.36	−0.71	−0.19	−0.36	+0.14	+0.91	+1.21	+0.78	+0.38
10–11	+3.13	+0.85	−0.35	−0.69	−1.03	−1.24	−1.28	−1.50	−1.13	−0.64	+1.12	+1.44	−0.11
11–12	+1.60	+0.80	−0.16	−0.83	−1.01	−1.22	−1.40	−1.93	−1.73	1.30	+0.04	+1.22	−0.49
12–13	−0.27	+0.10	−0.07	−1.00	−1.09	−0.98	−1.04	−1.39	−1.51	−1.20	−0.59	+0.26	−0.73
13–14	−1.35	−0.42	−0.56	−0.71	−0.55	−0.57	−0.72	−0.60	−0.73	−0.80	−0.65	−0.67	−0.69
14–15	−1.92	−0.59	−0.39	−0.39	+0.01	+0.30	+0.31	+0.33	−0.33	−0.62	−0.96	−0.89	−0.43
15–16	−1.57	−0.55	+0.01	−0.16	+0.74	+0.88	+0.61	+0.38	+0.10	0.22	−0.92	−0.97	−0.14
16–17	−0.89	−0.20	+0.29	+0.61	+0.73	+0.98	+0.92	+0.78	−0.11	−0.02	−0.34	−0.46	+0.19
17–18	+0.31	+0.18	+0.67	+0.82	+0.71	+0.69	+0.56	+0.47	+0.28	−0.06	+0.13	+0.11	+0.41
18–19	+0.27	+0.05	+0.67	+0.48	+0.08	+0.02	+0.21	−0.02	+0.45	+0.24	+0.27	+0.39	+0.29
19–20	+0.47	+0.38	+0.70	−0.04	+0.19	−0.23	−0.24	−0.26	+0.09	+0.44	+0.35	+0.48	+0.19
20–21	+0.42	+0.62	+0.24	+0.09	+0.14	−0.20	−0.01	+0.30	+0.12	+0.20	+0.36	+0.45	+0.2[illegible]
21–22	+0.43	+0.25	+0.08	+0.02	−0.25	+0.03	−0.11	+0.13	+0.21	+0.37	+0.33	+0.27	+0.1[illegible]
22–23	+0.21	+0.01	+0.16	−0.52	−0.48	−0.07	−0.12	+0.36	+0.16	−0.08	+0.14	+0.20	0.0[illegible]
23–24	−0.14	+0.14	−0.38	−0.41	−0.65	−0.18	−0.06	−0.54	−0.09	−0.29	+0.02	−0.09	−0.2[illegible]

either December or February. So it seems quite certain that the differences indicated by this figure are real and are not due either to accident or experimental error.

Passing around the clock of the year then we find a very definite change in the type of variation recorded. During the four winter months the hodograph consists of two nearly circular clockwise loops, a large one during the day and a smaller one at night, with not much difference in the amplitudes of the two components. During the four midsummer months the hodograph is decidedly elongated by reason of a large increase in the amplitude of the northward component and a tendency for the two components to remain in phase particularly during the midday hours. During this time the current-flow seems to be restricted to almost a straight line which it will be noted is not far from the magnetic meridian. The mean declination at Tucson is about 14° east. During June and July the progress of the vector during the midday minimum, that is, greatest flow of current toward the south, is slightly counterclockwise instead of clockwise as during the remainder of the year. The graphs for March, April, September, and October are of an intermediate character.

As mentioned before, the records for January and March are anomalous, each in a different aspect. January falls out—or rather jumps out—of the picture by reason of an *increase in the amplitude of both*

components, while the phase-relationship does not change appreciably from its regular winter character. During March, on the other hand, the northward component follows its normal trend toward increasing amplitude with height of Sun and the peculiarities of the graph are due almost entirely to the eastward component which becomes vague and irregular at this time each year.

In Table 5 are given the constants for the harmonic series $\Sigma c_n \sin (n\theta+\phi n)$ as determined for the mean monthly and mean annual calm-day diurnal-variation data. Although the hourly means are centered on the half-hour the phase-angles shown in Table 5 have been increased by $7.5\,n$ degrees so that $\theta=0°$ in the table corresponds to midnight (105° west meridian mean time). From this table the harmonic dials

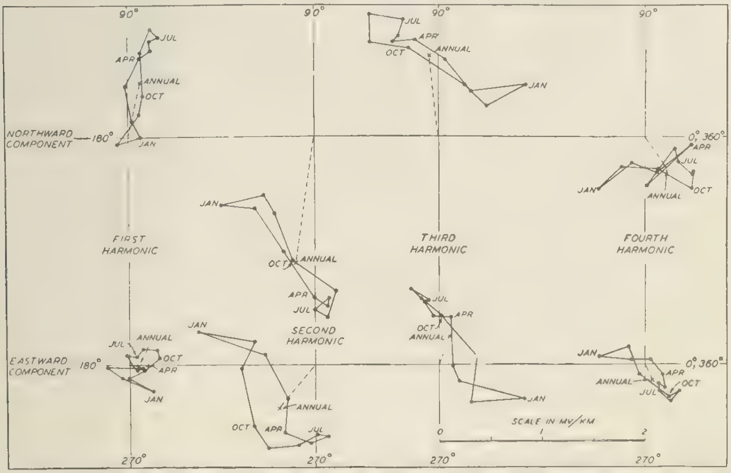


FIG. 3—SEASONAL CHANGE IN PHASE AND AMPLITUDE, PRINCIPAL HARMONICS OF DIURNAL-VARIATION CURVE, EARTH-CURRENT POTENTIAL-GRADIENT AT TUCSON, ARIZONA—MEAN VALUES SHOWN FOR THREE-YEAR PERIOD, 1932-1934, TEN CALM DAYS PER MONTH—0° CORRESPONDS TO MIDNIGHT 105° WEST MERIDIAN MEAN HOURS

in Figure 3 have been constructed. They indicate the seasonal changes in phase and amplitude of the first four harmonics for each component. All four harmonics are found to vary in the same manner. During the summer months the two components are in close phase-agreement and the amplitude of the northward component is much the larger. The records in winter for the northward component show some retardation in the phase-angles and a sharp decrease in amplitude *except* in the case of January. The harmonics of the eastward component show much less change in amplitude but a considerably greater retardation in phase. The points in winter for the eastward component will be seen to have dropped back in phase with reference to the corresponding points for the northward by something between 45° and 90° so that they fall in the next quadrant back. These changes are most readily apparent in the large second and third harmonics, although the type of change is about the same for all four.

Having observed the changes which take place in the diurnal varia-

TABLE 5—*Fourier analyses of diurnal variations of earth-current potential-gradient at Tucson, Arizona, for ten calm days per month, during three-year period, 1932-34*

Component	Month	Amplitudes				Phase-angles			
		c_1	c_2	c_3	c_4	ϕ_1	ϕ_2	ϕ_3	ϕ_4
		mv/km	*mv/km*	*mv/km*	*mv/km*	°	°	°	°
Northward	Jan.	0.12	1.13	0.97	0.67	357	216	31	228
	Feb.	0.16	0.90	0.53	0.28	74	230	54	243
	Mar.	0.49	1.15	0.74	0.36	92	255	85	290
	Apr.	0.77	1.56	0.96	0.46	81	270	104	351
	May	0.86	1.65	1.02	0.47	75	274	116	272
	June	0.94	1.57	1.05	0.31	77	275	112	340
	July	1.01	1.67	1.18	0.40	73	270	107	324
	Aug.	1.06	1.76	1.35	0.59	78	274	119	321
	Sep.	0.82	1.50	1.14	0.58	81	278	126	325
	Oct.	0.42	1.22	0.89	0.68	69	260	108	313
	Nov.	0.23	0.84	0.51	0.34	65	242	.72	292
	Dec.	0.13	0.74	0.54	0.36	221	229	32	233
	Mean	0.55	1.23	0.79	0.36	76	261	96	303
Eastward	Jan.	0.33	1.17	0.88	0.46	311	164	338	171
	Feb.	0.15	0.50	0.25	0.13	244	168	321	162
	Mar.	0.22	0.43	0.13	0.07	187	231	356	39
	Apr.	0.14	0.73	0.47	0.20	342	245	76	330
	May	0.08	0.76	0.47	0.30	325	266	97	311
	June	0.10	0.71	0.63	0.23	110	280	104	303
	July	0.12	0.68	0.64	0.29	46	269	100	299
	Aug.	0.20	0.80	0.78	0.44	50	258	111	305
	Sep.	0.32	0.93	0.67	0.42	26	240	106	322
	Oct.	0.30	0.85	0.47	0.39	11	225	88	304
	Nov.	0.15	0.72	0.37	0.11	339	182	12	243
	Dec.	0.13	0.63	0.47	0.23	259	160	321	134
	Mean	0.08	0.55	0.30	0.13	350	231	70	296

tions of earth-current potentials with season it is in order to look into their significance. There is always the possibility that variations observed in this sort of measurement may be due in part to changes in the resistance of the paths of current-flow. Any attempt to explain them on that basis, however, is pretty well ruled out by two facts. First, resistivity-surveys made at Watheroo, at Huancayo, and in other locations show that such changes as occur in the specific resistance of soil and rock with season are confined to a relatively shallow layer at the surface and that the average resistivity to depths such as are involved in the flow of earth-currents is practically constant. Hence effects as great as those observed could not be due to changes in conductivity And second, it would be extremely difficult to conceive changes in resistivity occurring in such a way as to produce many of the variations noted in the graphs, such, for instance, as the change from a rounded figure in winter to the restricted graph in summer, or the practical disappearance of the eastward component in March while the northward component is apparently not affected. Hence it appears that the changes must be due rather to variations in the phenomenon which causes the currents to flow—most probably the circulating currents in the upper atmosphere.

For further evidence that this is the case we return to the usual type of diurnal-variation curve and compare the results with those of *magnetic* measurements at Tucson. Many attempts have been made to demonstrate the exact relationship between earth-currents and variations in the Earth's magnetic field, but without success. Nevertheless certain general correlations have been observed from time to time. It has been found, for instance, that for middle-latitude stations the mean diurnal-variation curve of northward earth-current potential-gradient bears a strong resemblance to the first derivative of the diurnal-variation curve of Y, the eastward component of the horizontal magnetic field. In other words, a comparison of N and Y indicates a relationship much like that which would exist if the earth-currents were the effect of the variations in the magnetic field. If we compare the eastward earth-current component with X, the northward magnetic force, however, the opposite relationship is suggested since for these components the earth-current curve resembles, not the derivative of X, but the X-curve itself inverted. This general qualitative correspondence is found to exist at Tucson as will be seen in Figure 4, not only in the

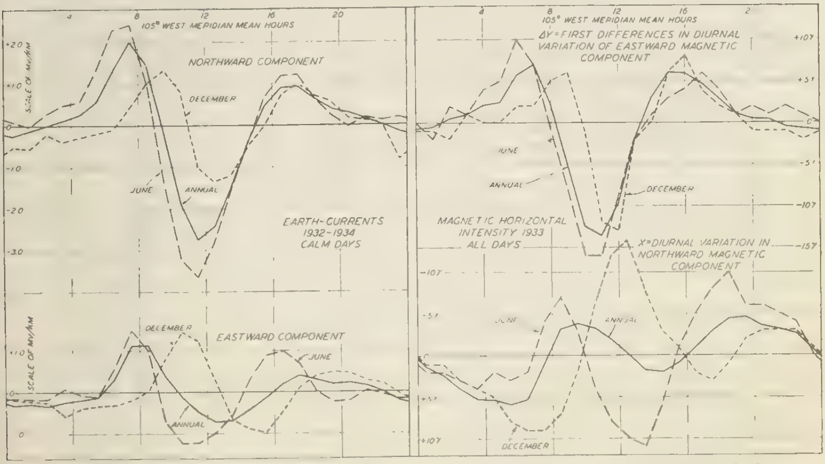


FIG 4—DIURNAL VARIATION IN JUNE AND DECEMBER AT TUCSON, ARIZONA, SHOWING PARALLEL CHANGES IN EARTH CURRENT POTENTIAL-GRADIENT AND MAGNETIC HORIZONTAL INTENSITY

mean annual diurnal-variation curves, but also in the months of greatest contrast in type, namely, June and December. On the left of Figure 4 are the diurnal-variation curves for the two components of earth-currents and on the right the two magnetic curves referred to. It will be noted that the seasonal changes in the curve for the northward earth-current component consisting chiefly of a change in amplitude accompanied by a marked shift in the time of the morning maximum, a smaller shift in the noon minimum, and practically no change in the time of the secondary maximum toward evening are duplicated point by point in the $\triangle Y$-curves to the right. The changes shown by the curves for the eastward earth-current component, on the other hand, are almost

entirely changes in phase, the amplitude in June differing but little from that in December, and the same holds for the X-curves which are here shown inverted to facilitate the comparison.

As stated previously, correspondence of this nature has been observed at other places between the mean diurnal-variation curves for earth-currents and those of the magnetic elements. The Tucson records, however, afford the first opportunity to compare records at a station where the type of diurnal variation changes radically with season and the parallelism of the changes in the two sets of records seems to be an important addition to the evidence linking the two phenomena together.

The magnetic data shown in Figure 4 are not absolutely simultaneous with those of the earth-currents being those for all days of the middle year 1933. Final reduction of the magnetic records for 1932-34 has not been completed. Comparison with calm-day magnetic records for the years 1925-26 has also been made and shows an agreement very like that shown by the records for 1933. The examination of the preliminary magnetic records was also extended to the months of January and March to see if the parallelism noted in Figure 4 extends to the anomalies found in the earth-current records in those months. If it does we should find in January an increase in amplitude over that for December in both the $\triangle Y$- and the X curves and in March an appreciable dimunition of the X-curve. In both cases that is exactly what the magnetic records of 1932-34 show.

Department of Terrestrial Magnetism,
Carnegie Institution of Washington,
Washington, D. C.

THEORY OF THE IONOSPHERE*

E. O. Hulburt

Introduction—During the past ten years the ionization in the reaches of the atmosphere at several hundred kilometers above sea-level has been explored by means of radio waves. Concurrently with the experimental investigations various possible causes of the ionization have been examined from the theoretical standpoint and this in turn has called for theoretical elaboration of the entire physics of the high atmosphere. The recent and detailed ionosphere-measurements of the National Bureau of Standards and of the Department of Terrestrial Magnetism of the Carnegie Institution of Washington permit a closer scrutiny of fact and theory than has been possible heretofore. In the following brief resumé a few of the more important conclusions are outlined. These support the view that ionization is caused mainly by the ultra-violet light of the Sun.

Meteorology of the high atmosphere—The calculations of Maris[1] of the rates of diffusion of the atmospheric gases show that because of winds the atmosphere is uniformly mixed up to levels of 100 km to 150 km. Therefore, except for water-vapor and ozone, the gaseous composition of the atmosphere is the same at 100 km to 150 km as it is at sea-level. The drift and distortion of meteor-trains give direct evidence of winds and air-currents up to 110 km. Above 150 km the distribution of the gases according to equilibrium under the action of gravity may be expected, the proportion of the lighter gases increasing with height.

Observations with sounding balloons to 33 km show that the temperature of the atmosphere increases from 219°K at 20 km to 226°K at 33 km. Above 33 km all estimates of the temperatures are theoretical. The calculations of Maris, Gowan[2], and others as to the possible warming of the high atmosphere by the absorption of solar and Earth-radiation may be summarized thus: The calculations, based on reasonable hypotheses, indicate that the high atmosphere is warm, but are unable to say how warm, or to decide whether it remains warm or grows cool at night.

The skip-distances of sound-waves have been interpreted by Whipple[3] to mean that the temperature increases with the height, being 280°K at 40 km and 350°K at 55 km.

Oxygen is important in the meteorology of the atmosphere above 200 km, for its molecular band absorbs ultra-violet light from 1850$\AA$ to 1250$\AA$. With the solar constant 1.35×10^6 erg cm^{-2} sec^{-1} and extrapolating the solar energy-curve into the ultra-violet, assuming that the Sun radiates as a black body at 6000°K, the energy in the spectral region from 1800$\AA$ to 1250$\AA$ falling vertically into the atmosphere is 10^3 erg cm^{-2} sec^{-1}. The energy is absorbed by atmospheric molecular oxygen with an average absorption-coefficient 10^{-17} per molecule and increases the temperature about 50°K per hour in levels from about 180 km to 300 km. The heating dissociates molecular oxygen to atomic oxygen; probably nitrogen-molecules are reduced to atoms also. The

*Presented April 26, 1935, in symposium on the upper atmosphere, general assembly of the American Geophysical Union, Washington, D. C.

[1] H. B. Maris, Terr. Mag., **34**, 45-53 (1929).

[2] E. H. Gowan, Proc. R. Soc. A., **128**, 531-550 (1930).

[3] F. J. W. Whipple, Q. J. R. Met. Soc., **57**, 331-335 (1931).

following calculation shows that due to the heating and the dissociation there is an upward expansion of the atmosphere during the day such that the pressure at 300 km, or even 350 km, at noon is at 240 km at night. Taking the temperature to be 360°K in levels above 100 km a table of the molecular densities was drawn up. Table 1 is a portion of the complete table and gives the total molecular density, which consists mostly of nitrogen, oxygen, and helium, assuming the oxygen above 200 km to be molecular and atomic in turn. If atomic nitrogen is assumed in addition to atomic oxygen, the total particle-densities in levels above 250 km are an order of magnitude greater than those of the last column of Table 1.

TABLE 1—*Total molecular densities of high atmosphere*

Height	With molecular oxygen	With atomic oxygen
km		
100	1.6×10^{14}	1.6×10^{14}
150	1.9×10^{12}	1.9×10^{12}
200	2.5×10^{10}	2.9×10^{10}
250	2.5×10^{8}	8.8×10^{8}
300	7.7×10^{6}	7.0×10^{7}
350	7.3×10^{5}	7.0×10^{6}
400	4.1×10^{5}	1.0×10^{6}

The evidence thus far presented indicates that the atmospheric levels from 100 km to 200 km are warm during the day, but does not show whether these levels remain warm or become cool at night; in the levels from 200 km to 350 km one would expect a pronounced expansion of the atmosphere by day and contraction at night. The radio ionospheric observations support and supplement these conclusions. Radio indicates that the average temperature below about 200 km is around 350°K and does not fluctuate as much as 30°K, for the heights of the E- and F_1-regions of ionization are observed to remain fairly constant for the day, night, latitude, and season. Above 200 km the observed diurnal and seasonal changes in the F_2-region are in keeping with daylight upward expansion suggested by theory.

Ionosphere data—The National Bureau of Standards[4] at Washington and the Department of Terrestrial Magnetism[5] of the Carnegie Institution of Washington have measured the ionization in the three main regions of the ionosphere over a period of years at Washington, D. C., U. S. A., and at Huancayo, Peru. Observations were also made during a total eclipse of the Sun. The data are the most accurate and detailed which have yet been obtained and may be regarded as the first approach to a world-wide survey of the ionosphere. A glimpse of the data is given in the figures.

The virtual heights Z' of the three regions of ionization at Washington and at Huancayo are plotted against local time in Figures 1 and 2.

[4] T. R. Gilliland, Bur. Stan. J. Res., **14**, 283-303 (1935); S. S. Kirby and E. B. Judson, Bur. Stan. J. Res. **14**, 469-486 (1935).

[5] L. V. Berkner and H. W. Wells, Terr. Mag., **39**, 215-230 (1934); H. W. Wells, Terr. Mag., **39**, 209-214 (1934).

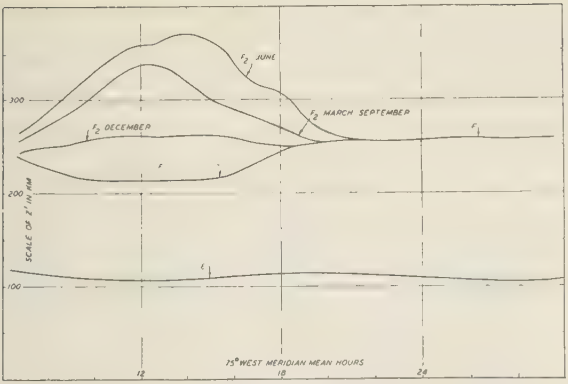


FIG 1—VIRTUAL HEIGHT Z' OF THE IONIZED REGIONS AT NATIONAL BUREAU OF STANDARDS, WASHINGTON, LATITUDE 39° 38' NORTH

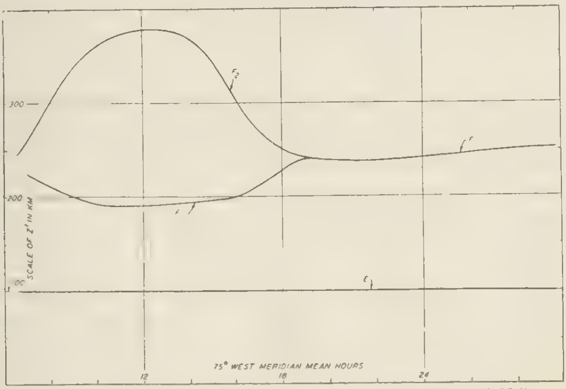


FIG 2—VIRTUAL HEIGHT Z' OF THE IONIZED REGIONS, DEPARTMENT OF TERRESTRIAL MAGNETISM, CARNEGIE INSTITUTION OF WASHINGTON, AT HUANGAYO MAGNETIC OBSERVATORY, PERU, LATITUDE 12° 03' SOUTH

Z' for E, F_1, and F at Washington and for E, F_1, and F at Huancayo has little or no seasonal variation. Z' for F_2 at Washington undergoes marked changes with the season, the maximum height, which occurs near noon, being about 350 km in midsummer and 250 km in midwinter. The true heights Z are less than Z', but probably not more than 20 km less for E, F_1, and F, and not more than 50 km less for F_2.

The electron- or ion-density of a region is denoted by y. In Figures 3 and 4 are given the values at noon of y for E and F_1, respectively, during a year at Washington averaged over the days of each month. The points for May, June, and July, 1933, were based on fewer data than the points for the other months and are to be given less weight.

Figure 5 shows the diurnal and seasonal values of y for F_2 and F at Washington in 1933. The maximum of y for F_2 is somewhere near noon in winter and shifts to sunset in summer. At Huancayo y for F_2 has an erratic maximum around 10 a. m., a minimum at noon and a broader, greater maximum at sunset. At Washington, y for F decreases during

the night, except that during winter nights y exhibits a recrudescence at about 4 a. m. and then decreases until its rise at dawn.

Radio-wave propagation —From the values of y and Z' at Washington the skip-distances of radio waves were calculated and are given in columns 2 and 3 of Table 2, for midday conditions for June 1933 and January 1934, an epoch of minimum solar activity. The average-day

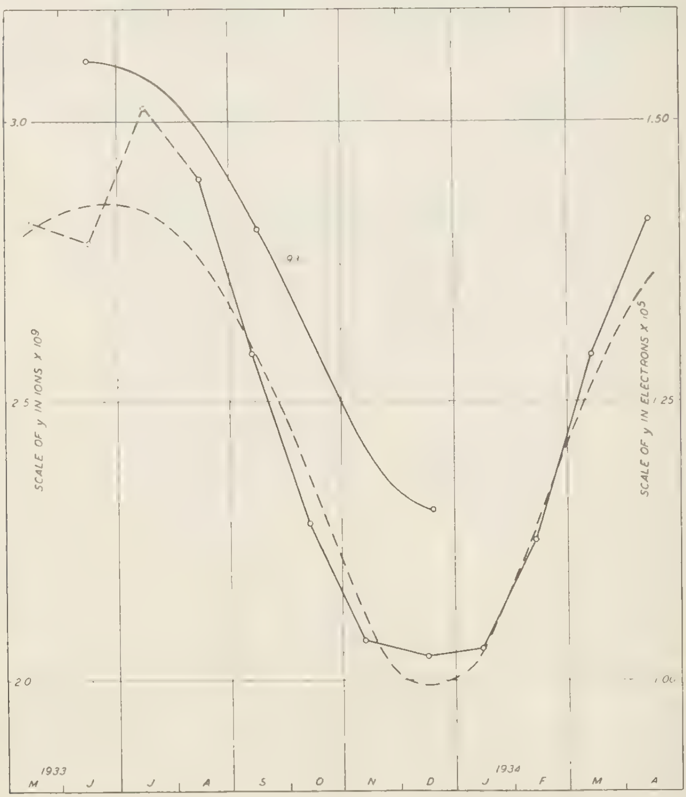


FIG. 3—MONTHLY AVERAGE NOON E-IONIZATION AT NATIONAL BUREAU OF STANDARDS, WASHINGTON; DOTTED CURVE, Y PROPORTIONAL TO $\sqrt{\cos \zeta}$

TABLE 2—*Skip-distances of radio waves*

Wave-length	June 1933	January 1934	Observed, 1923-24
meters	*km*	*km*	*km*
30	600	700	700
25	800	930	900
20	1100	1300	1200
16	2000	2000	2200

temperate-zone values[6] of 1923 and 1924, also a period of minimum solar activity, are in the last column of Table 2, and agree with the average of columns 2 and 3. From columns 2 and 3 the ratio of the summer to winter day skip-distances is about 1.2, which is the value observed.[7]

The skip-distance calculation showed that the skip-distance, and hence radio-wave propagation to long distances, was controlled by any one of the regions of ionization depending on the season and time of day. For example, during a summer day in temperate latitudes E controls long-distance short-wave propagation, the radio waves being refracted down by E without passing through to F_1 or F_2. Whereas, during a winter day F_2 controls the propagation. The molecular density in E is greater than in F_2 and therefore the absorption of energy from the radio wave is greater in E than it is in F_2. Thus we arrive at an explanation of the well-known fact that daylight long-distance short-wave radio communication is better in winter than in summer.

Theory of the ionosphere—If the ionization of a region is caused by the ultra-violet light of the Sun, y during the day is a function of the zenith-angle ζ of the Sun. In the case that the recombination-coefficient α of ions or electrons is of the form given by the three-body collision theory of Sir J. J. Thomson, it may be shown[7] that approximately

$$y = y_0\sqrt{\cos\ \zeta} \qquad (1)$$

where y_0 is the value of y for the Sun directly overhead. Equation (1) also results from the theory of recombination by collisions between electrons and positive ions given by Pannekoek[8] in his modification of the Saha theory of photoelectric equilibrium.

If the electrons disappear by attachment to oxygen-molecules we have

$$y = y_0 \cos\ \zeta \qquad (2)$$

(1) and (2) are true for the ideally simple case of photoelectric radiative equilibrium, no diffusion of the ionization, and no motion or changes in the upper atmosphere.

The E-region—Experiment has not determined definitely whether the E-ionization is mainly ionic or electronic; at present a majority of experiments indicates ions and a few experiments, electrons. In Figure 3 the dotted curve is plotted from (1) with $y_0 = 1.45\times10^5$ electrons or 2.9×10^9 ion-pairs and is seen to follow the observed points very closely. The agreement is favorable to the hypothesis that the ionization is caused by the ultra-violet light of the Sun and that the recombination-coefficient is of the form given by the three-body collision theory. On the basis of ions the observed values of y yield a recombination-loss $\alpha n = 10^{-12}$, where n is the molecular density at the E-level. The same value is obtained from theory. For electrons αn is of order 10^{-9} both from observation and theory. Thus, theory and facts are in agreement for either ions or electrons.

Since y of E is given by (1) it follows that the intensity i of the ionizing wave-lengths of the solar ultra-violet light is proportional to y^2. y of E for 1931 is plotted in figure 3; the values are about 15 per cent greater than y for 1934, a year of sunspot-minimum. Hence i decreased

[6] A. H. Taylor, Proc. Inst. Radio Eng., **13**, 677-683 (1925).
[7] E. O. Hulburt, Phys. Rev., **31**, 1018-1037 (1928); Physics, **4**, 196-201 (1933).
[8] A. Pannekoek, Amsterdam, Proc. Sci. K. Akad. Wet., **29**, 1165-1171 (1926).

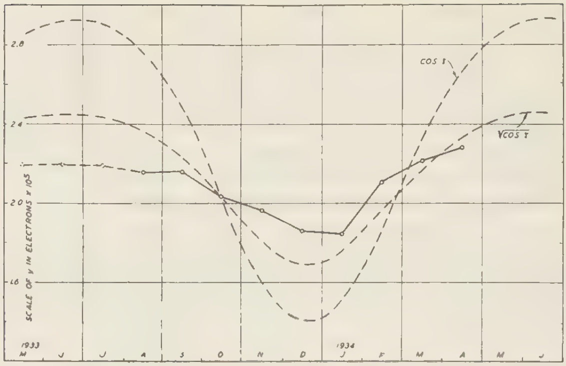


FIG 4—MONTHLY AVERAGE NOON F_1-IONIZATION AT NATIONAL BUREAU OF STANDARDS, WASHINGTON; DOTTED CURVES, y PROPORTIONAL TO COS χ AND $\sqrt{\cos \chi}$

about 30 per cent from 1931 to 1934. This is in keeping with the 50 per cent decrease in i from 1924, sunspot-minimum, to 1928, sunspot-maximum, derived from the skip-distances of short radio waves.

The F_1-region—Experiment shows that the ionization of the F_1-region is predominately electronic. Electrons may disappear either by recombination with positive ions or by attachment to neutral oxygen-molecules, and in Figure 4 the theoretical curves from both (1) and (2) are given. It is seen that neither of these curves agrees very well with the observed values. Therefore a simple calculation of ionization by ultra-violet sunlight in a static atmosphere does not give exact agreement with the facts. This is as it should be for, on theoretical grounds, the atmosphere at 200 km would not be expected to remain entirely unmoved during the day. There are two possible effects which are in the correct direction to bring theory into accord with observation, diffusion of the ionization and a relatively small vertical expansion of the F_2-region for small solar zenith-angles, the expansion being due to heating or the dissociation of oxygen- and nitrogen-molecules into atoms. These effects flatten out the theoretical curves of Figure 4 and bring them nearer to the observed curves. Although diffusion may be calculated, the expansion can not be estimated quantitatively so that one cannot determine the relative importance of diffusion and expansion nor whether (1) or (2) is to be preferred.

The F_2-region—For the F_2-region the hypothesis is made that the ionization is caused by the ultra-violet light of the Sun. The diurnal variation in y calculated from the hypothesis of a static atmosphere has a single maximum in the early afternoon. The maximum is greater for smaller solar zenith-angles; this does not agree with the curves of Figure 5 and the double maximum in y of F_2 at Huancayo. However, a daytime vertical expansion of the atmosphere above 250 km is assumed due to heating and dissociation of oxygen- or nitrogen-molecules by sunlight, just as in the case of F_1, except more pronounced. In fact F_1 may be regarded as being at the lower boundary of the expanding region.

In the absence of the vertical expansion the F_2-ionization would center about a 250-km level and would increase to a maximum in the

early afternoon. The expansion spreads the ionization through the levels from 250 km to 350 km and hence reduces the ionization density y. During a day in which ζ does not become less than 40° even at noon, the vertical expansion is slight and y reaches a maximum shortly after noon. This is in accord with the observed values of Z' and y for F_2 during a winter day at Washington, as shown in Figures 2 and 5. During a day in which ζ approaches 0° at noon, y begins to increase in the morning due to the increasing light-intensity. Soon, in spite of the fact that the total number of electrons in a vertical column of the atmosphere through F_2 continues to increase, the expansion is rapid enough to make y decrease. Hence y passes through a maximum during the morning. In the afternoon the march of events is reversed; the total ionization decreases with the lowering Sun, but the contraction of the atmosphere is sufficiently rapid to effect an increase in y for a time. Therefore y passes through another maximum in the afternoon. These conclusions are in accord with the observed variations in y during the day at Washington in summer and at Huancayo the year round. The morning maximum in summer is usually less pronounced at Washington than at Huancayo.

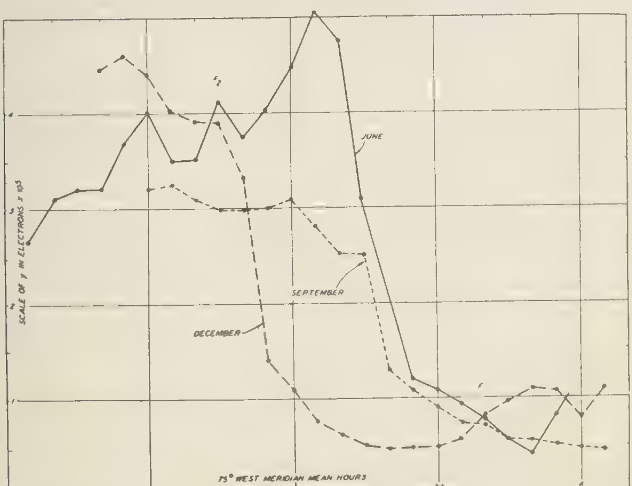


FIG. 5—DIURNAL AND SEASONAL VARIATIONS, F_2- AND F-IONIZATION AT NATIONAL BUREAU OF STANDARDS, WASHINGTON, 1933

There is another important effect. Due to the expansion of the F_2-atmosphere, winds in levels above 250 km blow away in all directions from the region directly beneath the Sun. Westward from the sub-solar point, and hence in the morning hemisphere, the stream of F_2-air moves against the rotation of the Earth, is checked and becomes turbulent as the whitecapping waves in a tide-rip, whereas eastward, and hence in the afternoon hemisphere, the stream moves with the rotation of the Earth and remains smooth and undisturbed. The eastward breeze may even displace the afternoon maximum of y an hour or so toward evening.

The effects are in accord with the observations of F_2 which at the equator record a greatly disturbed and erratic layer in the morning and a less disturbed ionization in the afternoon with a broad maximum at 6 or 8 p. m.

All in all, the two hypotheses, ionization by solar ultra-violet light and expansion of the F_2-atmosphere due to heating and dissociation by sunlight, are able to account for the main features in the rather complex variations in F_2-ionization. They do not, except by what seems at present considerable forcing, yield an explanation of the curious recrudescence of F-ionization observed in the small hours of a winter morning at Washington. There is, further, an uncertainty in the case of F_2 and F which may, or may not, turn out to be important. Because of diffusion it is difficult to see how a stable bank of ionization can exist much above 200 km. For example, if the ionization is produced in levels above 200 km, it has been shown[7] that the downward diffusion of the ionization is so rapid that the equilibrium position of the maximum of the bank is at 190 km; and yet the F-region appears to be considerably above 200 km. This may mean, among other possibilities, that the molecular densities above 200 km are greater than has been supposed or that the recombination-formulae are not applicable in the high levels.

NAVAL RESEARCH LABORATORY,
Washington, D. C.

COSMIC RAYS FROM NOVA HERCULIS?

By Victor F. Hess and Rudolf Steinmaurer

The appearance of a new star in the constellation of Hercules is of considerable interest not only to the astronomer but also to the physicist. W. Baade and F. Zwicky[1] recently estimated the energy of the so-called super-novae and advanced tentatively the hypothesis that cosmic rays are produced in the super-nova process. The few communications[2] on the Nova Herculis do not show—as far as they are accessible to us—whether this new star can be classed as of the super-nova type. At any rate its behavior seems to differ from that of a common nova. The light-emission shows an increase from the date of discovery, December 13, 1934, until December 25, after which period it decreases and seems to fluctuate rather irregularly.

W. Kolhörster kindly communicated to us his observations of a slight increase in the intensity of cosmic radiation on December 20 and asked whether our instruments on the Hafelekar and in Innsbruck showed a similar effect. He[3] later published a short note on his observations and reported that his measurements between December 22 and 31, 1934, with a double-coincidence Geiger-Mueller counter, indicated an increase of 1.7 per cent during the time when the Nova was high above the horizon (9^h to 16^h middle European time). He said that the detection of this effect by the ionization-method would be more difficult on account of the variations of the barometer-effect and the directional uncertainties of ionization-methods.

Nevertheless it seemed promising to determine whether accurate instruments such as our Steinke standard apparatus showed any effect ascribable to the appearance of the Nova. Two of our Steinke apparatus were located 2300 meters above sea-level (Hafelekar Observatory), and a third apparatus of the same type was in Innsbruck, under the roof of the University Building (600 meters above sea-level). They were kept at constant temperature ($15° \pm 0.2°$) and were permanently screened from local radiations by lead-shields of 10-cm thickness beneath and on the sides. Two of them (Nos. 3 and 8) were also shielded by 10 cm of lead from above while the third was not so shielded, so that softer components of the cosmic radiation might be admitted. These Steinke instruments automatically record the intensity of the cosmic radiation for every hour. The mean hourly values were reduced to the same barometric pressure (711 mm for Innsbruck, 580 mm for Hafelekar).

There are different methods of examining our observations for the effect in question. First, we compared the average intensities of a series of days before and after the appearance of the Nova. This is necessary on account of the irregular variations of the cosmic radiation ("variations of the second kind"). Thus we took the average of every day from December 1 to December 20, 1934, and calculated the mean values of these daily means for a group of days before and after the appearance of the Nova. These values are given in Table 1 together with the mean errors.

It is noted that a very small increase of ionization occurs in each apparatus. This effect, however, is well within the limits of the mean errors with exception of No. 8 where the increase ($0.013\ I = 13\ mI$) is

[1]Proc. Nat. Acad. Sci., **20**, 254-263 (1934).

[2]See Nature, **135**, 192 (1935), and K. Graff, Zirkular der Mittelmeerstation der Wiener Universitäts-sternwarte, Nr. 1 (1935).

[3]Zs. Physik, **93**, 429-431 (1935).

TABLE 1—*Average cosmic-ray intensities*

Period	Hafelekar (2300 meters)		Innsbruck (600 meters)
	Apparatus No. 9 (No lead on top)	Apparatus No. 3 (10 cm lead on all sides)	Apparatus No. 8 (10 cm lead on all sides)
1934	I	I	I
Dec. 1-12	4.464 ± 0.006	2.804 ± 0.004	1.770 ± 0.002
Dec. 13-20 (or 22)	4.465 ± 0.008	2.807 ± 0.006	1.783 ± 0.003
Effect of Nova	$+0.001 \pm 0.010$	$+0.003 \pm 0.007$	$+0.013 \pm 0.004$

three times as great as the mean error. If we take the average of the values from December 17 to December 20 (or 22) when the new star approaches its maximum brightness we obtain larger effects. These values are 4.477, 2.814, and 1.786 I for apparatus Nos. 9, 3, and 8, respectively. The increase (compared with the period December 1 to 12), therefore, would amount to 13, 10, and 16 mI for Nos. 9, 3, and 8. If we take the average of the daylight hours alone (December 17 to 20 and 22, respectively), we again obtain an increase for all three apparatus (9, 6, and 14 mI), while for the daylight hour means for December 13 to 20, the effect is positive only for apparatus No. 8. At any rate it must be kept in mind that all these effects are within the limits of the mean errors. If the increase mentioned is really due to the new star, we should expect to get a relatively larger increase by taking average values not for whole days but for hours when the Nova was high above the horizon.

In the week December 13 to 20, the Nova's culmination was practically at noon ($12^h\ 40^m$ M. E. T.). Now we must bear in mind that at this time of the day a maximum of the ionization was found to occur[4]. A possible effect of the Nova would naturally be superposed on this solar-noon maximum which, on the Hafelekar, exceeds the night minimum by about 10 mI; therefore in order to separate these effects we must compare the difference of ionization at noon and at night both before and after the appearance of the new star.

This was done by grouping our observations in three different ways. (*a*) We took the average values for the six-hour intervals 9^h to 15^h and 21^h to 3^h for each day from November 22 to December 22. These six-hour means were grouped separately for the time before and after the appearance of Nova Herculis. The mean errors of these groups were calculated. On the average they amounted to $\pm 6\ mI$ for apparatus No. 9, $\pm 4\ mI$ for No. 3, and $\pm 3\ mI$ for No. 8. (*b*) We next took the averages for 11^h to 13^h and compared these with the six-hour night-means (21^h to 3^h). Here the errors naturally are larger for the values at noon. (*c*) Finally we calculated the mean values of the four-hour intervals 10^h to 14^h and 22^h to 2^h. Here the calculated errors were about $\pm 10\ mI$ (No. 9), $\pm 5\ mI$ (No. 3), and $\pm 4\ mI$ (No. 8). The differences of average values before and after the appearance of the Nova, of course, can be in error by amounts of nearly twice those mentioned. The results of these three methods of comparison are given in Table 2.

[4] V. F. Hess, R. Steinmaurer, und H. Graziadei, Wien, SitzBer. Ak. Wiss., IIa, **143**, 313-338 (1934).

TABLE 2—*Difference of day-to-night ionziation before and after appearance of Nova Herculis*

Group	Apparatus No. 9		Apparatus No. 3		Apparatus No. 8	
	Nov. 22-Dec. 12	Dec. 13-20	Nov. 22-Dec. 12	Dec. 13-20	Nov. 22-Dec. 12	Dec. 13-22
	mI	*mI*	*mI*	*mI*	*mI*	*mI*
(*a*) Comparison mean values, 9^h-15^h and 21^h-3^h	+18	+18	+4	+11	+1	+2
(*b*) Comparison mean values, 11^h-13^h and 21^h-3^h	+19	+13	+6	+9	0	0
(*c*) Comparison mean values, 10^h-14^h and 22^h-2^h	+17	+13	+6	+6	+4	+2

The maximum at noon is well pronounced in nearly all cases. An increase of the day-to-night difference after the appearance of the Nova is clearly discernible in three cases only [No. 3 (*a*) and (*b*), No. 8 (*a*)]. Apparatus No. 9 on the Hafelekar which would also show effects of softer radiations does not indicate any positive effect of the Nova. Apparatus No. 3 screened on all sides by lead (10 cm) shows a slight positive effect for (*a*) and (*b*). Apparatus No. 8 (in Innsbruck, only 600 meters above sea-level) does not show any decisive effect. It must be remembered, however, that all these difference-effects (whether positive or negative) are again within the limits of the possible mean errors and that these are larger here where whole-day mean values were not taken.

It is very unfortunate that our measurements had to be discontinued on December 22 and, accordingly, no measurements at the time of the maximum brightness of the Nova were available. Therefore our results are not fully comparable with those of Kolhörster.

From the foregoing results we can only conclude that an effect of the appearance of Nova Herculis could not be definitely proved by the ionization-method using three Steinke apparatus placed at 2300 meters and 600 meters above sea-level during the period December 13-20. There are indications of a small increase of ionization after the appearance of the Nova when mean values of whole days before and after December 13 are taken. The effect, if it is real, certainly does not exceed 2 per thousand of the total radiation nor the limits of the mean errors.

INSTITUT FÜR STRAHLENFORSCHUNG DER UNIVERSITAET INNSBRUCK,
Innsbruck, Austria
February 22, 1935

NOTES

(See also page 208)

13. *A British magnetic-survey vessel*—Rear-Admiral I. A. Edgell, Hydrographer of the British Admiralty, has advised us of the decision to construct a magnetic-survey vessel as announced in the explanatory statement by the First Lord of the Admiralty presented with the navy estimates for 1935 to the House of Commons, March 6, 1935. The decision to build is consequent on representations made by the Astronomer Royal and the Hydrographer regarding the ignorance of the changes which are taking place in the Earth's magnetism at sea, particularly with regard to the South Indian Ocean and the South Atlantic. The need for early action was recognized by the Lords Commissioners of the Admiralty, and it is hoped that the new ship will be ready for commissioning sometime in 1936. It is not known whether it will be possible to undertake any other scientific work, but it is hoped that oceanographical meteorology, water-samples, sea-temperatures, and possibly some deep-sea soundings will be included in the programme.

In commenting on this matter NATURE of March 16, 1935, says: "The magnetic charts published by the British and other governments for use at sea have been based in recent years to an increasingly large extent upon the data provided by the *Carnegie*. There are some serious gaps in the present data, which would have been filled if the *Carnegie* had completed her last cruise. Due partly to these gaps and to a recent rapid change in the secular variation in the Indian Ocean, the extrapolated values of the magnetic elements in the southern Indian Ocean are now unreliable, and the possibility of serious errors in this and other areas in future charts has given rise to some concern. The Carnegie Institution, having definitely decided not to replace the *Carnegie*, and in view of the special interest of Great Britain, as the principal maritime nation, in the accuracy of the magnetic charts, the British Government has assumed the responsibility. A non-magnetic ship is to be constructed, primarily for the purpose of determining magnetic data at sea. Details of the design have not yet been decided upon, though it is probable that the new vessel will be larger than the *Carnegie*."

14. *Magnetic-survey work of Adelaide Observatory*—From the report of G. F. Dodwell, director of the Adelaide Observatory, to the Council of the Royal Astronomical Society, we note that during the year 1934 values of declination, inclination, and horizontal intensity of terrestrial magnetism were obtained at ten stations, namely, Victor Harbour, Goolwa, Strathalbyn, Kuitpo, Nairne, Ceduna, Yalata, Flinders, Adelaide (West Park), and Belair. It was found that the declinations at these stations, after being practically stationary for several years, began to increase easterly in 1923, and the values are now nearly half a degree higher than in that year. Corresponding changes were found in the values of inclination and horizontal intensity.

15. *Magnetic station on Cape Chelyuskin*—In addition to a considerable increase in meteorological and aerological investigations at a large number of polar stations in the U. S. S. R. during 1934, both by pilot-balloons and the Molchanov meteorograph, the Arctic Institute has reorganized the polar station at Cape Chelyuskin into a magnetic observatory. The program of geophysical work at this station embraces terrestrial magnetism, aurora, atmospheric electricity, and radioactivity of the air and rocks.

16. *Magnetic-survey work in China*—The Magnetic Section of the Observatory of Zi-ka-wei, in cooperation with the National Academy at Peiping, has begun work looking toward a magnetic survey of China. Despite unfavorable weather and instrumental difficulties, eight stations have been occupied in southern and southwestern China during the past winter. The values of declination obtained indicate a considerable decrease in the annual change which is approaching zero in these regions while those of horizontal intensity show an annual increase of 16 to 18 gammas. These rates of change agree well with those derived from recent observations made by F. C. Brown for the Department of Terrestrial Magnetism of the Carnegie Institution of Washington, in western China. The Observatory of Zi-ka-wei plans, during the present year, to secure a considerable number of additional stations in cooperation with the National Academy or the Academia Sinica.

CONCERNING THE MAGNETIC NUMERICAL CHARACTER-NUMBERS

By J. Egedal

In a communication presented to the Assembly of the Association of Terrestrial Magnetism and Electricity held at Lisbon September 1933[1] the author gave the results of a comparison of numerical character-numbers with international character-numbers. It was shown that the numerical character-number corresponding to the international character-number 0.2 for the magnetic observatory at Rude Skov was about 120 in the winter (December) and about 240 in the summer (June).

In the present note the magnitude of the numerical character-numbers for magnetically quiet days for observatories at different latitudes is examined. The numerical character-numbers for the five international quiet days of June and of December for the years 1932 and 1933 are considered.

In Table 1 the ratios between the mean of the numerical character-numbers for the quiet days of June and that of the numerical character-numbers for the quiet days of December are given for the two years considered. For the better understanding of the data considered it should be stated that the mean international character-numbers for the five quiet days of June and December 1932 and June and December 1933 are all the same, namely, 0.05.

Table 1—*Ratios between means of numerical character-numbers of quiet days of June and of December for the years 1932 and 1933*

Station	Latitude	1932	1933
	° ′		
Rude Skov	55 51 N	2.3	2.2
Ebro	40 49 N	2.0	1.9
Cheltenham	38 44 N	1.6	1.7
Kakioka	36 14 N	1.6	1.0
Kuyper	6 02 S	1.0	1.4
Huancayo	12 03 S	1.0	1.0
Watheroo	30 19 S	0.4	0.5[2]

From Table 1 it will be seen that in the main the ratios steadily fall from 2.3 for Rude Skov to 0.4 for Watheroo. This variation must be due to the diurnal variation of the horizontal- and vertical-intensity components because the disturbances for the quiet days considered have little influence on the magnitude of the numerical character-numbers. In June the diurnal variation of the horizontal- and vertical-intensity components is large in the Northern Hemisphere and in December it is large in the Southern Hemisphere, and according to this the ratios for the observatories lying in the Northern Hemisphere and at more than 25° from the equator are greater than 1.0 and the ratio for the Watheroo Observatory lying in the Southern Hemisphere at latitude 30° is smaller than 1.0.

[1]J. Egedal, C.-R., Assemblée Lisbonne 1933, Union Géod. Géophys. Internat., Ass. Mag. Electr. Terr. Bull. No. 9, 286-288 (1934).

[2]For Watheroo, values of January 1934 are used instead of those for December 1933, the latter not being published.

In the above-mentioned article a comparison of numerical and international character-numbers for the Rude Skov Observatory showed that the numerical character-number corresponding to the international character-number 0.2 had a value which for June was twice that of December (compare the ratios of Table 1). In the present note it has been shown (Table 1) that the disagreement between the numerical and international character-numbers also exists for other observatories lying at some distance from the equator and, further, that the relation between the numerical and international character-numbers in June and December for an observatory lying at a certain latitude in the Northern Hemisphere is equal to the relation respectively in December and June for an observatory lying at the same southern latitude.

In order to bring out still more the dependence on latitude of the numerical character-numbers some numerical character-numbers for quiet days are given in Table 2 where the values for Rude Skov have been put equal to 1.0.

TABLE 2—*Relative numerical character-numbers*

Station	$HR_H/10000$				$(HR_H+ZR_Z)/10000$			
	June		December		June		December	
	1932	*1933*	*1932*	*1933*	*1932*	*1933*	*1932*	*1933*
Rude Skov*	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Ebro	1.0	1.0	1.5	1.5	1.1	0.9	1.2	1.1
Cheltenham	1.1	1.0	1.3	2.0	0.9	1.0	1.2	1.3
Kakioka	...	0.7	...	2.4	1.0	0.7	1.3	1.6
Kuyper	3.1	3.4	6.5	5.2	1.4	1.6	3.0	2.6
Huancayo	3.2	3.5	6.8	8.8	1.2	1.4	2.5	3.0
Watheroo	0.7	0.9	2.0	2.1[2]	0.7	0.8	3.5	3.2[2]

* Values for Rude Skov have been put equal to 1.0.

Considering the relative values of HR_H 10000 given in Table 2, the influence of the diurnal variation of the horizontal forces will be noticed. Thus the small ratio 0.7 for Kakioka June 1933 may be due to the rather small diurnal variation of the horizontal force at that Observatory. For Huancayo the values are greater than those for Kuyper although the latter observatory lies nearer to the equator. Referring to Chapman's analysis of diurnal variations it has been considered, in another connection[3], as doubtful whether the diurnal variation of the horizontal force should be as great as observed. Possibly the addition of small terms, depending on magnetic latitude, to Chapman's potential function might be sufficient for the removal of the discrepancy. The values for Watheroo of June (southern winter) are below 1.0 but the values of December (southern summer) are more than 2.

The relative values of the numerical character-number, (HR_H+ZR_Z) 10000, influenced by diurnal variations both of the horizontal and the vertical components, show a rather even variation, but it is a characteristic feature that the values for the observatories near the equator are greatest. This fact is due as well to the magnitude of the

[3] *l. c.*, p. 197.

diurnal variation of the horizontal force as to the magnitude of the horizontal force itself, the vertical force being small.

It is clear that the numerical character-numbers will be influenced by the diurnal variation, this not being eliminated before the determination of the character-number, but from the above it furthermore can be concluded that in certain cases a greater part of the magnitude of the numerical character-numbers of lower value in the main depends on the diurnal variation, while the character-number of higher value in the main depends on disturbances. Therefore the separation proposed by J. Bartels[4] of the two parts of magnetic activity, due to diurnal variation and to disturbances respectively, should be of great importance.

[4] *l. c.*, p. 286.

Copenhagen, Denmark.

NOTES

(See also page 204)

17. *International Meteorological Organization*—The Conference of Directors will take place at Warsaw, Poland, September 6 to 13 or 14, 1935. The opening date has been set with the understanding that the presidents of the commissions who desire may assemble their commissions during the days preceding the Conference of Directors. These meetings, with the exception of that of the Commission of Maritime Meteorology which will take place at De Bilt in mid-July, will be held partly at Danzig following the meeting of the German Meteorological Society, August 26 to 29, and partly at Warsaw. The date and place of the meeting of the Commission of Terrestrial Magnetism and Atmospheric Electricty are tentatively set for September 2 to 5, at Warsaw.

18. *Errata*—In the March number of the JOURNAL the following corrections are to be made in the article by P. W. Glover: Page 61, delete last sentence of first paragraph of abstract; page 64, in Figure 3 reduce by one the numbers of equations indicated against the various graphs except to let the graph marked equation (1) stand and to change graph marked equation (3) to equation (2*a*); page 66, add "in collected form" at the end of the first paragraph; page 67, in Table 3, for the entry in column ΔR against year 1909 read "47" instead of "48."

19. *Personalia*—We have just learned of the death, on August 19, 1933, of the Reverend Father *Edmund Goetz*, S.J., aged 67 years. Father Goetz was for many years in charge of the observatory at Bulawayo, Rhodesia. In May 1914, he was enengaged, in participation with Mr. Wood of the Johannesburg Observatory, in a magnetic and astronomical survey of Barotseland, Goetz being responsible for the magnetic part of the work. The last years of his life were spent in teaching mathematics and French at St. George's College in Salisbury, Rhodesia.

Dr. *Charles E. St. John*, research associate at the Mount Wilson Observatory of the Carnegie Institution of Washington, and well known for his contribution to solar physics, died April 26, 1935, aged seventy-eight years.

Veryl R. Fuller, professor of physics, Alaska Agricultural College and School of Mines, known to readers of this JOURNAL for his work on the aurora, died suddenly May 30, 1935, aged 39 years.

Professor *Jules Schokalsky*, honorary president of the Russian Geographical Society, was elected an honorary member of the Washington Academy of Science May 14, 1935. This distinction was conferred upon him not only because of his outstanding contributions to oceanography in general but particularly because of the valuable assistance rendered the Department of Terrestrial Magnetism of the Carnegie Institution of Washington in connection with the oceanographical work of the *Carnegie*.

COMPUTED AND OBSERVED RATES OF SMALL-ION PRODUCTION IN THE ATMOSPHERE

By G. R. Wait

Abstract—By the use of equations derived by Whipple from a theoretical basis Scrase has computed values of the coefficients of combinations between small ions and condensation-nuclei, charged and uncharged, using data from Kew. The rate of small-ion production also has been computed from the same data, also on the basis of Whipple's equations. In a similar manner, the combination-coefficients and rate of small-ion production have been computed by the author, using data taken at Washington, D. C. The results, both for combination-coefficients and rate of small-ion production, are in good agreement with those from Kew, but the production-rate is not in good agreement with that determined by means of a thin-wall ionization-chamber. Computation of combination-coefficient between small ions and large ions on the basis of simultaneous data on small ions, large ions, and rate of small-ion production by means of a thin-wall chamber, yield a value many times larger than that derived on the basis of Whipple's formula, and somewhat higher than that deduced by Harper from theory. The coefficient so obtained shows a definite diurnal variation somewhat similar in character to that for the rate of small-ion production, or small-ion content. The similarity to the diurnal variation in the reciprocal of large-ion numbers is even more pronounced. Nolan's square-root law, connecting rate of ionization, small-ion numbers, and large-ion numbers is not consistent, however, with the results here obtained. A new relationship between formation and destruction of small ions is found from the present results. Additional experiments are needed to test whether or not such a relationship has universal application.

In a recent paper on the charged and uncharged nuclei in the atmosphere and their part in atmospheric ionization, Scrase[1] applied formulas developed by Whipple[2] for computing values of the combination-coefficients between small ions and charged and uncharged condensation-nuclei and for computing the rate of small-ion production in the atmosphere. The equilibrium-conditions for small ions is customarily expressed by the equations

$$q = an_1n_2 + \eta_{12}n_1N_2 + \eta_{10}n_1N_0 \qquad (1)$$
$$q = an_1n_2 + \eta_{21}n_2N_1 + \eta_{20}n_2N_0 \qquad (2)$$

where q = rate of small-ion production (pair per cc per sec), n_1 = number of small positive ions per cc, n_2 = number of small negative ions per cc, N_1 = number of large positive ions per cc, N_2 = number of large negative ions per cc, N_0 = number of uncharged nuclei per cc, η_{12} = combination-coefficient between positive small ions and negative large ions, η_{21} = combination-coefficient between negative small ions and positive large ions, η_{10} = combination-coefficient between positive small ions and uncharged nuclei, η_{20} = combination-coefficient between negative small ions and uncharged nuclei.

The relationship between combination-coefficients, as derived by Whipple is

$$\eta_{12} = \eta_{10} + 4\pi ew_2 \qquad (3)$$
$$\eta_{21} = \eta_{20} + 4\pi ew_1 \qquad (4)$$

where w_1 and w_2 refer to the mobilities of the positive and negative small ions, respectively. The values of η_{10} and η_{20} were computed by Scrase on the basis of the formulas

$$\eta_{10} = [4\pi ew_1(1 + w_2n_2N_0/w_1n_1N)]/[(N_0/N)^2 - 1] \qquad (5)$$
$$\eta_{20} = [4\pi ew_2(1 + w_1n_1N_0/w_2n_2N)]/[(N_0/N)^2 - 1] \qquad (6)$$

[1]London, Met. Office, Geophys. Mem., No. 64 (1935).
[2]Proc. Phys. Soc., 45, 367-380 (1933).

where $N=(N_1+N_2)/2$. The rate of small-ion production, q, is usually assumed to be given approximately by

$$q_B=2\eta_{12}Nn_{12} \tag{7}$$

whereas that derived by Whipple, is given by

$$q_A=4\pi(\lambda_1+\lambda_2)N_0/[(N_0/N)-1] \tag{8}$$

where $n_{12}=(n_1+n_2)/n$ and λ_1 and λ_2 refer to the positive and negative conductivity of the air, respectively. The average values for the combination-coefficients computed by Scrase from data taken at Kew were: $\eta_{10}=0.58\times10^{-6}$; $\eta_{20}=0.76\times10^{-6}$; $\eta_{12}=2.35\times10^{-6}$; and $\eta_{21}=2.96\times10^{-6}$. These values are considerably lower than the corresponding values derived from experiments. J. J. Nolan and de Sachy[3] obtained values based on experiment as follows: $\eta_{12}=8.7\times10^{-6}$; $\eta_{21}=9.7\times10^{-6}$, $\eta_{10}=6.8\times10^{-6}$; $\eta_{20}=8.7\times10^{-6}$. P. J. Nolan and C. O'Brolchan[4] determined the value of η_{12} as 5.7×10^{-6}, but this was reduced to 1.9×10^{-6} by P. J. Nolan.[5] Harper[6] maintains that Whipple's treatment leads to incorrect results, since he fails to make due allowance for Brownian movements of the ions. Harper proceeds to derive a theoretical value for the coefficient η_{12}, arriving at the value 7.2×10^{-6} for this quantity.

Notwithstanding the objection raised by Harper to the derivation by Whipple of a theoretical value for the combination-coefficient, it was considered of value to make further tests similar to that applied by Scrase to the Kew data but using data at Washington, D. C. From simultaneously observed values of uncharged condensation-nuclei, large ions, and small-ion conductivity, values of the combination-coefficients and of the rate of small-ion production have been computed, using equations derived by Whipple. The rate of small-ion production also has been computed, using equation (7), from the same data. Both q_A and q_B are then compared with results obtained from measurements, thus providing a rough test for the equations. Equation (7) also has been applied to simultaneous data on small ions, large ions, and rate of small-ion production, for a determination of the combination-coefficient η_{12}.

Previously published data[7] on number of total condensation-nuclei, uncharged nuclei, large-ion numbers as measured by means of an electrometer, ratio of uncharged nuclei-number to large-ion number are given in Table 1. In addition to the list as previously published there has been added a column giving the ratio of positive to negative conductivity of the air for the indicated hour of day, the ratio being the mean of two months' daily records with one apparatus recording during alternate weeks on the two conductivities. The total conductivity as given is based on the observations on one sign together with the indicated ratio of positive to negative conductivity and correction due to the fact that the conductivity near the ground where the large ions and the nuclei were measured was 2.6 times that where the conductivity was recorded (on top of the main laboratory building). For each value of the ratio of uncharged nuclei to large ions and the ratio of conductivities, values of

[3]Proc. R. Irish Acad., A, **37**, 71-94 (1927).
[4]Proc. R. Irish Acad., A, **38**, 40-48 (1929).
[5]Proc. R. Irish Acad., A, **51**, 61-69 (1933).
[6]Phil. Mag., **18**, 97-113 (1934).
[7]Terr. Mag., **39**, 47-64 (1934).

`ABLE 1—*Observed condensation-nuclei, large ions, and conductivity of the atmosphere at Washington, D. C., with computed values of coefficient of combination between small ions and uncharged nuclei*

te	75° west meridian mean time		Nuclei per cc		N_0/N_{12}	$\lambda_1+\lambda_2$	λ_1/λ_2	η_{10}	η_{20}	Ion-pairs per cc per sec		N_+	N_-
			Total Z	Uncharged N_0						q_A	q_B		
31	*h*	*m*				*ESU*							
2	14	20	21000	13650	7.2	1.00×10^{-4}	1.30	0.23×10^{-6}	0.40×10^{-6}	2.8	8.6	....	1882
	14	48	11620	10000	6.9	1.10	1.30	0.24	0.42	2.4	7.2	....	1446
	15	55	23000	18860	9.1	1.29	1.23	0.18	0.29	3.8	12.2	2079	....
	16	18	21210	18480	8.3	0.83	1.38	0.18	0.36	2.7	8.4	....	2231
. 3	10	09	22890	15960	3.2	0.65	1.29	0.62	1.00	6.0	14.8	4993	....
	10	30	14700	8610	2.8	0.99	1.29	0.72	1.16	6.0	13.7	....	3040
	10	54	13440	6720	2.1	1.08	1.29	1.05	1.64	8.3	15.4	3140	....
	11	18	11730	8860	4.0	1.03	1.11	0.52	0.68	3.8	10.4	....	2210
	11	42	13680	11620	4.3	1.07	1.11	0.47	0.62	4.7	13.1	2700	..
	12	06	8860	6210	3.6	1.09	1.25	0.54	0.85	3.3	8.5	....	1720
. 9	10	54	36540	24780	6.5	1.05	1.29	0.26	0.45	6.0	18.3	3830	.
	11	14	34230	23520	6.5	1.07	1.11	0.29	0.39	5.8	17.5	....	3590
	11	36	29400	24570	5.6	1.04	1.11	0.35	0.46	7.0	20.8	4390	. ..
	11	59	24150	18480	4.1	0.99	1.11	0.50	0.66	7.4	20.1	....	4470
	12	18	21420	14910	3.6	1.28	1.25	0.54	0.86	9.3	24.4	4180	....
	12	37	27300	18480	4.7	1.00	1.25	0.39	0.62	6.3	17.8	....	3920
. 10	11	25	23520	13650	4.8	0.99	1.11	0.42	0.55	4.5	12.9	2860	..
	11	46	10350	7700	3.3	0.99	1.11	0.65	0.84	4.2	10.4	....	2300
	12	16	12540	8400	3.7	1.00	1.25	0.52	0.81	4.0	10.2	2250	...
	12	38	15410	11380	4.2	0.89	1.25	0.45	0.71	2.9	11.0	....	2710
. 16	9	46	22890	11760	6.4	1.40	1.41	0.25	0.49	3.8	11.8	1847	..
	10	16	13230	8400	6.0	1.56	1.29	0.29	0.49	3.2	10.0	....	1409
	10	42	16680	12540	7.7	1.54	1.29	0.21	0.38	3.6	11.3	1620	....
	11	04	18900	10500	2.6	1.64	1.11	0.90	1.15	13.6	30.2	....	4049
	14	18	13860	7140	3.0	0.80	1.30	0.68	1.11	3.6	8.8	2410	..
	14	52	16440	10120	5.4	0.80	1.30	0.32	0.55	2.3	6.8	....	1870
	15	14	14910	14280	7.9	0.46	1.23	0.22	0.34	1.2	3.8	1810	...
t. 17	11	08	22680	15750	6.9	0.78	1.11	0.27	0.36	2.6	8.1	2280	....
	11	33	20160	14910	8.4	0.70	1.11	0.22	0.29	1.8	5.6	....	1770
v. 5	11	15	27300	21420	8.1	1.02	1.11	0.23	0.31	3.9	12.3	2660	....
	11	52	29860	22410	8.4	1.02	1.11	0.22	0.29	3.9	12.4	....	2680
	14	36	29650	25720	9.4	0.87	1.30	0.17	0.30	3.4	10.8	2730	...
	15	20	26250	24360	10.0	1.00	1.23	0.16	0.27	3.4	11.1	....	2430
	15	47	28240	17490	7.7	1.02	1.23	0.22	0.35	3.4	10.5	2270	. .
	16	11	32020	20480	8.4	0.79	1.38	0.18	0.36	2.7	8.8	....	2440
v. 21	10	48	36330	29820	4.9	....	1.29	0.37	0.62	....	....	6140	....
Means....................................								0.39×10^{-6}	0.60×10^{-6}	4.5	12.5	..	.

η_{10} and η_{20} were computed using equations (5) and (6). The rate of small-ion production in the column headed q_A was computed by equation (8), while that in column headed q_B was computed by the simplified equation (7), in which the value of η was assumed to be 7.2×10^{-6} as deduced by Harper[6] from theoretical considerations alone. The value of n in this case was not observed, but was computed from the observed conductivity assuming constant mobility.

The mean values calculated for η_{10} and η_{20} from the data are 0.42×10^{-6} and 0.62×10^{-6}, respectively; they are in good agreement with the corresponding values computed by Scrase from the Kew data. The individual values of the quantity q_A in Table 1 vary considerably but

the mean value is 4.5 ion-pairs per cc per sec; this is in exact agreement with the average value found for Kew by Scrase, who stated why he regards such a value as reasonable for Kew. For Washington, on the other hand, this cannot be regarded as a more or less average value for q. Data on the rate of small-ion production in the free atmosphere are available for the past two years, as a result of continuous records with a thin-walled ionization-chamber. The results indicate a considerably higher value for q than 4.5. The mean for August 1933 was over 7, while that for October 1934 was over 12. The ionization-chamber is sealed from the outside air and the wall has a stopping power of about 1.5 cm of air for α particles. A reduction-factor for reducing the ionization inside the chamber to that in the outside free atmosphere has been secured. The values given above are on the basis of this provisional reduction-factor.

Results with a thin-walled ionization-chamber together with those secured with the small-ion and large-ion recorders have made it possible to compute values for η_{12} by equation (7). Values of q, n, and N measured simultaneously, meaned over each hourly interval, were used in the computations. The results with the small-ion counter were corrected for errors due to collection of intermediate ions. The small ions and intermediate ions were recorded during alternate hours with this apparatus. The small ions, during the hour the intermediate ions were recorded, were filtered from the air-stream by a charged cylindrical condenser-system. The reduction-factor used for reducing ionization-rates inside the ionization-chamber to that for outside air was secured by coating the wall of the chamber with paraffin to a thickness of about 0.2 mm. If one denotes the rate of ionization per cc inside the chamber by I before applying the paraffin and that after applying it by I_c, then $(I-I_c)$ represents the ionization inside the chamber due to the α particles in the air (assuming that the γ-ray, β-particle, cosmic-ray, and residual ionizations did not change in the process of applying the paraffin). It was deduced, from the geometry of the chamber, together with the α-particle absorbing-power of its wall, that the ionization per cc inside the chamber due to α particles in the air was one-half that for the outside air due to the same process. Consequently $2(I-I_c)$ represents the rate of ionization per cc for outside air due to α particles and $[2(I-I_c)+I_c]$ the total ionization (q) for outside air except for the residual ionization of the chamber, which according to the results seems sufficiently small to be neglected in most considerations. The value for I_c amounted to 4 which is from 20 to 50 per cent of the total ionization.

In Figure 1 are shown diurnal-variation curves for various elements resulting from measurements upon air in an unoccupied closed room at the laboratory of the Department of Terrestrial Magnetism. Tests on air in the closed room were found advantageous over tests on outside air, especially because of a much smaller value for the large-ion content and because of its daily variation and the reverse conditions for both small-ion content and the rate of small-ion formation. The character of diurnal variation of the small-ion formation (q) resembles somewhat its character in the atmosphere outside the room, the time of maximum occurring approximately at the same time of day. In a general way the same may be said of the small-ion content, the time of the principal maximum in the room and outside occurring at about the same time. The large-ion

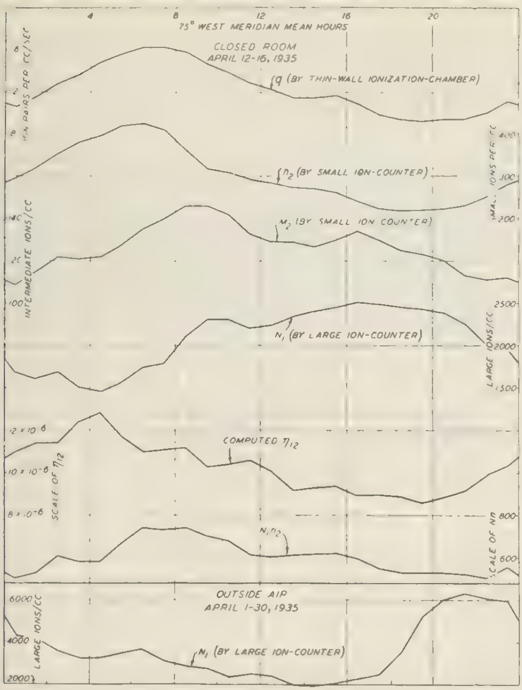


FIG 1— IONS AND IONIZATION IN CLOSED ROOM AND IN OUTSIDE AIR, WASHINGTON, D.C.

daily curve has a maximum for indoor air at a totally different time than in the outside air, as may be seen by comparing the curve for indoor air with the curve for April 1933 shown in Figure 1. The curve for intermediate ions is somewhat similar to that for large ions, just as is the case for outside air. The combination-coefficient, η_{12}, as computed on the basis of equation (7) shows a definite daily variation, somewhat similar in character to the curves for q_1 and n_1 but still more similar to the curve for the reciprocal of the large ions.

Attention was first directed by P. J. Nolan[3] to a variation in η_{12} with the reciprocal of the number of condensation-nuclei in the air. As a result of this relationship he has suggested the relationship between ionization (q), small-ion numbers (n), and number of condensation-nuclei (Z) is $q=\zeta n\sqrt{N}$. He found that only a formula of this type leads to a constant computed value for q. The value of the factor ζ may be obtained from the measured values of q, n, and N as the slope of the $(n\sqrt{N})$-curve. The values so obtained change but little through the day, the average being 59×10^{-5} which agrees closely with the value 55×10^{-5} obtained by Nolan. By extrapolating the $[q, (n\sqrt{N})]$-curve back to zero-value of $(n\sqrt{N})$ a residual ionization for the chamber of 3 pairs of ions per cc per sec is obtained. Since I_c was found to be 4 pairs

[3] Proc. R. Irish Acad., A, 38, 49-59 (1929).

of ions per cc per sec, only one pair could be attributed to ionization due to cosmic and gamma rays together with that due to beta particles. This of course is not the case, since the cosmic-ray ionization alone is about double this. It, therefore, seems impossible to interpret the results of this test in accordance with the equation $q=\zeta n\sqrt{N}$.

Turning our attention now to the curve connecting I and (nN) it is found that by extrapolation back to zero-value of (nN) and allowing 4 pairs of ions per cc per sec as the rate of ionization due to the cosmic and gamma rays and to beta particles, the residual ionization is negligibly small. That it is small, may be realized from the fact that the chamber consists essentially of an open-mesh brass screen covered with thin cellophane. The ends of the chamber consist of wood and the central electrode of a brass rod of small diameter. In other words, the results seem to indicate that equation (7) may be regarded as a justifiable relationship between q, n, and N and further that the diurnal variation in η_{12} is real.

By plotting all values of the slopes $dq/d(Nn)$ and $(1/N)$ a considerable scatter in the points was found, yet a definite tendency toward a linear relationship appeared. The resulting equation connecting the slopes η_{12} and $(1/N)$ is $\eta_{12}=0.50+(18600/N)$, which value leads to a relationship among q, N, and n as follows

$$q=1.0\times 10^{-6}\,Nn+0.0372n \qquad (9)$$

Values of q computed from (9) are in most cases in close agreement with observed values of q. If such a relationship has universal application, as well as in this particular investigation, theory must be altered accordingly. Interpretation of the meaning of such a relationship may be varied. It immediately suggests combination between small and large ions with a coefficient of 1.0×10^{-6} which may be difficult to reconcile with theory. In addition, it is suggested that the small ions disappear rapidly through other processes, for example, through diffusion to solids, dust-particles, etc. The value of this factor may well depend upon the particular conditions under which any investigation is made. Additional experiments must be made to determine this as well as to test the entire suggested relationship between formation and removal of small ions from the atmosphere.

The value of 1.0×10^{-6} as the coefficient of combination between small ions and large ions as required on the basis of these experimental results, is in rather close agreement with the value to be obtained by applying Whipple's formula but is not at all in agreement with that obtained from the formula derived by Harper. Likewise, the value is in much better agreement with the revised value by Nolan than with the value originally announced by him. Besides pointing to a low value for the coefficient, the results indicate necessity of considering an additional term, which is proportional to the first power of small-ion numbers.

DEPARTMENT OF TERRESTRIAL MAGNETISM,
CARNEGIE INSTITUTION OF WASHINGTON,
Washington, D. C.

FINAL RELATIVE SUNSPOT-NUMBERS FOR 1934

By W. Brunner

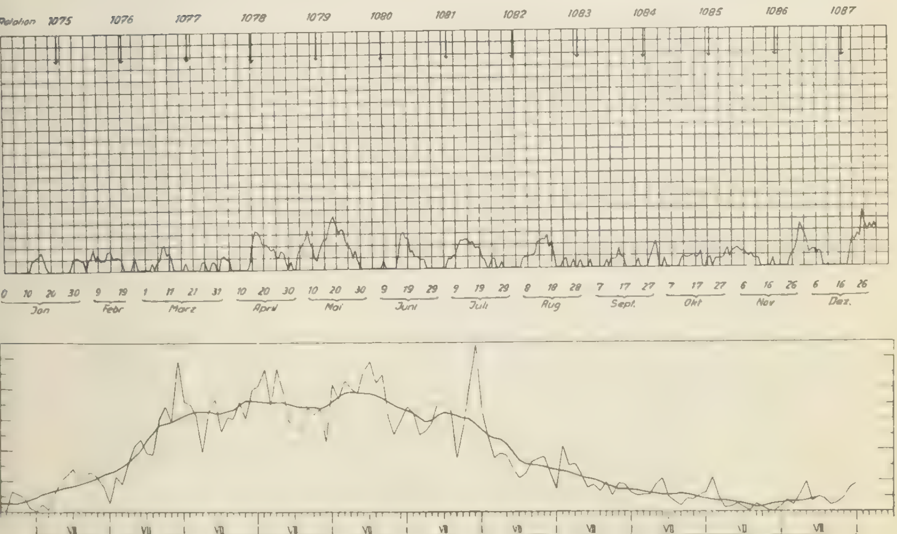
The following Tables contain the final sunspot-numbers for 1934, for the whole disc of the Sun, based on observations made at the Zürich Observatory, supplemented by series furnished by other cooperating observatories for days (indicated by asterisks) on which no observations were possible at Zürich.

Table 1 gives the yearly means of the relative numbers, R, since the last minimum 1923 and the number of days without spots.

Figure 1 gives a graphical representation of the daily relative sunspot-numbers for 1934, the times being plotted as abscissas and the relative

Table 1—*Yearly means of relative sunspot-numbers, R*

Year	R	Increase	No. spotless days
1923	5.8	......	200
1924	16.7	+10.9	116
1925	44.3	+27.6	29
1926	63.9	+19.6	2
1927	69.0	+ 5.1	0
1928	77.8	+ 8.8	0
1929	65.0	−12.8	0
1930	35.7	−29.3	3
1931	21.2	−14.5	43
1932	11.1	−10.1	108
1933	5.7	− 5.4	240
1934	8.7	+ 3.0	154



Figs. 1 and 2

Table 2—*Final relative sunspot-numbers for the whole disc of the Sun for 1934*

Day	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
1	0	11	0*	0	7	0	0	0	0	15	13	27
2	0	9[a]	0	11	0	0	0	0	0	0	14	20
3	0*	9	0	12	0	0	0	0	7	0	16	12
4	0	0	6	9	*M*14[c]*	0	0	0*	0	8	14	14
5	0	*E*10[c]	0	8	17	0	0	0	0	0	12	*W*14[c]
6	0	11	*E*7[c]	0	21[a]	0	8[d]	8[d]	0	0	14*	10
7	0	*E*19[c]	9	0	26	0	8	9	0	0	10	13
8	0	8	19	0	34	0	11	9	0	0	8	11
9	0	14	22[a]	0*	23	7	11	10	0	0	10	0
10	0	8	12	0	19	0	18	11	7*	0	7	0
11	0	9	15	0	10	0	24[d]	12	0	7	7	0
12	*M*8[c]	9	7	0	7	0	24[b]	*M*21[ac]	7	8	7	0
13	11	16	0	0	15[d]	0	25	22	7	8	0	0
14	12	17	0	7[d]	21	0[d]	24	24	8	8	0	0
15	13	10	0	21	26	11	20	24	*W*16[c]	9*	0	0*
16	16*	11	0	33	25	27	23	27	9	9*	0	0
17	11	11	0	32	35[d]*	31	17	17	8	8	0	0
18	4*	11	7	29	41	26	18	22	0	8	7	0
19	0	8	0	22	46	25	17	13	0	14	0*	0*
20	0*	0	0	21	37[a]	14[b]	9	0	0	0	0	*E*14[c]
21	0	0	0	22	29	16	8	0	0	0	0	24*
22	0*	0	0	18[b]	34	10	0	0	0	7	0	21
23	0*	0	0	16	33[a]	10	0	7	7	8	0	27*
24	0	10	0	19	23	8	8	8	0	0	0	25[a]
25	0	7	7	11	19	8	8	0	0	7	7	*E*47[cc]
26	0	0	7	10	17	8	0	0	0	7	*M*11[c]	37
27	0	0	0	16	9	0	0	7*	0	7	14	29
28	0	0	0	14	16	0	6	0	9	7	23	35[a]
29	*E*8[c]		7	7	8	0	0	0	14	9	*W*35[c]	31
30	11		7	0	0	0	0	7	21	16	31	36[a]
31	12*		0		0		0	0		8		31
Mean	3.4	7.8	4.3	11.3	19.7	6.7	9.3	8.3	4.0	5.7	8.7	15.4

*No obsrvations at Zürich.
[a] Passage of an average-sized group through the central meridian.
[b] Passage of a larger group or spot through the central meridian.
[c] New formation of a centre of activity: *E* on the eastern part of the Sun's disc; *W* on the western part; *M* in the central circle zone.
[d] Entrance of a large or average-sized centre of activity on the east limb.

numbers as ordinates. The limits of the successive solar rotations are indicated by vertical arrows in the upper edge of the Figure. The secondary maxima and minima succeeding the rotation-periods do not represent real fluctuations in sunspot-activity, but are rather to be attributed to the influence of solar rotation, to a certain stability of the centres of activity for spots, and to the special distribution of these centres of activity in the direction of rotation.

Figure 2 shows the observed and smoothed monthly relative numbers for 1923 to 1934 (1923 year of the last sunspot-minimum). The purpose of smoothing is to eliminate the secondary variations. The method of smoothing is as follows: For obtaining the mean of the epoch

July 1, the average of the monthly means of the twelve months January to December is taken (m_1), and for the epoch August 1, the average of the monthly means for February to January (m_2). The mean of these $m=(m_1+m_2)/2$, which represents the smoothed relative number for the middle of July, is used for the construction of the curve.

The epoch for the last minimum has been determined as 1933.8.

Table 3 contains data on the epochs of maxima and minima of sunspot-activity since 1610, together with the greatest and smallest monthly Wolf's numbers of the smoothed sunspot-curve. Epochs with weight-numbers 1 to 5 are doubtful, whereas those with weights 8, 9, and 10 may be taken as accurate.

TABLE 3—*Maximal and minimal relative sunspot-numbers, 1610-1933*

Maxima				Minima			
Epoch	Wt.	Period	Greatest smoothed monthly number	Epoch	Wt.	Period	Smallest smoothed monthly number
1615.5	2		....	1610.8	5		
1626.0	5	10.5a	....	1619.0	1	8.2	
1639.5	2	13.5	....	1634.0	2	15.0	
1649.0	1	9.5	...	1645.0	5	11.0	
1660.0	1	11.0	....	1655.0	1	10.0	
1675.0	2	15.0	....	1666.0	2	11.0	
1685.0	2	10.0	....	1679.5	2	13.5	
1693.0	1	8.0	....	1689.5	2	10.0	
1705.5	4	12.5	....	1698.0	1	8.5	
1718.2	6	12.7	....	1712.0	3	14.0	
1727.5	4	9.3	....	1723.5	2	11.5	
1738.7	2	11.2	....	1734.0	2	10.5	
1750.3	7	11.6	92.6	1745.0	2	11.0	
1761.5	7	11.2	86.5	1755.2	9	10.2	8.4
1769.7	8	8.2	115.8	1766.5	5	11.3	11.2
1778.4	5	8.7	158.5	1775.5	7	9.0	7.2
1788.1	4	9.7	141.2	1784.7	4	9.2	9.5
1805.2	5	17.1	49.2	1798.3	9	13.6	3.2
1816.4	8	11.2	48.7	1810.6	8	12.3	0.0
1829.9	10	13.5	71.7	1823.3	10	12.7	0.1
1837.2	10	7.3	146.9	1833.9	10	10.6	7.3
1848.1	10	10.9	131.6	1843.5	10	9.6	10.5
1860.1	10	12.0	97.9	1856.0	10	12.5	3.2
1870.6	10	10.5	140.5	1867.2	10	11.2	5.2
1883.9	10	13.3	74.6	1878.9	10	11.7	2.2
1894.1	10	10.2	87.9	1889.6	10	10.7	5.0
1906.4	10	12.3	64.2	1901.7	10	12.1	2.6
1917.6	10	11.2	105.4	1913.6	10	11.9	0.0
1928.4	10	10.8	78.1	1923.6	10	10.0	5.6
				1933.8	10	10.2	3.4

PROVISIONAL SUNSPOT-NUMBERS FOR MARCH TO MAY, 1935

(Dependent alone on observation at Zürich Observatory and its station at Arosa)

Day	March	April	May	Day	March	April	May
1	19	0	17	17	43	. .	8
2	. .	0	26	18	33	20	0
3	20[a]	0	*M*46[c]	19	*E*40[c]	11	0
4	. .	7	56[b]	20	35	16	0
5	17	0	56[da]	21	25	15	0
6	15	0	56	22	17	16	0
7	8	0	*E*56[c]	23	12	9	0
8	8	*M*11[c]	62	24	8	7	17
9	17[d]	24[a]	54	25	8	8	7?
10	*E*27[c]	22	41	26	8	0	8[d]
11	*W*34[c]	16	46[aa]	27	7	0	8
12	44	*E*22[c]	41	28	0	0	8
13	56	34	41	29	. .	8[d]	17[d]
14	72[da]	30	49	30	0	16[d]	30
15	68[aa]	37[b]	32	31	. .		38
16	*M*50[c]	29?	. .				
				Means. .	25.6	12.3	27.3
				No. days	27	29	30

Mean for quarter January to March, 1935, 21.8 (73 days)

[a]Passage of an average-sized group through the central meridian.
[b]Passage of a larger group through the central meridian.
[c]New formation of a large or average sized center of activity: E, on the eastern part of the Sun's disc; *W*, on the western part; *M*, in the central-circle zone.
[d]Entrance of a large or average-sized center of activity on the east limb.

EIDGEN. STERNWARTE,
Zürich, Switzerland

W. BRUNNER

PROVISIONAL SOLAR AND MAGNETIC CHARACTER-FIGURES, MOUNT WILSON OBSERVATORY JANUARY, FEBRUARY, AND MARCH, 1935

Greenwich mean time						Range hor. int.
Beginning			Ending			
1935	*h*	*m*	*d*	*h*	*m*	γ
Jan. 27	14	51	28	22	. .	116
Feb. 13	4	. .	14	17	. .	150

The storm of January 27 began with a sudden commencement in which the horizontal intensity increased by 20 gammas. No solar activity of an unusual nature was recorded during or immediately preceding either of these storms.

Day	January 1933								February 1933															
	K_2 A	K_2 B	$H\alpha B$ A	$H\alpha B$ B	$H\alpha D$ A	$H\alpha D$ B	No. groups	Mag'c char.	K_2 A	K_2 B	$H\alpha B$ A	$H\alpha B$ B	$H\alpha D$ A	$H\alpha D$ B	No. groups	Mag'c char.	K_2 A	K_2 B	$H\alpha B$ A	$H\alpha B$ B	$H\alpha D$ A	$H\alpha D$ B	No. groups	Mag'c char.
1	1.5	1	1.5	0.5	0.5	0.5	7	0.5	0.5	0.5	0.5	0	0.5	0	1	0.5	2	2	2[i]	2[i]	1	0.5	1	0.5
2	....	....	....	....	....	....	......	0.5	....	....	....	....	....	....	......	1	....	....	....	....	....	....	......	0.5
3	1.5	0.5	1.5	0.5	0.5	0	3	0	....	....	....	....	....	....	......	0.5	....	....	....	....	....	....	....[a]	0
4	....	....	....	....	....	....	......	0.5	....	....	....	....	....	....	......	0	....	....	....	....	....	....	......	0
5	....	....	....	....	....	....	......	0	....	....	....	....	....	....	......	0	1	1	1.5	1	1	0.5	1	0
6	1	1	1.5	1	0.5	0	2	0	....	....	....	....	....	....	......	0	....	....	....	....	....	....	1	0
7	1	1	1	1	0.5	0	2	0	....	....	....	....	....	....	......	0	....	....	....	....	....	....	......	0
8	....	....	....	....	....	....	....[d]	0	....	....	....	....	....	....	......	0	....	....	....	....	....	....	......	0
9	....	....	....	....	....	....	......	0	....	....	....	....	....	....	2	0	....	....	....	....	....	....	2	0
10	....	....	....	....	....	....	......	0	1.5	0.5	2	0.5	0.5	0.5	2[g]	0	....	....	....	....	....	....	3	0
11	....	....	....	....	....	....	......	0	....	....	....	....	....	....	2	0	1.5	0	1.5	0	1	1	4	0
12	....	....	....	....	....	....	2	0	....	....	....	....	....	....	3	0	2	1	2	1	1	0	4	0
13	....	....	....	....	....	....	......	0	....	....	1.5	1	0	0.5	3	1	....	....	....	....	....	....	5	0.5
14	1	0	1	0	0.5	0	2	0	....	....	....	....	....	....	......	1	2	2	2	2.5	1?	0?	6	1
15	....	....	....	....	....	....	1	0	2	1	....	....	....	....	2[h]	0.5	2	2.5	2	2.5	1	0	5[a,b]	1
16	1	1.5	1	1.5	0.5	0	1	0	2	1	2	0.5	0.5	0	2	0	2	2	2	2	1	0.5	4	0.5
17	1	1.5	1.5	2	0.5	1	1	1	2	1.5	2	2	0.5	0	1	0.5	2	0.5	2	0.5	1.5	0.5	5	0.5
18	....	....	....	....	....	....	1[c]	0	2	2	2	2	1	0	2	0.5	....	....	....	....	....	....	......	0
19	....	....	....	....	....	....	1	......	2	2.5	2	2.5	1	0	3[c]	0	....	....	....	....	....	....	......	0
20	1.5	0.5	1.5	0.5	0.5	0.5	1	......	1.5	2	2	2	1	0	4	0	2	0.5	2	0.5	2	0.5	3	0
21	1.5	0.5	2	0	0.5	0	1	......	1.5	0.5	2	0	0.5	0	4	0.5	....	....	....	....	....	....	......	0.5
22	....	....	....	....	....	....	2	0	1.5	0.5	2	0	1	0	3	0	1	0.5	1	0	2.5	2	1	0
23	....	....	....	....	....	....	2	0.5	1.5	1	1.5	1	1	0	1	0	1	1	1	1	3	2	1	0
24	2	1	2	1	1.5	0.5	2	0.5	....	....	....	....	....	....	......[f]	0	1	1.5	1	1.5	2	0.5	1	0.5
25	2	1	2	1	1	1	2	0	1	1	1	1	1.5	0	2	0.5	1	1	1	1.5	3	1	1	0
26	....	....	....	....	....	....	......	0	1.5	0.5	1.5	0.5	1	0	2	0.5	1	1	1.5	1.5	3	0	1	0
27	....	....	....	....	....	....	2	0.5	1.5	0	1.5	0	1	0	2	0	1	1	2	1.5	2	0	1	0
28	2	1	2?	1?	1?	0.5?	2[e]	0.5	....	....	....	....	....	....	......	0	1.5	-0.5	2	1	1	0	0	0
29	1	0.5	1	0	1	0	1	0									1	1	1.5	1	0.5	0	0	0
30	1	0	1	0	0.5	0	1	0									1.5	2	....		....	....	0	0.5
31	1	0	1	0	0.5	0	1	0.5									1.5	2	1.5	2	0.5	0	0	0
Mean	1.3	0.7	1.4	0.7	0.7	0.3	1.8	0.2	1.6	1.0	1.7	0.9	0.8	0.1	2.3	0.2	1.5	1.2	1.6	1.3	1.6	0.5	2.2	0.2

NOTE—For an explanation of these tables see this JOURNAL, **35**, 47-49 (1930).
?Indicates an uncertain value which should be given low weight.
[a, b, c, d, e]Passage of an average sized group through the central meridian within 10°, 15°, 20°, 30°, 35° of the center of the disc, respectively. [f, g, h]Passage of a small group through the central meridian within 20°, 30°, 35° of the center of the disc, respectively. [i]Small areas very bright $H\alpha$ in the southeast group.

Carnegie Institution of Washington, Mount Wilson Observatory
Pasadena, California

SETH B. NICHOLSON
ELIZABETH E. STERNBERG

AMERICAN *URSI* BROADCASTS OF COSMIC DATA[1] JANUARY TO MARCH, 1935

The data for terrestrial magnetism, sunspots, solar constant, and auroræ are the same as given in previous tables.

The first three columns of the Table give (1) the magnetic character according to the scale 0-2 of the International Commission of Terrestrial Magnetism and Atmospheric Electricity, (2) the type featuring the day other than normal by the letters *b*, *p*, *o*, and *i* for days marked by bay, rapid pulsations, long-period oscillations, and irregular oscillations, respectively, and (3) the hour and minute of Greenwich mean time marking the beginning of a storm, the end of the storm being indicated in the foot-

[1]For previous announcements see Terr. Mag., **35**, 184-185 and 252-253 (1930); **36**, 54, 141, 258-259, and 358-360 (1931); **37**, 85-89, 189-192, 408-411, and 484-487 (1932); **38**, 60-63, 148-151, 262-265, 335-339 (1933); **39**, 73-77, 159-163, 244-247, 353-356 (1934), and **40**, 111-115 (1935).

Summary American URSI daily broadcasts of cosmic data, January to March, 1935

Date	January							February							March							Date
	Magnetism			Sun-spot		Solar constant		Magnetism			Sun-spot		Solar constant		Magnetism			Sun-spot		Solar constant		
	Character	Type	G. M. T. begin. distur.	Groups	Number	Value	Character	Character	Type	G. M. T. begin. distur.	Groups	Number	Value	Character	Character	Type	G. M. T. begin. distur.	Groups	Number	Value	Character	
			h m			*cal*				*h m*			*cal*				*h m*			*cal*		
1[a]	1	...	.. ..	7*	20*	......	...	0	...	.. ..	1	1	1.944	*s*	0	...	.. ..	1	8	......	...	1
2	0	...	.. ..	...	...	1.945	*f*	1	...	.. ..	...	...	1.935	*f*	1	...	.. ..	...				2
3	0	...	.. ..	3	10	1.958	*s*	1	...	.. ..	...	...	1.946	*f*	0	...	.. ..	...	...	1.937	*f*	3
4	0	...	.. ..	...	...	1.952	*f*	0	...	.. ..	...	...	......	...	1	...	.. ..	...	...	1.950	*f*	4
5	0	...	.. ..	...	...	1.952	*s*	0	...	.. ..	...	...	......	...	0	...	.. ..	1	3	1.948	*f*	5
6	0	...	.. ..	2	3	1.959	*s*	0	...	.. ..	...	...	......	...	0	...	.. ..	1	1	1.951	*f*	6
7	0	...	.. ..	2	4	1.950	*f*	0	...	.. ..	...	...	1.951	*u*	0	...	.. ..	...	...	1.949	*s*	7
8	0	...	.. ..	...	...	1.955	*s*	0	...	.. ..	...	...	......	...	1	...	.. ..	...	...	......	...	8
9	0	...	.. ..	...	...	1.957	*s*	0	...	.. ..	2	9	......	...	1	...	.. ..	2	3			9
10	0	...	.. ..	...	...	1.946	*s*	0	...	.. ..	2	4	......	...	0	...	.. ..	3	8	......	...	10
11	0	...	.. ..	...	...	1.948	*s*	0	...	.. ..	2	2	1.949	*s*	0	...	.. ..	4	15			11
12	0	...	.. ..	2	5	......	...	0	...	.. ..	3	3	......	...	0	...	.. ..	4	25			12
13	0	...	.. ..	...	...	......	...	1	...	.. ..	3	5	1.954	*f*	1	...	.. ..	5	23			13
14	0	...	.. ..	2	15	......	...	2	...	15 00	...	...	1.956	*u*	1	...	.. ..	6	36			14
15	0	...	.. ..	1	8	......	...	1	...	.. ..	2	4	1.952	*f*	1	...	.. ..	5	25			15
16	0	...	.. ..	1	10	......	...	0	...	.. ..	...	...	......	...	1	...	.. ..	4	19			16
17	1	...	.. ..	1	8	1.955	*f*	1	...	.. ..	1	2	......	...	1	...	.. ..	5	16			17
18	0	...	.. ..	1	22	......	...	0	...	.. ..	2*	3*	......	...	1	...	.. ..	...	...			18
19	0	...	.. ..	1	2	......	...	0	...	.. ..	3	8	1.953	*u*	1	...	.. ..	...	...			19
20	0	...	.. ..	1	5	......	...	0	...	.. ..	4	8	......	...	0	...	.. ..	3	12			20
21	0	...	.. ..	1	1	1.958	*f*	1	...	.. ..	4*	6	1.951	*f*	1	...	.. ..	...	...	......	...	21
22	1	...	.. ..	2*	2	1.949	*f*	1	...	.. ..	3	5	1.947	*f*	1	...	.. ..	1	3	....		22
23	1	...	.. ..	2	2	......	...	0	...	.. ..	1	6	......	...	0	...	.. ..	1	5	1.954	*s*	23
24	0	...	.. ..	2	18	......	...	0	...	.. ..	...	...	......	...	0	...	.. ..	1	2	1.935	*u*	24
25	0	...	.. ..	2	8	......	...	1	...	.. ..	2	3	......	...	1	...	.. ..	1	4			25
26	0	...	.. ..	...	...	1.953	*f*	1	...	.. ..	2	3	......	...	0	...	.. ..	1	2			26
27	0	...	.. ..	2	8	......	...	0	...	.. ..	2	3	......	...	0	...	.. ..	1*	2*			27
28	1	0	14 52	2	6	......	...	0	...	.. ..	...	...	......	...	0	...	.. ..	0	0			28
29	0	...	.. ..	1	2	1.944	*f*								0	...	.. ..	0	0			29
30	0	...	.. ..	1	2	......	...								0	...	.. ..	0	0	1.955	*u*	30
31	0	...	.. ..	1	2	......	...								1	...	.. ..	0	0	1.951	*f*	31
Mean	0.2	...	.. ..	1.8	7.4	1.952	...	0.4	...	.. ..	2.3	4.4	1.949	..	0.5	...	.. ..	2.2	9.2	1.948		Mean

*A revision of value originally broadcast.

Greenwich mean time for ending of storms: 3 h, January 29; 7 h, February 15. [a]Auroral data, January 1: Character, 1; duration, 4 hours; cloudiness, 0; form without rays, *HV*; area covered .0.2; average altitude, 20°; position, *NW-N-E*; G. M. T. greatest disturbance, 11 h.

Kennelly-Heaviside Layer heights, Washington, D. C., January to March, 1935
(Nearest hour, Greenwich mean time, of all observations is 17)

Date	Frequency	Height	Date	Frequency	Height	Date	Frequency	Height
1935	*kc/sec*	*km*	*1935*	*kc/sec*	*km*	*1935*	*kc/sec*	*km*
Jan. 2	2,650	140	Jan. 30	3,900	370	Mar. 6	3,000	120
" "	2,900	280	" "	4,900	280	" "	3,200	120
" "	3,100	140, 350	" "	5,500	300	" "	3,300	200
" "	3,170	230	" "	6,100	300, 450	" "	3,800	200
" "	4,000	280	" "	6,500	330	" "	4,000	250
" "	4,700	280	" "	6,900	400	" "	4,200	280
" "	5,900	260	" "	7,100	680	" "	4,500	280
" "	6,700	280	Feb. 6	2,500	130	" "	4,700	300
" "	7,100	290, 370	" "	2,890	310	" "	5,100	280
" "	7,500	300	" "	3,030	190	" "	6,100	280
" "	7,900	320	" "	3,180	250	" "	7,300	290
" "	8,100	380	" "	3,400	210	" "	7,500	300
" "	8,300	*	" "	4,100	350	" "	7,500	340
" 9	2,700	130	" "	4,500	270	" "	7,900	320
" "	3,170	160, 340	" "	5,300	300	" "	7,900	490
" "	3,350	240	" "	5,700	310, 430	" "	8,700	390
" "	4,150	300	" "	6,100	320	" 13	2,500	110
" "	4,400	260	" "	6,500	380	" "	3,000	120
" "	4,700	280	" 13	2,500	120	" "	3,300	130
" "	5,300	250	" "	2,900	120	" "	3,300	220
" "	6,100	280, 480	" "	3,200	140, 230	" "	3,500	230
" "	6,900	320	" "	3,500	220	" "	3,600	230
" "	7,300	290	" "	3,800	240	" "	4,200	450
" "	7,500	710	" "	4,100	300	" "	4,300	240
" 16	2,700	140	" "	4,300	400	" "	4,300	510
" "	2,970	160	" "	4,900	300	" "	4,500	250
" "	2,980	*	" "	5,500	300	" "	4,500	440
" "	3,000	380	" "	5,900	330	" "	4,900	320
" "	3,300	220	" "	6,500	330, 400	" "	4,900	570
" "	4,000	280	" "	6,900	350	" "	5,300	490
" "	4,400	270	" "	7,300	430	" "	5,500	490
" "	4,700	240	" 20	4,500	280	" "	5,700	530
" "	5,700	260	" "	5,500	270	" 20	2,500	120
" "	6,300	270, 350	" "	6,500	270	" "	3,000	100
" "	6,900	320	" "	7,500	290	" "	3,100	*
" "	7,500	360	" "	7,900	210	" "	3,600	*
" "	7,700	720	" "	8,100	320, 360	" "	3,700	210
" 23	3,000	120	" 27	2,600	120	" "	4,000	200
" "	3,050	*	" "	2,800	160	" "	4,500	330
" "	3,100	250	" "	2,900	130	" "	4,700	330
" "	3,200	220	" "	3,200	200	" "	4,900	300
" "	4,200	310	" "	3,400	180	" "	6,300	360
" "	4,700	240	" "	3,600	180	" "	6,500	340
" "	5,900	290	" "	3,900	260	" "	6,500	450
" "	6,300	320, 400	" "	4,400	250	" "	7,100	420
" "	6,900	340	" "	4,700	300	" 27	4,500	450
" "	7,100	380	" "	5,300	280	" "	5,100	330
" "	7,700	310	" "	6,100	290	" "	5,500	340
" "	8,100	350	" "	6,700	300, 350	" "	5,500	410
" 30	2,850	140	" "	7,300	340	" "	5,900	360
" "	2,880	360	" "	7,900	540	" "	6,700	470
" "	3,300	210	Mar. 6	2,500	110			

* = No value obtained.

note to the Table. The next two columns give the data relating to sunspots: (1) the number of groups of spots and (2) the total number of spots. It is to be noted that sunspot-numbers such as those from Zürich can be obtained from the number of groups and spots given in the Table by the formula $N=k\ (10g+s)$, where k for Mount Wilson is about 0.7.

Mount Wilson Observatory is now supplying corrections and additions to the sunspot-data which are broadcast in the *URSI*gram. So far as possible, these additional and corrected values will be used in this tabular summary and will be designated as such in footnotes to the Table.

Beginning January 1, 1934, the magnetic information of the *URSI*-gram is for Cheltenham, Maryland, instead of Tucson, Arizona. In addition to this change in observatory, the data cover the 24 hours ending 8 A. M., 75° west meridian mean time, instead of the 24 hours ending at 7 A. M., 105° west meridian mean time.

The columns headed solar constant show (1) the value in calories of the solar constant, and (2) by letters s, f, and u whether the determination was satisfactory, fair, or unsatisfactory, respectively.

In accordance with information received from Dr. C. G. Abbot, Secretary of the Smithsonian Institution, transfer from Table Mountain to Montezuma solar-constant values was made as of October 23, 1934. Table Mountain for a considerable time has been 0.012 calorie above Montezuma, and above the scale of 1913 to 1930. Hence the value of October 23 and succeeding values are on a scale 0.012 calorie lower than previous ones.

On January 1, 1935, Professor Fuller reported by radio: "Since today's auroral data conclude the period covered by the original grant of funds for auroral study and as there have been no further grants made for continuation of the study, today's message will be the last until such time as additional funds are made available."

The table of Kennelly-Heaviside Layer heights is self-explanatory.

C. C. Ennis

Department of Terrestrial Magnetism,
Carnegie Institution of Washington,
Washington, D. C.

PRINCIPAL MAGNETIC STORMS

Sitka Magnetic Observatory

January to March, 1935[1]

(*Latitude 57° 03′.0 N., longitude, 135° 20′.1 or 9ʰ 01ᵐ.3 W. of Gr.*)

March 14—This storm began about 6^h, with a gradual increase of about 35′ in east declination, accompanied by small increases in the other elements, and followed by similar decreases in all the elements. About $8^h.5$ the fluctuations became violent, characterized by small, short-period oscillations superimposed on slow, large movements. The severest part of the storm ended about 18^h, but the record continued moderately stormy for about 24 hours longer. A lesser disturbance had occurred about 20 hours before the beginning of the storm, lasting for several hours. The ranges were: Declination, 99′; horizontal intensity, 786 gammas; vertical intensity, 528 gammas.

John Herschenberger, *Observer-in-Charge*

[1]Communicated by R. S. Patton, Director, United States Coast and Geodetic Survey.

Cheltenham Magnetic Observatory
January to March, 1935[1]

(Latitude 38° 44′.0 N., longitude 75° 50′.5 or 5^h 07^m.4 W. of Gr.)

February 1-2—After the daily rise in D the trace gradually dropped off until 3^h 30^m on February 2 when there was a rather sharp decrease in westerly declination of 12′, at which level it remained substantially for about an hour when there was a sudden return almost as great as the fall and thence a gradual rise to a normal position. During the same time horizontal intensity underwent a gradual drop and at the time D took its sudden fall went up sharply about 25 gammas, held this level a half-hour or so, and then at about 4^h dropped suddenly around 50 gammas for about a half-hour and then increased 70 or more gammas almost as suddenly as it had dropped, after which there was a gradual rise to normal.

February 13—Between 0^h and 1^h there was a gradual increase in H. At 14^h there began a remarkable series of sharp and increasingly violent drops or peaks a few hours apart, in step with which there were corresponding decreasing westerly changes in declination so that the two curves matched in style and sharpness. The last and greatest peak came at 23^h, and it was 24^h before normal conditions were reached; the peaks, reasonably sharp in the case of H, came in the middle of this period or at about 23^h 30^m; in this last drop H had decreased 90 gammas and westerly declination had decreased 25′.

March 13-18—There were faint signs of this disturbance several hours in advance of the definite disturbance at 3^h 30^m March 13, when horizontal intensity increased abruptly some 40 gammas and continued at this value for an hour and then fell with only a slightly less precipitance to normal. Meanwhile declination, slightly anticipating this sharp rise of H, showed a decrease in westerly declination of 18′ and almost as sharply a return to normal so that the peak thus formed served as a pointer to the starting place of the high value of H. Thereafter the three elements—marked by slight perturbations of D and H—proceeded normally until 19^h, at which time all leveled out and ran parallel with the base-line for 11 hours. There then began around 6^h March 14 a series of steep peaks, particularly manifested in H, which varied sharply up and down with a range of 80 gammas in two hours and less, and with the jagged lines of D and H being crowded with tiny serrations; the disturbance maintained this style but slightly lessened in intensity, disappearing around 2^h March 18. Particularly noteworthy was the sharp vertical spurt in H at 1^h 39^m March 15, when it increased 40 gammas at once and fell back almost as quickly and in half an hour again reached nearly as far above the first high; apparently at the time H showed this initial increase D suddenly showed an increase of westerly declination amounting to 15′, after which it dropped steeply to normal. This disturbance is coincident with increased sunspot-activity.

Eoline R. Hand, *Officer-in-Charge*

[1]Communicated by R. S. Patton, Director, United States Coast and Geodetic Survey.

Huancayo Magnetic Observatory
January to March, 1935

(Latitude 12° 02′.7 S., longitude 75° 20′.4 or 5ʰ 01ᵐ.4 W. of Gr.)

Greenwich mean time						Range		
Beginning			Ending			Decl'n	Hor. int.	Ver. int.
1935	*h*	*m*	*d*	*h*	*m*	′	γ	γ
Jan. 27	14	49	28	22	..	8	360	63
Feb. 1	9	06	2	23	..	?	(160)	?
Feb. 12	21	46	15	5	..	7	212	49
Mar. 1	9	..	2	22	..	6	165	31
Mar. 12	16	37	15	22	..	8	216	50
Mar. 30	12	13	30	20	..	3	128	34

January 27-28—This storm, of comparatively short duration, is conspicuous not only for a sudden commencement in all three elements, but also for two subsequent sudden and large changes in *H*. The sudden commencement occurred at $14^h\,49^m$, G. M. T., with an increase, within a period of five minutes, of 105 gammas in *H*, an increase of 4 gammas in *Z* and an increase of 2′ in *D*. There followed minor peaks and bays in *H*, while *D* and *Z* were little disturbed, until $18^h\,17^m$. Then, in six minutes, *H* decreased 102 gammas, *Z* decreased 3 gammas, and *D* first increased 1′ and then decreased 2′. Again, from $19^h\,46^m$ to $19^h\,50^m$ there was a sudden decrease in *H* of 71 gammas, and of 1′ in *D*, but no effect in *Z*. Thereafter *H*, showing various peaks and bays, remained lower than normal for several hours, reaching a minimum at $0^h\,58^m$ on January 28 then gradually recovering to normal values by 12^h. Between 12^h and 22^h on January 28, various small peaks and bays were superposed on the normal diurnal trend. *D* and *Z* were little disturbed subsequent to the second sudden change described above.

February 1-2—The commencement of this storm was inconspicuous, at about $9^h\,06^m$, G. M. T.; in a period of ten minutes *H* increased 15 gammas, while *Z* increased 2 gammas and *D* showed a very slight decrease. The *H*-decrease, maintained at a steady value for about three hours, was followed by a deep bay which persisted from 12^h to 15^h. Thereafter a tendency was shown to return to a normal trend, but the usual maximum was depressed and showed minor bays and peaks. Beginning with 0^h on February 2, instead of the usual more or less steady value of *H*, the latter increased steadily and quite smoothly until 11^h, after which the usual diurnal maximum was replaced by several large peaks and bays, averaging less than normal magnitude, many of the bays and peaks representing a change of 25 to 75 gammas in periods of ten to twenty minutes. A final deep bay at 19^h was followed by rapid recovery to normal conditions by 23^h.

February 12-15—The beginning of this storm is somewhat indefinite, although it may be taken as having its commencement at $21^h\,46^m$ on February 12, G. M. T., with a sudden increase in *H* of 3 or 4 gammas. There was no coincident change either in *D* or *Z*. After 0^h on February 13, and until $11^h\,30^m$, *H* is very irregular, though the bays and peaks are small and of long period. After $11^h\,30^m$, the changes in *H* become more rapid and larger, with several peaks and bays having a range of 50 to 100 gammas with periods of 30 to 40 minutes. The last peak of this group is followed by a fairly regular decline in *H*, when, from $18^h\,18^m$ to

$23^h\ 15^m$ there is a decrease of 212 gammas. Thereafter for 24 hours *H* is somewhat lower than normal and very irregular, although general features of the normal diurnal trend are seen. *D* and *Z* are quite disturbed throughout the storm, but the diurnal trend in each element is well maintained.

March 1-2 -This disturbance, given character 1, is chiefly noteworthy for the large variations in *H* during the daylight period from 12^h to 19^h, on both March 1 and 2. Several bays and peaks of 30- to 40-minute period, having ranges of 50 to 100 gammas, occurred on each day during the period mentioned. *D* and *Z* were unusually irregular during the same period. For the other portions of the disturbance minor fluctuations were superposed on the usual diurnal trend.

March 12-15 -This disturbance, all three days of which were given character 1, appears to have its beginning with a sudden decrease in *H* and *D* and an increase in *Z* at $16^h\ 37^m$, G. M. T., on March 12. The changes in *D* and *Z* at that time are just perceptible, while *H* changed 23 gammas in six minutes. For seven hours after the commencement all three elements were undisturbed; then, after 0^h on March 13, greater irregularities are shown. *H* showed fairly large, long-period, irregularities in the night hours, and rapid, large ones in the daylight hours, between 12^h and 19^h, G. M. T., on March 13, 14, and 15. Bays and peaks in *H* in the daylight hours were of duration 20 to 40 minutes, with ranges of 50 to 100 gammas. *D* and *Z* showed quite large variations during the daylight hours.

March 30 This storm, of duration less than eight hours, is conspicuous for a sudden commencement and a sudden termination in all three elements. At the commencement, *H* increased 47 gammas in five minutes, *Z* increased 3 gammas in two minutes, and *D* decreased $0'.8$ in two minutes. At the termination *H* decreased 71 gammas in six minutes, *Z* decreased 3 gammas, and *D* decreased $1'.0$. During the storm-period two large peaks and one bay, with a range of 100 gammas in each, are seen, on which are superposed rapid, minor fluctuations. *D* and *Z* were quite irregular, especially in the first half of the storm-period.

O. W. Torreson, *Observer-in-Charge*

Apia Observatory

January to March, 1935

(Latitude 13° 48'.4 S., longitude 171° 46'.5 or $11^h\ 27^m.1$ W. of Gr.)

January 27 A slight disturbance began at $14^h\ 50^m$, G. M. T., with a sudden increase of 22 gammas in the horizontal intensity. A minimum value was shown at $5^h\ 40^m$ on January 28, the range of the disturbance being approximately 94 gammas. The vertical intensity increased initially by 4 gammas and then underwent slight irregular changes.

J. Wadsworth, *Director*

Watheroo Magnetic Observatory

October, 1934, to March, 1935

(Latitude 30° 19'.1 S., longitude 115° 52'.6 or $7^h\ 43^m.5$ E. of Gr.)

October 24, 1934 This small disturbance began with a sudden commencement at $1^h\ 02^m$, G. M. T., the horizontal intensity showing a de-

crease of 5 gammas in two minutes. The declination changed 0′.7 westerly, while the commencement was not well marked in vertical intensity.

December 3-4, 1934—This disturbance began at approximately $23^h\ 50^m$, G. M. T., on December 3. The initial impulse was of the nature of a sudden commencement, but owing to fluctuations in the trace it is difficult to decide upon the magnitudes of the first movement. There was a striking movement in horizontal intensity on December 4 at $1^h\ 55^m$, when it increased 68 gammas in ten minutes. During the same period sudden increases in westerly declination and vertical intensity were apparent. The disturbance which followed this large movement was not remarkable for its violence, although the range in horizontal intensity was approximately 120 gammas. Normal conditions were resumed at approximately 16^h on December 4.

December 29-31, 1934 -This very moderate disturbance began at $10^h\ 57^m$, G. M. T., on December 29 with a very small kick in all three elements. The slightly disturbed condition prevailed with short intervening quiet periods until 5^h December 31. The disturbance was noteworthy for its duration rather than its intensity.

January 27-28, 1935—This moderate disturbance began with a marked sudden commencement at $14^h\ 50^m$, G. M. T., when horizontal intensity suddenly increased 2 gammas and then very suddenly increased 46 gammas in less than four minutes. Vertical intensity was but moderately affected, decreasing 8 gammas. Declination swung 1′ easterly. Irregular fluctuations followed the sudden commencement, and all elements returned to normal by midnight January 28.

February 1-2, 1935--This very moderate disturbance probably began at 8^h on February 1, and deserves notice because of some very rapid oscillatory motions about 15^h, G. M. T., on February 2. The movements were shown to advantage by the magnetogram from the la Cour rapid magnetograph. They appeared as two groups of waves, the first group centering at $15^h\ 05^m$ and consisting of three complete oscillations of period 50 seconds, the second group centering at $15^h\ 14^m$ and being more irregular, having, however, the same period as the first group. The trace in all elements resumed its normal course at approximately 20^h on February 2.

February 13-14, 1935—This disturbance, with an ill-defined sudden commencement, exhibited a very marked impulse in the three elements at $15^h\ 31^m$, G. M. T., on February 13, though the preceding sixteen hours had shown irregular fluctuations. The impulse increased the horizontal intensity by 40 gammas in nine minutes and decreases in vertical intensity and westerly declination took place during the same interval. The very moderate fluctuations of this disturbance were superimposed upon the normal diurnal variation. However, it is probable that the maximum fluctuations occurred between the hours of 8 and 12 on February 14. Normal conditions were resumed at 16^h on the same day.

March 30, 1935—There was a sudden commencement at $12^h\ 13^m\ 40^s$, G. M. T. Horizontal intensity increased 22 gammas in four minutes, while declination decreased 1′ and vertical intensity decreased 5 gammas. A small disturbance followed, but the magnetograms had resumed normally quiet conditions at 21^h on March 30.

W. C. PARKINSON, *Oberver-inCharge*

LIST OF RECENT PUBLICATIONS

By H. D. Harradon

A—Terrestrial and Cosmical Magnetism

Akiyosi, T. On the observations of magnetic variation during the total solar eclipse on the 14th February 1934. Hydrogr. Bull., Tokyo, 14th Year, No. 2, 1935 (41-48 with figs.). [Text in Japanese language.]

Baldwin, J. M. Report on magnetic results obtained at Melbourne Observatory. Proc. Fifth Pacific Sci. Cong., Canada, 1933, v. 3, Toronto, Univ. Press, 1934 (1813).

Copenhagen, Det Danske Meteorologiske Institut. Magnetisk aarbog 1ste del: Danmark (undtagen Grönland)—Annuaire magnétique, lère partie; le Danemark (excepté le Groenland). 1933. Köbenhavn, G. E. C. Gad, 1934 (37). 32 cm.

De Bilt, Institut Météorologique des Pays-Bas. Caractère magnétique numérique des jours. Tome XII. Juillet-septembre 1934. De Bilt, 1935, 20 pp. 24 cm. [Published under the auspices of the Association of Terrestrial Magnetism and Electricity of the International Union of Geodesy and Geophysics.]

Dimmler. Die wöchentlichen magnetischen Beobachtungen in Jinsen, Taihoku, Otomari und Palau 1933. Ann. Hydrogr., Berlin, Jahrg. 63, Heft 4, 1935 (177-178).

Eblé, L. Sur la variation annuelle du champ magnétique terrestre. Paris, C.-R. Acad. sci., T. 200, No. 15, 1935 (1342-1343).

Eblé, L., et G. Gibault. Valeurs des éléments magnétiques à la Station du Val-Joyeux (Seine-et-Oise) au 1er janvier 1935. Paris, C.-R. Acad. sci., T. 200, No. 11, 1935 (957-958).

Ferraro, V. C. A. On the radial limitation of the Sun's magnetic field. London, Mon. Not. R. Astr. Soc., v. 95, No. 3, 1935 (280-289).

Fleming, J. A. Terrestrial magnetism and electricity. Article in Amer. Year Book for 1934, New York, N. Y., 1935 (736-743).

The distribution and need of additional magnetic observatories and secular-variation stations in the Pacific Region. Proc. Fifth Pacific Sci. Cong., Canada, 1933 v. 3, Toronto, Univ. Press, 1934 (1675-1683).

The relations of magnetic and electric work in the Pacific Ocean to the Polar-Year campaign, 1932-33. Proc. Fifth Pacific Sci. Cong., Canada, 1933, v. 3, Toronto, Univ. Press, 1934 (1685-1690).

Gallo, J. Magnetic results in the Pacific Region. Proc. Fifth Pacific Sci. Cong., Canada, 1933, v. 3, Toronto, Univ. Press, 1934 (1855-1859).

Green, J. W. Magnetic secular-variation in the Pacific Region. Proc. Fifth Pacific Sci. Cong., Canada, 1933, v. 3, Toronto, Univ. Press, 1934 (1669-1674).

Hazard, D. L. Results of observations made at the United States Coast and Geodetic Survey Magnetic Observatory at Sitka, Alaska, in 1927 and 1928. Washington, D. C., U. S. Dept. Comm., Coast Geod. Surv., 1935 (107 with 14 figs.). 28 cm.

Herbert, W. H. Magnetic secular-change tables for British Columbia and Yukon Territory. Proc. Fifth Pacific Sci. Cong., Canada, 1933, v. 3, Toronto, Univ. Press, 1934 (1697-1701).

Jackson, W. E. W. Solar activity and terrestrial magnetism. Proc. Fifth Pacific Sci. Cong., Canada, 1933, v. 2, Toronto, Univ. Press, 1934 (1057-1061).

Diurnal variations at Meanook. Proc. Fifth Pacific Sci. Cong., Canada, 1933, v. 3, Toronto, Univ. Press, 1934 (1863-1869).

Johnston, H. F. The equipment and work of the Huancayo Magnetic Observatory. Proc. Fifth Pacific Sci. Cong., Canada, 1933, v. 3, Toronto, Univ. Press, 1934 (1835-1840).

Jouravsky, G., P. Charczenko, et G. Choubert. Sur le magnétisme rémanent induit des roches éruptives. Paris. C.-R. Acad. sci., T. 200, No. 7, 1935 (541-543).

Keys, D. A. The application of magnetometric measurements to the location and determination of geological structure. Proc. Fifth Pacific Sci. Cong., Canada, 1933, v. 3, Toronto, Univ. Press, 1934 (1909-1917).

Kraeber, L. Magnetische Messungen als Hilfsmittel bei aufbereitungstechnischen Untersuchungen an Eisenerzen. Naturw., Berlin, Jahrg. 23, Heft 10, 1935 (160-164).

Kuwahara, S. Weekly magnetic observations at Zinsen, Taihoku, Otomari, and Palau in 1933. Hydrogr. Bull., Tokyo, 14th Year, No. 1, 1935 (1-6). [Text in Japanese language.]

Liu, C.-Y. A new period of terrestrial magnetism. Proc. Fifth Pacific Sci. Cong., Canada, 1933, v. 3, Toronto, Univ. Press, 1934 (1815-1830).

Lutzow-Holm, O. Carta magnetica de la República Argentina. Confeccionada para la época del 1º de enero de 1931 con un mapa y tres gráficos ilustrativos. Buenos Aires, Ministerio de Agricultura, Dirección de Meteorologia, Geofisica, e Hidrologia, 1933 (15 con 1 mapa y 3 figs.). 24 cm. [This isogonic map of Argentina is accompanied by a historical introduction and tables of values of declination at various stations, reduced to the epoch of 1931. There are three figures showing respectively the secular-variation curve of the declination at Paris 1500-1940, the sunspot-numbers and the annual range of the magnetic declination at Pilar 1905-1930, and the harmonic curve of the secular variation of the magnetic declination in Pilar 1905-1930.]

Muramoto, A. Outstanding features of the results of the magnetic observations at the Zinsen, Taihoku, Otomari, and Palau observatories. Proc. Fifth Pacific Sci. Cong., Canada, 1933, v. 3, Toronto, Univ. Press, 1934 (1831-1834).

Parkinson, W. C. Terrestrial magnetism at the Watheroo Magnetic Observatory. Proc. Fifth Pacific Sci. Cong., Canada, 1933, v. 3, Toronto, Univ. Press, 1934 (1719-1720).

Peters, J. H. The Honolulu Magnetic Observatory of the United States Coast and Geodetic Survey. Proc. Fifth Pacific Sci. Cong., Canada, 1933, v. 3, Toronto, Univ. Press, 1934 (1937-1942).

Procopiu, St. Détermination des éléments magnétiques en Roumanie et cartes magnétiques de la Roumanie, dressées pour le 1er juillet 1934. Jassy, Ann. sc. Univ., T. 21, fasc. 1-4, 1935 (270-302 avec 3 pls.). [Values of the magnetic elements were determined at 27 stations in Moldavia, Bessarabia, and Bucovina during 1931 and 1934. On the basis of these values together with those of the two previous surveys of 1890 and 1901, all reduced to the same epoch (1934.5), magnetic charts of declination, inclination, and horizontal intensity were constructed for Rumania for that epoch.]

Procopiu, St., G. Vasiliu, et N. Calinicenco. Mesures magnétiques en Bessarabie et Bucovine en 1933 et 1934. Bucuresti, Bull. Sect. Sci., Acad. Roumaine, 17e année, No. 1-2, 1934 (1-6).

Reich, H. Ueber die neue magnetische Aufnahme des Deutschen Reiches. Beitr. angew. Geophysik, Leipzig, Bd. 5, Heft 1, 1935 (133-140).

Rose, N. V. General magnetic survey of U. S. S. R. Trans. 7th Conf. Baltic Geod. Comm., Leningrad, No. 4, 1934 (3-9) [Russian text]. Abstract, Geophys. Abstr., Washington D. C., No. 72, Apr. 1935 (1408-1409).

Rössiger, M. Die Aufzeichnung der erdmagnetischen Elementarwellen mittels des Kriechgalvanometers. Zs. Geophysik, Braunschweig, Jahrg. 11, Heft 1/2, 1935 (75-82).

Sandoval, R. O. On a remarkable magnetic wave. Proc. Fifth Pacific Sci. Cong., Canada, 1933, v. 3, Toronto, Univ. Press, 1934 (1861-1862).

Sodankylä. Ergebnisse der Beobachtungen des Magnetischen Observatoriums zu Sodankylä im Jahre 1931. Von E. Sucksdorff. (Veröff. Mag. Observatoriums der Finnischen Akad. Wiss. Nr. 18.) Kuopio, Osakeyhtiö Kirjapaino Sanan Valta, 1935 (57 mit 4. Tafeln). 29 cm.

Stagg, J. M. The diurnal variation of magnetic disturbance in high latitudes. London, Proc. R. Soc., A, v. 149, No. 867, 1935 (298-311).

Stonyhurst College Observatory. Results of geophysical and solar observations, 1934. With report and notes of the Director, Rev. J. P. Rowland. Blackburn, Thomas Briggs, Ltd., 1935 (xx+42). 18 cm.

Ulrich, F. P. Outstanding features of magnetic results from the Sitka Magnetic Observatory. Proc. Fifth Pacific Sci. Cong., Canada, 1933, v. 3, Toronto, Univ. Press, 1934 (1871-1882).

VISSER, S. W. On anomalies of terrestrial magnetism. Proc. Fifth Pacific Sci. Cong., Canada, 1933, v. 3, Toronto, Univ. Press, 1934 (1883-1891).

Magnetic anomalies in the Netherlands East Indies. Proc. Fifth Pacific Sci. Cong., Canada, 1933, v. 3, Toronto, Univ. Press, 1934 (1893-1907).

WADSWORTH, J. Terrestrial magnetism in Samoa. Proc. Fifth Pacific Sci. Cong., Canada, 1933, v. 3, Toronto, Univ. Press, 1934 (1841-1843).

B—Terrestrial and Cosmical Electricity

ACKEMANN, M. Ein neuer Sekundäreffekt der kosmischen Ultrastrahlung. Zs. Physik, Berlin, Bd. 94, Heft 5/6, 1935 (303-316).

ALVAREZ, L. W. Artificial radioactivity induced by cosmic rays. Phys. Rev., Lancaster, Pa., v. 47, No. 4, 1935 (120-121).

ANDERSON, C. D., AND S. H. NEDDERMEYER. Fundamental processes in the absorption of cosmic-ray electrons and photons. Internat. Conf. Phys., London, 1934, Papers and Discussions, v. 1, 1935 (171-187).

AUGER, P. Sur l'absorption du rayonnement cosmique. Paris, C.-R. Acad. sci., T. 200, No. 9, 1935 (739-742).

AUGER, P., AND L. LEPRINCE-RINGUET. The latitude-effect for cosmic rays. Internat. Conf. Phys., London, 1934, Papers and Discussions, v. 1, 1935 (188-194).

Some measurements of cosmic radiation at high altitudes. Internat. Conf. Phys., London, 1934, Papers and Discussions, v.1, 1935 (195-198).

AUGER, P., ET A. ROSENBERG. Sur les effets secondaires des rayons cosmiques. Paris, C.-R. Acad. sci., T. 200, No. 6, 1935 (447-449).

AUGER, P., A. ROSENBERG, ET F. BERTEIN. Caractère des deux composantes corpusculaires du rayonnement cosmique. Paris, C.-R. Acad. sci., T. 200, No. 13, 1935 (1022-1024).

BARANOV, V. I., and M. D. KRAVTCHENKO. Concerning rapid fluctuations of the concentration of easily movable ions in the air. J. Geophys., Leningrad, v. 4, No. 3, 1934 (379-385). [Russian text with English summary.]

BARBIER, D., D. CHALONGE, ET E. VASSY. Effet de la température de la stratosphère sur le spectre de l'ozone. Paris, C.-R., Acad sci., T. 200, No. 13, 1935 (1063-1065).

BARNÓTHY, J. AND M. FORRÓ. Cosmic rays from Nova Herculis? Nature, London, v. 135, Apr. 20, 1935 (618).

BEALS, C. S. Audibility of the aurora and its appearance at low atmospheric levels. Proc. Fifth Pacific Sci. Cong., Canada, 1933, v. 2, Toronto, Univ. Press, 1934 (1083-1088).

BENNETT, R. D., G. S. BROWN, AND H. A. RAHMEL. Frequency and magnitude of cosmic-ray bursts as a function of elevation. Phys. Rev., Lancaster, Pa., v. 47, No. 6, 1935 (437-443).

BLACKETT, P. M. S. The absorption of cosmic rays. Internat. Conf. Phys., London, 1934, Papers and Discussions, v. 1, 1935 (199-205).

BOSSOLASCO, M. Ueber die Anzahl der Kondensationskerne in Mogadischu. Beitr. Geophysik, Leipzig, Bd. 44, Heft 1, 1935 (1-15).

BOWEN, I. S., R. A. MILLIKAN, AND H. V. NEHER. A very high altitude survey of the effect of latitude upon cosmic-ray intensities and an attempt at a general interpretation of cosmic-ray phenomena. Internat. Conf. Phys., London, 1934, Papers and Discussions, v. 1, 1935 (206-224).

CHAPMAN, S. The ionized regions in the upper atmosphere. Proc. Fifth Pacific Sci. Cong., Canada, 1933, v. 3, Toronto, Univ. Press, 1934 (2181-2184).

CHVOSTIKOV, I., AND A. LEBEDEV. Intensity variation of the auroral green line in the night sky. Leningrad, C.-R. Acad. sci., v. 1, No. 2/3, 1935 (118-124). [Russian and English text.]

CLAY, J. Ionization by cosmic and radioactive radiation at different gas pressures; the influence of the wall of the vessel; disintegration of heavy atoms by cosmic rays. Physica, The Hague, v. 2, No. 2, 1935 (111-124).

Results of the Dutch Cosmic-Ray Expedition 1933: V. The variation of the radiation with atmospheric pressure; a barometer paradox; the influence of clouds. Physica, The Hague, v. 2, No. 3, 1935 (189-195).

Results of the Dutch Cosmic-Ray Expedition 1933: VI. The variation of the penetrative power of the radiation with magnetic latitude; the energy-distribution of the particles. Physica, The Hague, v. 2 No. 4, 1935 (299-308).

Clay, J., and P. M. van Alphen. Results of the Dutch Cosmic-Ray Expedition 1933: IV. Variation of the intensity of the radiation at different heights. Physica, The Hague, v. 2, No. 3, 1935 (183-188).

Compton, A. H. Cosmic rays. Nature, London, v. 135, May 4, 1935 (695-698).

Compton, A. H., and R. D. Bennett. A study of cosmic-ray bursts at different altitudes. Internat. Conf. Phys., London, 1934, Papers and Discussions, v. 1, 1935 (225).

Cravath, A. M., and L. B. Loeb. The mechanism of the high velocity of propagation of lightning discharges. Physics, Lancaster, Pa., v. 6, No. 4, 1935 (125-127).

Danforth, W. E., and W. F. G. Swann. On the electrostatic deflection of cosmic radiation. Phys. Rev., Lancaster, Pa., v. 47, No. 5, 1935 (421).

Deutschländer, H. Verfahren zur Registrierung der elektrischen Raumladungsdichte der Luft mittels Potentialmessungen in der Nähe von Gebäuden. Beitr. Geophysik, Leipzig, Bd. 44, Heft 1, 1935 (16-39).

Dines, L. H. G. The potential acquired in the natural electric field by a vertical rod standing on the ground, insulated at the bottom and carrying a collector at the top. London, Proc. Phys. Soc., v. 47, No. 259, 1935 (218-234).

Eckersley, T. L. Collision-frequency and molecular density in the F_1-layer of the ionosphere. Nature, London, v. 135, Mar. 16, 1935 (435).

Fink, D. G. Methods of electrical prospecting. Elec. Eng., New York, N. Y., v. 54, No. 3, 1935 (293-296).

Flach, E. Ergebnisse von Freiluftionen-Untersuchungen im westsächsischen Mittelgebirge. Bioklim. Beibl., Braunschweig, Jahrg. 2, Heft 1, 1935 (12-21).

Gibrat, R., et G. Viel. Sur la relation entre les conductibilités électriques de l'air et le danger provenant de la foudre. Paris, C.-R. Acad. sci., T. 200, No. 14, 1935 (1233-1235).

Giorgi. Observation d'une foudre globulaire. Paris, C.-R. Acad. sci., T. 200, No. 8, 1935 (690). [Note regarding a case of ball lightning observed in Corsica in September 1875 or 1876.]

Gott, J. P. Phenomena attending the splashing of water in an electric field. Cambridge, Proc. Phil. Soc., v. 31, Pt. 1, 1935 (85-93).

Grieger, H. Ein neuer Apparat zur Messung des Ionenspektrums. Zs. Instrumentenk., Berlin, Jahrg. 55, Heft 3, 1935 (116-122).

Guizonnier, R. Phase de la composante diurne du gradient de potentiel électrique terrestre. Paris, C.-R. Acad. sci., T. 200, No. 10, 1935 (852-853).

Amplitude de la composante semi-diurne du gradient de potentiel électrique terrestre et activité solaire. Paris, C.-R. Acad. sci., T. 200, No. 14, 1935 (1235-1236).

Harang, L., and L. Vegard. Interferometer-measurements of the red auroral line. Nature, London, v. 135, Apr. 6, 1935 (542-543).

Hess, V. F. Entgegnung auf die Abhandlung von Arthur Wagner: "Kritische Bemerkungen zum täglichen Gang der kosmischen Ultrastrahlung (nach Aufzeichnungen auf dem Hafelekar)." Wien, SitzBer. Ak. Wiss., Abt. IIa, v. 144, No. 1 und 2, 1935 (53-64).

Hess, V. F., and R. Steinmaurer. Cosmic rays from Nova Herculis? Nature, London, v. 135, Apr. 20, 1935 (617-618).

Hoffman, G. The connexion between cosmic radiation and atomic disintegration. Internat. Conf. Phys., London, 1934, Papers and Discussions, v. 1, 1935 (226-232).

Hogg, A. R. Continuous observations of the rate of production of small ions in the atmosphere. Beitr. Geophysik, Leipzig, Bd. 43, Heft 4, 1935 (359-378).

HUMMEL, J. N. Unterlagen der geoelektrischen Aufschliessungsmethoden. Beitr. angew. Geophysik, Leipzig, Bd. 5, Heft 1, 1935 (32-132).

Die elektrische Leitfähigkeit von Aggregaten in bezug auf die Leitfähigkeiten ihrer Bestandteile. Zs. Geophysik, Braunschweig, Jahrg. 11, Heft 1/2, 1935 (92-95). [Auf die Leitfähigkeit der Gesteine sind sechs Faktoren von Einfluss: 1. Das Widerstandsverhältnis der Bestandteile; 2. die Raumteile der Bestandteile; 3. die Gleichberechtigung oder Nichtgleichberechtigung der Bestandteile; 4. die Gestalt der Bausteine; 5. die Orientierung der Bausteine gegen die Stromrichtung und 6. die Anordnung der Bausteine im Raume.]

JOHNSON, T. H. Evidence for a positron-negatron component of the primary cosmic radiation. Phys. Rev., Lancaster, Pa., v. 47, No. 4, 1935 (318-319).

JOLIOT, F., ET L. KOWARSKI. Sur la production d'un rayonnement d'énergie comparable à celle des rayons cosmiques mous. Paris, C.-R. Acad. sci., T. 200, No. 10, 1935 (824-827).

JUDSON, E. B. Ionosphere studies. Phys. Rev., Lancaster, Pa., v. 47, No. 6, 1935 (509).

KASSNER, C. Gewitter, Donner, und Wetterleuchten in Potsdam. Met. Zs., Braunschweig, Bd. 52, Heft 3, 1935 (114-116).

KRAVCHENKO, M. D. The real number of light-ions according to the observations in Kuchino. J. Geophys., Leningrad, v. 4, No. 3, 1934 (386-397). [Russian text with English summary.]

LANDSBERG, H. Observations of condensation-nuclei in the atmosphere. Washington, D. C., Mon. Weath. Rev., v. 62, No. 12, 1934 (442-445). [Summary: The results of 1,051 observations of condensation-nuclei at four different locations are given. Connections between the number of nuclei and convection and turbulence are shown; and correlations between nuclear counts, relative humidity, visibility, and wind are mentioned. The relationship between nuclei-contents and air-mass is discussed, and the former is found to be a valuable help in air-mass analysis. The effect of nuclei on human health, and their absorption of radioactive emanation, principally with the occurrence of surfaces of subsidence, are discussed.]

LEMAITRE, G., M. S. VALLARTA, AND L. BOUCKAERT. On the north-south asymmetry of cosmic radiation. Phys. Rev., Lancaster, Pa., v. 47, No. 6, 1935 (434-436).

LEPRINCE-RINGUET, L. Los rayos cósmicos. Ibérica, Barcelona, Año 22, Núm. 1059, 1935 (105-111).

McCREA, W. H. Cosmic rays and novae. Nature, London, v. 135, Mar. 9, 1935 (371-372).

MILLIKAN, R. A. What to believe about cosmic rays. Science, New York, N. Y., v. 81, Mar. 1, 1935 (211-215).

MILLINGTON, G. Ionization-charts of the upper atmosphere. Part II. London, Proc. Phys. Soc., v. 47, No. 259, 1935 (263-276). [This paper discusses an envelope-correction to some ionization-charts previously published, and in addition presents the new charts on a circular projection to help in the study of conditions in the polar regions.]

MONTGOMERY, C. G., AND D. D. MONTGOMERY. The variation with altitude of the production of bursts of cosmic-ray ionization. Phys. Rev., Lancaster, Pa., v. 47, No. 6, 1935 (429-434).

MONTGOMERY, C. G., D. D. MONTGOMERY, AND W. F. G. SWANN. The enhancement of cosmic-ray nuclear-bursts by the presence of subsidiary material. Phys. Rev., Lancaster, Pa., v. 47, No. 6, 1935 (512-513).

MORRIS, W. G. Lightning storms and fires on the national forests of Oregon and Washington. Washington, D. C., Mon. Weath. Rev., v. 62, No. 10, 1934 (370-375).

NAKAYA, U., AND T. TEREDA. On the electrification of dust-particles blown by an air-blast. Phil. Mag., London, v. 19, No. 124, 1935 (115-123).

NEHER, H. V. The east-west and longitude effects. Phys. Rev., Lancaster, Pa., v. 47, No. 5, 1935 (417-418).

Neuberger, H. Zur Methodik der Kernzählung. Met. Zs., Braunschweig, Bd. 52, Heft 3, 1935 (118-119).

Nolan, J. J., and V. H. Guerrini. Atmospheric condensation-nuclei. Nature, London, v. 135, Apr. 27, 1935 (654).

Parkinson, W. C. Problems of terrestrial electricity which future observations in the Pacific Region will help to solve. Proc. Fifth Pacific Sci. Cong., Canada, 1933, v. 3, Toronto, Univ. Press, 1934 (1845-1852).

Pickering, W. H. A note on the production of cosmic-ray showers. Phys. Rev., Lancaster, Pa., v. 47, No. 5, 1935 (423).

Priebsch, J. A. Zur statistischen Ermittlung des Barometereffekts der Ultrastrahlung. Wien, SitzBer. Ak. Wiss., Abt. IIa, Bd. 144, Heft 1 und 2, 1935 (65-75).

Rossi, B. Some results arising from the study of cosmic rays. Internat. Conf. Phys., London, 1934, Papers and Discussions, v. 1, 1935 (233-246).

Sawyer, J. H., Jr. The secondary and tertiary particles produced by cosmic rays. Phys. Rev., Lancaster, Pa., v. 47, No. 7, 1935 (515-521).

Shower producing cosmic-ray primaries. Phys. Rev., Lancaster, Pa., v. 47, No. 8, 1935 (635).

Schlumberger, C., M. Schlumberger, et P. Charrin. Etudes géophysiques par les méthodes électriques in U. S. S. R. Rev. Pétrolifère, Paris, No. 614, 1935 (77-83); No. 615, 1935 (110-116). Abstract, Geophys. Abstr., Washington, D. C., No. 72, Apr., 1935 (1413-1414).

Schlumberger, C., M. Schlumberger, et H. G. Doll. L'exploration électrique des sondages. Rev. Pétrolifère, Paris, No. 609, 1934 (1525-1530); No. 610, 1934 (1557-1562).

Scholz, J. Luftelektrische Parallelmessungen auf zwei voneinander entfernten arktischen Stationen. Met. Zs., Braunschweig, Bd. 52, Heft 2, 1935 (54-57).

Das luftelektrische Forschungslaboratorium auf Franz-Josephs-Land. Met. Zs., Braunschweig, Bd. 52, Heft 3, 1935 (110-113).

Kernzahlmessungen auf Franz-Josephs-Land. Beitr. Geophysik, Leipzig, Bd. 43, Heft 4, 1935 (419-423).

Ergebnisse der Messungen des Potentialgradienten und vertikalen Leitungsstromes auf Franz-Josephs-Land. Beitr. Geophysik, Leipzig, Bd. 43, Heft 4, 1935 (424-436).

Messungen der Ionenzahlen auf Franz-Josephs-Land unter gleichzeitiger Bestimmung von Potentialgradient und vertikalem Leitungsstrom. Beitr. Geophysik, Leipzig, Bd. 44, Heft 1, 1935 (86-97).

Untersuchungen von Apparaten, die die Bestimmung des luftelektrischen Potentialgefälles gestatten, und ihrer Wirkungsweise unter extremen aüsseren Bedingungen. Zs. Instrumentenk., Berlin, Jahrg. 55, Heft 4, 1935 (157-166).

Scrase, F. J. The charged and uncharged nuclei in the atmosphere and their part in atmospheric ionisation. London, Met. Office, Geophys. Mem., No. 64, 1935 (15 with 1 pl.). 31 cm.

Shiratori, K. Atmospheric electrical conductivity in Formosa. Proc. Fifth Pacific Sci. Cong., Canada, 1933, v. 3, Toronto, Univ. Press, 1934 (1703-1718).

Stevens, W. R. Meteorological conditions preceding thunderstorms on the national forests. Washington, D. C., Mon. Weath. Rev., v. 62, No. 10, 1934 (366-370).

Stevenson, A. F. On the theoretical determination of earth-resistance from surface potential measurements. Phil. Mag., London, v. 19, No. 125, 1935 (297-306).

Stevenson, E. C., and T. H. Johnson. Coincidence-counter studies of the variation of intensities of cosmic-ray showers and vertical rays with barometric pressure. Phys. Rev., Lancaster, Pa., v. 47, No. 8, 1935 (578-580).

Street, J. C., and R. T. Young. Shower-groups in the cosmic radiation. Phys. Rev., Lancaster, Pa., v. 47, No. 7, 1935 (572-573).

Swann, W. F. G. The nature of the cosmic radiation. Phys. Rev., Lancaster, Pa., v. 47, No. 8, 1935 (575-577).

THELLIER, O. Mesures simultanées de divers éléments d'électricité atmosphérique. Paris, C.-R. Acad. sci, T. 200, No. 13, 1935 (1124-1127).

VALLARTA, M. S. On the longitude effect of cosmic radiation. Phys. Rev., Lancaster, Pa., v. 47, No. 9, 1935 (647-651).

WADSWORTH, J. Atmospheric electricity in Samoa. Proc. Fifth Pacific Sci. Cong., Canada, 1933, v. 3, Toronto, Univ. Press, 1934 (1691-1696).

WALKE, H. J. Absorption of cosmic rays. Nature, London, v. 135, Mar. 23, 1935 (472).

WALTER, B. Ueber die von B. F. J. Schonland und H. Collens entdeckten neuartigen Vorgänge in der Blitzbahn. Ann. Physik, Leipzig, Bd. 32, Heft 4, 1935 (421-424)

WRIGHT, H. L. Small ions and condensation-nuclei at Eskdalemuir. London, Q. J. R. Met. Soc., v. 61, No. 258, 1935 (93-95).

C—Miscellaneous

APPLETON, E. V. A method of measuring the collisional frequency of electrons in the ionosphere. Nature, London, v. 135, Apr. 20, 1935 (618-619).

Progress in radio communication. Nature, London, v. 135, May 4, 1935 (745-747).

BARNETT, S. J. Gyromagnetic and electron-inertia effects. Rev. Modern Phys., Lancaster, Pa., v. 7, No. 2, 1935 (129-166).

BURROWS, C. R. Radio propagation over spherical earth. New York, Proc. Inst. Radio Eng., v. 23, No. 5, 1935 (470-480).

CHELTENHAM MAGNETIC OBSERVATORY. Celebration of completion of one-third century of continuous observations, October 10, 1934. Washington, D. C., U. S. Coast Geod. Surv., Field Eng. Bull., No. 8, 1934 (83-87).

DÜLL, T., UND B. DÜLL. Neue Untersuchungen über die Beziehungen zwischen der Zahl der täglichen Todesfälle und dem magnetischen Störungscharakter. Bioklim. Beibl., Braunschweig, Jahrg. 2, Heft 1, 1935 (24-31).

GOODALL, W. M. The ionosphere. Bell. Lab. Rec., New York, N. Y., v. 13, No. 7, 1935 (194-199). [Popular article on the ionosphere.]

HADFIELD, R. Special steels. Nature, London, v. 135, May 4, 1935 (741-745).

HEIDKE, P. Die russische Beteiligung am Zweiten Internationalen Polarjahr 1932-33. Ann. Hydrogr., Berlin, Jahrg. 62, Heft 12, 1934 (482-488). [Contains list of principal Russian Polar-Year stations, 92 in all, with indication of observations made at each, together with list of the 10 expeditions at sea with pertinent notes.]

HOPPE, J. Das Geheimnis des grünen Sonnenstrahls. Weltall, Berlin, Jahrg. 34, Heft 3, 1934 (43-44).

HULBURT, E. O. Radio transmission and the ionosphere. Proc. Fifth Pacific Sci. Cong., Canada, 1933, v. 3, Toronto, Univ. Press, 1934 (2185-2193).

Solar ultra-violet light or corpuscular radiation as the cause of certain phenomena in the upper atmosphere. Proc. Fifth Pacific Sci. Cong., Canada, 1933, v. 3, Toronto, Univ. Press, 1934 (2229-2231).

INTERNATIONAL CONFERENCE ON PHYSICS. International Conference on Physics, London, 1934. A joint conference organized by the International Union of Pure and Applied Physics and the Physical Society. Paper and discussions. Vol. I. Nuclear physics. Vol. II. The solid state of matter. Cambridge, University Press, 1935, 2 v. 26 cm.

International Conference on Physics, London, 1934. International Union of Pure and Applied Physics. Reports on symbols, units, and nomenclature. Approved by the General Assembly of the Union at its meeting in London on 5th October 1934. Published by the Physical Society. Cambridge, University Press, 1935 (v+40). 26 cm.

IONESCU, T., ET C. MIHUL. Propagation des ondes électriques dans le champ magnétique terrestre. Paris, C.-R. Acad. sci., T. 200, No. 15, 1935 (1301-1303).

ISRAËL, H., UND F. BECKER. Die Bodenemanation in der Umgebung der Bad Nauheimer Quellenspalte. Beitr. Geophysik, Leipzig, Bd. 44, Heft 1, 1935 (40-55).

JEFFREYS, H. Constitution of the Earth. Nature, London, v. 135, May 4, 1935 (678-680).

JONES, H. S. General astronomy. Second edition. London, Edward Arnold and Co., 1934 (viii+437 with 27 pls.). 22 cm. [In chapter 7, on the Sun, sunspots and terrestrial relationships and the magnetic field of sunspots are discussed.]

Kirby, S. S., T. R. Gilliland, and E. B. Judson. Ionosphere studies during partial solar eclipse. Phys. Rev., Lancaster, Pa., v. 47, No. 8, 1935 (632-633).

Kirby, S. S., and E. B. Judson. Recent studies of the ionosphere. Washington, D. C., J. Res. Bur. Stan., v. 14, No. 4, 1935 (469 486).

Lugeon, J. Tables crépusculaires donnant l'altitude au zénith des rayons rasants du Soleil pour toutes les latitudes de degré en degré. Warszawa, Institut National Météorologique, 1934 (xxxviii+438). 26 cm. [These tables designed largely for use in interpreting observations of the upper atmosphere and the ionosphere as well as in the study of radiotelegraphic transmission, are preceded by an introduction explaining their elaboration and use.]

Macek, O., und W. Illing. Messung des Emanationsgehaltes der Luft nach der Spitzenmethode. Beitr. Geophysik, Leipzig, Bd. 43, Heft 4, 1935 (388-418).

Martyn, D. F. The propagation of medium radio waves in the ionosphere. London, Proc. Phys. Soc., v. 47, No. 259, 1935 (323-339).

Dispersion- and absorption-curves for radio wave propagation in the ionosphere according to the magneto-ionic theory. Phil. Mag., London, v. 19, No. 126, 1935 (376-388).

Martyn, D. F., R. O. Cherry, and A. L. Green. Long-distance observations of radio waves of medium frequencies. London, Proc. Phys. Soc., v. 47, No. 259, 1935 (340-351).

Meetham, A. R., and G. M. B. Dobson. The vertical distribution of atmospheric ozone in high latitudes. London, Proc. R. Soc., v. 148, No. 865, 1935 (598-603). [Observations to determine the vertical distribution of ozone in the atmosphere—similar to those recently made at Arosa—have been carried out at Tromsö, Norway, in May and June, 1934. They show that the average height of the ozone is very slightly lower in the higher latitude, and indicate that at Tromsö the ozone is more concentrated in a region centered at a height of 21 km above sea-level, whereas in Switzerland it is more uniformly distributed through the lower 30 km.]

Meinesz, F. A. V. Gravity-expeditions at sea 1923-1932. Vol. II. Report of the gravity-expedition in the Atlantic of 1932 and the interpretation of the results. With the collaboration of J. H. F. Umbgrove and Ph. H. Kuenen. Delft, N. V. Technische Beokhandel en Drukkerij J. Waltman Jr., 1934 (208 with 5 pls.). 31 cm. [Publication of the Netherlands Geodetic Commission.]

Mimno, H. R., and P. H. Wang. Continuous records of the heights of the Kennelly-Heaviside layers. Proc. Fifth Pacific Sci. Cong., Canada, 1933, v. 3, Toronto, Univ. Press, 1934 (2195-2208).

Pacific Science Association. Proceedings of the Fifth Pacific Science Congress. Toronto, University of Toronto Press, 1934, 5 v. 24 cm. [This Congress was held in Victoria and Vancouver, British Columbia, Canada, in June, 1933.]

Pawsey, J. L. Further investigations of the amplitude variations of downcoming wireless waves. Cambridge, Proc. Phil. Soc., v. 31, Pt. 1, 1935 (125-144).

Schafer, J. P., and W. M. Goodall. Some recent studies of the ionization of the ionosphere. Proc. Fifth Pacific Sci. Cong., Canada, 1933, v. 3, Toronto, Univ. Press, 1934 (2171-2179).

Ionosphere measurements during the partial eclipse of the Sun of February 3, 1935. Nature, London, v. 135, Mar. 9, 1935 (393-394).

Schmid, F. La lumière zodicale du 47° latitude nord au 39° latitude sud. Arch. Sci. Phys., Genève, v. 16, 1934 (279-282).

Störmer, C. Measurements of luminous night clouds in Norway 1933 and 1934. Astroph. Norvegica, Oslo, v. 1, No. 3, 1935 (87-114 with 17 pls.).

Thellier, E. Appareil d'induction pour la mesure des faibles moments magnétiques. Paris, C.-R. Acad. sci., T. 200, No. 9, 1935 (736-738).

Toshniwal, G. R. Three-fold magneto-ionic splitting of the radio echoes reflected from the ionosphere. Nature, London, v. 135, Mar. 23, 1935 (471-472).

Whipple, F. J. W. Progress of knowledge of the upper air. Nature, London, v. 135, May 4, 1935 (698-702). [Stratosphere, transmission of sounds to great distances, ozone, aurora.]

Wolff, G. Ueber die Abhängigkeit des Gesundheitszustandes von plötzlichen Eruptionen auf der Sonne und die Existenz einer 27 tägigen Periode in den Sterbefällen Naturw., Berlin, Jahrg. 23, Heft 13, 1935 (210-213).

Worley, F. P. The green flash at sunset. Nature, London, v. 135, May 4, 1935 (760).

AGENDA FOR THE WARSAW MEETING OF THE INTERNATIONAL COMMISSION OF TERRESTRIAL MAGNETISM AND ATMOSPHERC ELECTRICITY, SEPTEMBER 1935[1]

By D. la Cour

Following the call of President Maurain the first session of the Commission will take place at Warsaw September 2, 1935, at 10 a. m. in the Staszic Palace, Nowy Swiat 72 (entrance by main door facing Copernik Place).

Professor Kalinowski has kindly invited the members of the Commission to visit the Swider Observatory, near Warsaw. Since a general excursion to the Jablonna Meteorological Observatory is scheduled for September 8, it is recommended that those present at Warsaw September 1, visit Swider on that day. Information as to the time of departure for Swider will be posted at the Bureau of the Conference.

The following items have been proposed for the agenda of the Warsaw meeting:

(1) Report of the President
(2) Revision of the list of members
(3) Innsbruck resolutions
(4) Report by G. van Dijk on the publication of the "Caractère magnétique de chaque jour"
(5) Report by S. Chapman on his work relative to the effect of the Moon on geophysical phenomena
(6) Communications on the activity of various observatories since the meeting at Innsbruck
(7) Publication of the diurnal variation during disturbed days; proposed by G. C. Simpson
(8) Publication of the diurnal variation during days according to zonal time instead of days according to Greenwich time; proposed by G. C. Simpson
(9) Proposal relative to the non-cyclic change; proposed by G. C. Simpson
(10) Frequent comparisons by means of instruments sent by mail; memorandum by D. la Cour
(11) Magnetic character of each half-day; proposed by G. Fanselau
(12) Magnetic activity deduced from the mean values instead of instantaneous values; proposed by G. Fanselau
(13) Determination of the effect of the co-oscillation of the air; proposed by G. Fanselau
(14) Publication of the secular variation in the magnetic yearbooks; proposed by G. Fanselau
(15) Designation of the unit of moment of inertia; proposed by G. Fanselau
(16) Magnetic year-books after the Polar Year; remarks by D. la Cour
(17) Centralization of magnetic data collected after the Polar Year; note by D. la Cour
(18) Net of stations obtaining continuous rapid magnetic registration; proposed by D. la Cour

Proposals by N. Rose:

(19) Organization of an international service of unified magnetic charts of the Globe
(20) Unification of magnetic publications
(21) Organization of an international service of information on the variations of the magnetic field

[1]Abstracted by H. D. Harradon from Secretary la Cour's circular letter No. 121, June 4, 1935.

Proposals by J. A. Fleming:

(22) Uniform methods and codes to adequately describe magnetic disturbances and perturbations

(23) Scale-value determinations at observatories: (*a*) Magnetic methods and reliability, (*b*) electrical methods, facility and reliability

(24) Need of improved design of variometer for recording vertical intensity at observatories

(25) Advisability of publishing hourly ranges in declination for magnetic observatories in or near the auroral zone

(26) Desirability of one or more additional non-magnetic vessels for continuation of magnetic, electric, and oceanographic observations at sea, since the extent of the oceans makes more intensive work of this kind essential to advance the understanding of terrestrial magnetism and electricity

(27) Charts of lines of equal auroral frequency untilizing up-to-date information

(28) Research in the ionosphere: (*a*) Instruments and methods; (*b*) compilations and reductions

(29) Methods of observation and procedure and encouragement for additional records of diurnal variation in rate of atmospheric ionization near the ground

(30) Desirability of more measurements of atmospheric-electric elements in the free atmosphere to better establish general character of variation with altitude and to ascertain the extent of electric stratification and reversals and the dependence of these on meteorological factors

(31) Desirability of further atmospheric-electric observations at sea to determine characteristics of diurnal, seasonal, and possible geographical variations in fuller detail and with better quantitative precision

(32) Desirability of short series of earth-current observations at numerous well-distributed secondary stations to roughly determine the general characteristics of the diurnal-variation vectors and their dependence on structural features of the Earth's crust

(33) Adoption of definite convention as regards the sense of earth-current components, preferably positive for south to north and for west to east

Various questions

Among the questions proposed for the agenda are some which seem rather to concern the Association of Terrestrial Magnetism and Electricity. Although it is necessary to avoid a duplication of work, the Bureau did not wish to omit these proposals from the provisional list of subjects to be discussed at Warsaw for it is deemed just to leave to the Commission the decision as to which questions shall be referred to the Association.

The members of the Commission may send to Copenhagen all the documents they desire to have multigraphed for distribution at Warsaw. It is however suggested that, after August 1, members themselves multigraph, as far as possible, the documents they desire distributed before the opening of the sessions.

Members intending to attend the meeting at Warsaw are requested to so notify the Bureau of the Commission, Toldbodvej 15, Copenhagen.

Terrestrial Magnetism *and* *Atmospheric Electricity*

VOLUME 40 SEPTEMBER, 1935 No. 3

EARTH-POTENTIAL MEASUREMENTS MADE DURING THE INTERNATIONAL POLAR YEAR

BY G. C. SOUTHWORTH

Abstract—Data are presented covering the normal diurnal variation of earth-potentials as measured at about a dozen different points, mostly in eastern United States. These data are arranged in graphical form for the convenience of the casual reader and also in numerical form for the use of the correlator. The data for Wyanet (Illinois), Houlton (Maine), and New York (New York) are based on nearly continuous recordings extending over a period of one or two years. This period includes the International Polar Year. At other points less extensive data were taken. These show the general characteristics peculiar to the location in question.

The data taken at Wyanet, Houlton, and New York have been analyzed for harmonic content. At New York the fundamental and to a large extent the harmonics also, are directed along a northwest-southeast line. At Wyanet and Houlton these components tend to rotate with time. The pronounced directive effect noted near New York appears to prevail rather generally along the eastern part of the United States from Massachusetts to Florida and possibly into Cuba. The rotary effect noted in the Houlton and Wyanet data is also found in data taken in the southern part of the Mississippi Valley. Most of the data point toward the generally accepted view that there is a close relation between earth-resistivity and the direction and magnitude of earth-potentials. However, there are some inconsistencies noted which tend to make this less definite.

Introduction

This paper gives the results of some experimental work on earth-potentials done by the Bell System[a] during and immediately following the International Polar Year. This work had its inception more than ten years ago when studies were begun of the relation between radio transmission and various solar and terrestrial phenomena. Several papers were presented from time to time during that period.[b] It was extended somewhat in 1932 in order to assist the Polar-Year Program. This latter work was confined mainly to the measurement of normal diurnal-variation of earth-potentials. This was, of course, a field in which the Telephone Company, with its rather extended network of wire-lines, could contribute most readily.

The principle object of this paper is to make available the data so far collected. Although a few conclusions have been drawn where such were more or less obvious, it has left to others, or perhaps to a later date when more data are available, the more extended task of association with similar information in related fields. A previous paper [11] on this subject gave the more important details concerning the method

[a]The Bell System consists of the American Telephone and Telegraph Company and some 24 associated and operating companies, each doing business in a different part of the United States. The research of the Bell System is conducted mainly by a subsidiary company known as Bell Telephone Labarotories, Inc. While most of its work has to do with improvements in materials and technique, some study is also made of the various natural phenomena that affect the communication-business. The work described in this and other papers cited in the attached bibliography are examples.

[b]See attached bibliography to which the references by numbers in brackets apply.

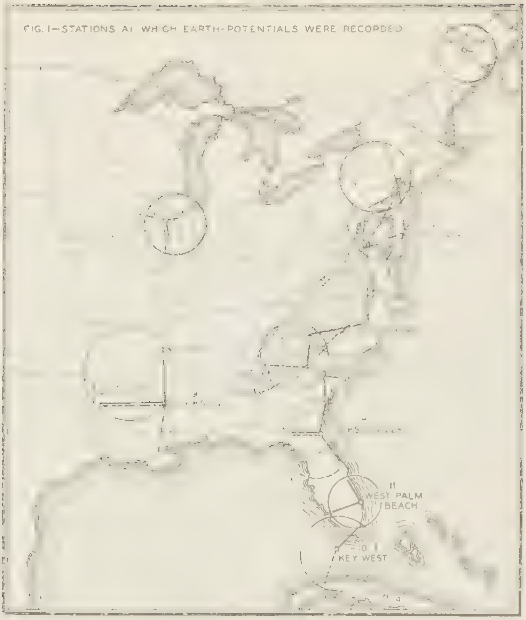
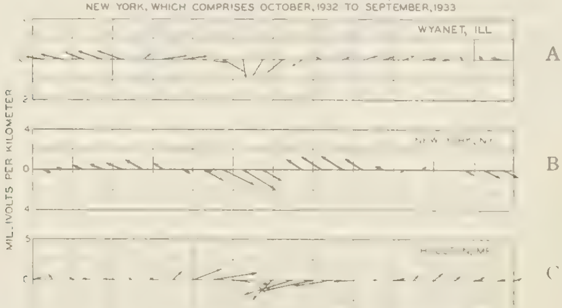


FIG. 1—STATIONS AT WHICH EARTH-POTENTIALS WERE RECORDED

of measurement and the analysis of data. As already explained in the earlier paper, this work is based on continuous records of the voltages prevailing between points in the Earth's crust, perhaps 75 kilometers

FIG. 2—DIURNAL VARIATION OF EARTH-POTENTIAL GRADIENT
VECTORS ARE MEANS FOR THE YEAR ENDING JUNE 30, 1933 EXCEPT NEW YORK, WHICH COMPRISES OCTOBER, 1932 TO SEPTEMBER, 1933



apart. These continuous records give not only the hourly means which are reported in this paper, but also some very interesting details about the earth-current disturbances that took place during the Polar Year. Reports on these disturbances are being left to some later date.

Details concerning the locations of the various observing-stations, the distances between electrodes, and the nature of the wire-lines involved are given by the map shown as Figure 1 and also by Table 1.

TABLE 1—*Information relative to observation-points*

Observation-point	Location of associated grounds: Place	Latitude north	Longitude west	Distance to grounds	Character of wire-line	Period of observation
		° ′	° ′	*km*		
yanet, Ill., (1)[a]	Wyanet, Ill.	41 22	89 35			July 1932
	Marseilles, Ill.	41 22	88 42	73 (120)[b]	Cable	to
	Petersburg, Ill.	40 01	89 40	149 (259)[b]	Cable	June 30, 1935
oulton, Me., (2)	Houlton, Me.	46 07	67 54			June 1, 1932
	Southampton, N.B.	46 01	67 16	51	Open wire	to
	Danforth, Me.	45 31	67 46	68	Open wire	Aug. 31, 1933
ew York, N. Y., (3)	Mendham, N. J.	40 46	74 40			Oct. 1, 1932
	Lebanon, Pa.	40 28	76 24	151 (159)[c]	Cable	to
	Fonda, N. Y.	42 58	74 41	243	Cable	Dec. 31, 1933
ew York, N. Y., (4)	Mendham, N. J.	40 46	74 40			Jan. 9, 1933
	Fairfax, Va.	38 44	77 21	322	Cable	to
	Cape May, N. J.	38 56	74 57	209	Cable	Mar. 1, 1933
oldsboro, N. C., (5)	Goldsboro, N. C.	35 23	78 02		Cable except	Mar. 9, 1933
	Gaffney, S. C.	35 03	81 40	330	Goldsboro	to
	McKenney, Va.	36 59	77 43	189	to Petersburg, which was open wire	Apr. 6, 1933
ape May, N. J., (6)	Cape May, N. J.	38 56	74 57			Apr. 6, 1933
	Fairfax, Va.	38 44	77 21	209	Cable	to
	McKenney, Va.	36 59	77 43	326	Cable	May 29, 1933
enmark, S. C., (7)	Fairfax, S. C.	32 57	81 15			June 22, 1933
	Charleston, S. C.	32 55	80 07	106	Open wire	to
	Savannah, Ga.	32 10	81 10	88	Open wire	Sept. 1, 1933
acksonville, Fla., (8)	Jacksonville, Fla.	30 57	81 45			
	Madison, Fla.	30 28	83 25	158	Open wire	Oct. 24, 1933 to Jan. 2, 1934
(1)	Daytona, Fla.	29 11	81 01	157	Open wire	
(2)	Savannah, Ga.	32 10	81 10	198	Open wire	Jan. 15 to Feb. 8, 1934
acksonville, Miss. (9)	Jacksonville, Miss.	32 21	90 24			
	Shreveport, La.	32 31	93 35	300	Open wire	Mar. 19 to May 21, 1934
(1)	Memphis, Tenn.	35 07	89 52	311	Open wire	
(2)	LaPlace, La.	30 04	90 30	254	Open wire	May 21 to June 22, 1934
ey West, Fla., (10)	Key West, Fla.	24 33	81 48			
	Miami, Fla.	25 46	80 12	220	Open wire	Aug. 1 to
	Havana, Cuba	23 11	82 23	170	Submarine cable	Oct. 11, 1934
est Palm Beach, Fla., (11)	W.PalmBeach,Fla.	26 46	80 03			
	Ft. Myers, Fla.	26 38	81 52	180	Open wire	Oct. 28 to
	Melbourne, Fla.	28 05	80 36	159	Open wire	Dec. 30, 1934
tlanta, Ga., (12)	Augusta, Ga.	33 28	82 10			
	Atlanta, Ga.	33 44	84 08	185	Open wire	Mar. 12 to
	Gaffney, S. C.	35 03	81 40	185	Open wire	Apr. 5, 1935

[a]Numerals refer to stations shown on map of Fig. 1.

[b]On January 1, 1934, Marseilles and Petersburg grounds were replaced by grounds located near Wilton, Iowa (41° 3′ N, 91° 00′ W) and near Gillespie, Illinois (39° 03′ N, 89° 55′ W).

[c]On September 7, 1933, the Lebanon ground was moved a short distance (40° 24′ N, 76° 29′ W).

TABLE 2—*Diurnal variation in millivolts per kilometer of earth-potential for selected quiet days*
Northward component at Wyanet, Illinois

Month	Mean values for 90° west meridian hours											
	0	1	2	3	4	5	6	7	8	9	10	11
1932–3												
Jul	0.191	0.110	0.535	0.416	0.931	0.820	0.595	0.211	−0.277	−0.876	−1.225	−0.856
Aug	−0.027	0.011	0.611	0.434	0.947	0.613	0.189	−0.011	−0.591	−1.320	−1.192	−0.906
Sep	−0.029	0.418	0.536	0.488	0.603	0.221	0.103	−0.431	−0.720	−1.029	−1.131	−0.719
Oct	0.204	0.556	0.534	0.414	0.316	0.030	0.179	0.012	−0.273	−0.593	−0.791	−0.565
Nov	0.087	0.591	0.327	0.606	0.466	−0.106	0.080	0.143	0.096	−0.459	−0.970	−1.145
Dec	0.122	0.116	0.119	0.499	0.180	−0.010	0.003	0.086	0.284	0.003	−0.476	−0.938
Jan	0.063	0.015	0.205	0.228	0.254	−0.106	0.135	0.424	0.519	−0.001	−0.811	−1.183
Feb	0.087	0.170	0.268	0.277	0.117	0.106	0.142	0.158	0.212	−0.310	−0.754	−0.785
Mar	0.261	0.165	0.146	0.554	0.229	0.033	0.117	0.223	0.030	−0.356	−0.904	−1.113
Apr	0.093	0.201	0.338	0.233	0.375	0.405	0.004	0.049	−0.255	−0.593	−1.131	−0.955
May	−0.121	0.097	0.328	0.398	0.614	0.511	0.305	0.180	−0.259	−0.802	−1.220	−0.915
Jun	0.256	0.102	0.307	0.501	0.815	0.656	0.301	0.179	−0.308	−0.795	−1.270	−1.126
Mean	0.099	0.213	0.355	0.421	0.487	0.265	0.179	0.102	−0.129	−0.594	−0.989	−0.934
1933–4												
Jul	0.140	0.169	0.372	0.461	0.421	0.309	0.216	0.061	−0.127	−0.662	−1.118	−0.779
Aug	0.194	0.139	0.474	0.530	0.455	0.262	0.290	0.152	−0.169	−0.973	−1.379	−0.937
Sep	0.235	0.024	0.395	0.627	0.113	0.190	0.201	−0.004	−0.516	−0.943	−1.266	−0.751
Oct	0.197	0.244	−0.132	0.496	0.193	−0.209	0.120	0.020	−0.206	−0.865	−1.151	−0.648
Nov[a]	0.087	0.591	0.327	0.606	0.466	−0.106	0.080	0.143	0.096	−0.459	−0.970	−1.145
Dec[a]	0.122	0.116	0.119	0.499	0.180	−0.010	0.003	0.086	0.284	0.003	−0.476	−0.938
Jan	0.128	0.084	0.344	0.521	0.375	0.381	0.304	0.301	0.199	−0.440	−1.346	−1.552
Feb	0.284	0.239	0.493	0.403	0.333	0.225	0.054	−0.115	0.014	−0.465	−0.868	−1.064
Mar	0.292	0.325	0.662	0.511	0.499	0.406	0.454	0.251	−0.241	−0.903	−1.189	−1.399
Apr	0.128	0.313	0.452	0.455	0.544	0.617	0.390	−0.140	−0.149	−1.294	−1.383	−1.047
May	0.292	0.261	0.444	0.455	0.660	0.640	0.316	0.114	−0.553	−1.098	−1.284	−0.977
Jun	0.147	0.199	0.319	0.432	0.815	0.822	0.588	0.305	−0.563	−0.808	−0.917	−1.068
Mean	0.187	0.225	0.354	0.500	0.421	0.295	0.251	0.098	−0.161	−0.743	−1.112	−1.025
1934–5												
Jul	0.185	0.359	0.405	0.466	0.690	0.860	0.681	0.062	−0.500	−1.302	−1.410	−1.166
Aug	0.003	−0.074	0.350	0.172	0.608	1.018	0.692	0.209	−0.707	−1.574	−1.533	−1.163
Sep	0.175	0.263	0.541	0.389	0.536	0.578	0.547	−0.197	−0.992	−1.366	−1.441	−0.902
Oct	0.131	0.186	0.418	0.339	0.252	0.180	0.323	0.040	−0.306	−0.839	−1.094	−0.982
Nov	0.157	0.015	0.291	0.513	0.162	0.418	0.298	−0.025	0.152	−0.858	−1.115	−0.960
Dec	−0.104	0.014	0.051	0.334	0.193	0.342	0.463	0.106	−0.018	−0.509	−1.149	−1.097
Jan	0.071	0.320	0.312	0.333	0.299	0.249	0.006	0.188	0.146	−0.498	−1.035	−1.043
Feb	0.165	0.103	0.296	0.196	0.435	0.237	0.356	0.188	0.005	−0.483	−1.192	−1.119
Mar	0.428	0.287	0.324	0.364	0.632	0.471	0.382	−0.071	−0.306	−1.042	−1.094	−1.043
Apr	0.307	0.278	0.270	0.383	0.460	0.611	0.416	0.151	−0.408	−1.115	−1.290	−0.806
May	0.325	0.239	0.312	0.337	0.861	0.595	0.474	0.129	−0.557	−1.141	−1.208	−0.824
Jun	0.259	0.037	0.254	0.415	0.890	0.935	0.818	−0.032	−0.773	−1.457	−1.527	−1.086
Mean	0.168	0.169	0.319	0.353	0.502	0.541	0.455	0.062	−0.355	−1.015	−1.257	−1.016

[a]Values for November and December of 1932.

Diurnal variation of earth-potential

The numerical data based on the hourly means for selected quiet days are given in Table 2. They are also shown in convenient graphical form in Figures 2 to 7, 13, 14, and 15. In these Figures, the lengths of the various vectors are proportional to the resultant magnitude of the potential-gradient for that hour. Their directions conform to the usual cartographic conventions. For purposes of more convenient comparison, mean hourly values for the International Polar Year for the three stations mentioned above have been plotted in Figure 2.

As pointed out in the previous paper, the data for Wyanet (Illinois) and Houlton (Maine) show a pronounced rotary effect with time whereas the potential-gradients for New York and vicinity are constrained largely to the northwest-southeast direction. This is shown by the yearly means shown in Figure 2. Wyanet is located in the north central part of the Mississippi Valley where the geological formation might be

TABLE 2—*Diurnal variation in millivolts per kilometer of earth-potential for selected quiet days*
Northward component at Wyanet, Illinois—Concluded

Month	Mean values for 90° west meridian hours												No. days
	12	13	14	15	16	17	18	19	20	21	22	23	
1932–3													
Jul	−0.692	−0.416	0.118	0.021	0.094	0.135	0.220	−0.055	−0.082	0.070	0.036	0.024	25
Aug	−0.605	−0.359	−0.174	0.177	0.257	0.314	0.074	0.180	0.235	0.262	0.184	0.356	28
Sep	−0.421	−0.115	0.457	0.366	0.172	0.019	0.091	0.200	0.374	0.293	−0.111	0.345	23
Oct	−0.398	−0.240	0.159	0.013	−0.162	−0.046	0.116	0.010	0.050	0.119	0.135	0.219	25
Nov	−0.751	−0.228	0.350	0.066	0.133	0.099	0.154	0.168	0.039	0.038	0.052	0.166	23
Dec	−0.725	−0.597	−0.139	0.065	0.303	0.298	0.363	0.212	−0.136	0.123	0.041	0.201	28
Jan	−0.953	−0.356	0.175	0.190	0.300	0.161	0.412	0.195	−0.133	0.253	0.127	−0.114	19
Feb	−0.586	−0.324	0.139	0.156	−0.044	−0.072	0.317	0.066	0.179	0.116	0.239	0.116	19
Mar	−0.748	−0.443	−0.013	0.167	0.118	0.142	0.346	0.303	0.002	0.294	0.470	−0.042	23
Apr	−0.616	−0.334	0.010	0.396	0.289	0.273	0.417	0.137	0.039	0.305	0.310	0.010	15
May	−0.864	−0.243	0.103	0.406	0.390	0.262	0.250	0.207	−0.218	0.151	0.271	0.170	23
Jun	−0.863	−0.440	−0.079	0.161	0.300	0.214	0.336	0.059	0.097	0.257	0.459	−0.104	21
Mean	−0.685	−0.341	0.092	0.182	0.179	0.149	0.258	0.140	0.037	0.190	0.184	0.112	
1933–4													
Jul	−0.639	−0.417	−0.013	0.050	0.130	0.334	0.378	−0.033	0.056	0.164	0.244	0.287	18
Aug	−0.672	−0.268	0.272	0.358	0.294	0.250	0.108	0.213	−0.027	0.122	0.150	0.154	22
Sep	−0.426	0.086	0.357	0.385	0.266	0.010	0.208	0.342	−0.054	0.093	0.084	0.347	14
Oct	−0.528	−0.102	0.241	0.176	0.042	0.130	0.219	0.429	0.056	0.327	0.454	0.494	8
Nov[a]	−0.751	−0.228	0.350	0.066	0.133	0.099	0.154	0.168	0.039	0.038	0.052	0.166	23
Dec[a]	−0.725	−0.597	−0.139	0.065	0.303	0.298	0.363	0.212	−0.136	0.123	0.041	0.201	28
Jan	−1.127	−0.487	0.123	0.427	0.434	0.140	0.233	0.422	0.240	0.134	0.134	0.026	22
Feb	−0.944	−0.698	−0.437	0.007	0.337	0.248	0.248	0.416	0.481	0.242	0.291	0.281	18
Mar	−1.160	−0.569	−0.351	0.247	0.210	0.044	0.316	0.350	0.241	0.200	0.324	0.476	18
Apr	−1.056	−0.460	−0.053	0.300	0.444	0.336	0.303	0.239	0.289	0.343	0.086	0.356	24
May	−0.735	−0.426	−0.124	0.259	0.262	0.228	0.130	0.117	0.120	0.283	0.298	0.315	20
Jun	−0.738	−0.737	−0.343	0.065	0.331	0.194	0.141	0.119	0.235	0.113	0.158	0.199	19
Mean	−0.792	−0.409	−0.010	0.200	0.265	0.193	0.233	0.249	0.128	0.182	0.193	0.275	
1934–5													
Jul	−0.663	−0.364	−0.154	0.147	0.417	0.411	0.122	0.112	0.093	0.180	0.209	0.171	19
Aug	−0.581	−0.127	0.146	0.467	0.273	0.508	0.088	0.226	0.136	0.396	0.359	0.114	16
Sep	−0.247	0.223	0.538	0.452	0.018	0.152	0.200	0.072	0.225	0.124	0.056	0.046	14
Oct	−0.365	0.185	0.279	0.253	0.073	0.185	0.031	0.215	0.248	0.093	0.018	0.129	25
Nov	−0.582	0.060	0.273	0.382	0.208	0.154	0.050	0.162	0.116	0.103	−0.106	0.119	24
Dec	−0.727	−0.037	0.245	0.453	0.466	0.338	0.126	0.181	0.158	0.025	0.091	0.062	21
Jan	−0.946	−0.543	−0.183	0.137	0.290	0.227	0.269	0.335	0.289	0.221	0.332	0.229	19
Feb	−1.029	−0.803	−0.235	0.309	0.542	0.495	0.257	0.452	0.370	0.153	0.183	0.118	19
Mar	−0.952	−0.666	−0.192	0.229	0.221	0.207	0.069	0.355	0.448	0.460	0.318	0.164	21
Apr	−0.668	−0.413	0.008	0.210	0.217	0.370	0.247	0.063	0.156	0.196	0.053	0.295	22
May	−0.904	−0.609	0.001	0.161	0.248	0.510	0.210	0.235	0.185	0.042	0.139	0.247	23
Jun	−0.795	−0.283	0.188	0.294	0.477	0.484	0.204	0.070	0.280	0.106	0.022	0.218	20
Mean	−0.705	−0.281	0.076	0.291	0.288	0.337	0.156	0.206	0.225	0.175	0.140	0.159	

[a]Values for November and December 1932.

expected to be fairly homogeneous. Houlton is north of the principal section of the Appalachian Mountains and appears also to be located in a moderately homogeneous formation. The lines connecting the electrodes for the New York measurements both cross a region known to have high earth-resistivity. This would seem to indicate that the constrained potential-gradient pattern shown by Figure 2B is of geologic origin. However, some of the data to be discussed later are not altogether in accord with this view. The mean potential-gradients as measured at Houlton, New York, and Wyanet are respectively in the approximate ratios of 10:6:2.

An attempt was made to correlate the magnitudes and directions of earth-potentials such as represented in Figures 3 to 7 with the time of day and the season of the year. This was only partially successful. For instance, the data of Figure 4 would seem to indicate that the time in the morning when the gradient is directed northward is intimately

TABLE 2—*Diurnal variation in millivolts per kilometer of earth-potential for selected quiet days*
Eastward component at Wyanet, Illinois

Month	Mean values for 90° west meridian hours											
	0	1	2	3	4	5	6	7	8	9	10	11
1932–3												
Jul	−0.105	−1.506	−1.206	−1.348	−1.468	−0.052	1.655	1.772	2.000	1.278	−0.395	−1.376
Aug	−1.298	−1.476	−1.272	−1.540	−1.687	0.707	2.278	2.117	1.748	0.651	−0.356	−1.318
Sep	−1.223	−1.359	−1.542	−1.342	−1.434	0.885	2.238	2.003	1.464	1.061	−0.229	−1.120
Oct	−1.145	−1.446	−1.528	−1.744	−1.715	0.231	1.074	1.214	1.591	0.749	0.329	−0.420
Nov	−1.410	−1.461	−1.919	−1.707	−1.868	0.338	1.006	0.548	1.815	1.472	0.907	0.277
Dec	−1.032	−1.642	−1.346	−1.500	−1.739	0.098	0.764	0.097	0.513	0.968	1.233	0.945
Jan	−1.199	−1.545	−1.480	−1.562	−1.632	0.084	0.733	0.588	0.895	1.804	1.308	0.057
Feb	−0.427	−0.507	−0.778	−0.733	−0.813	0.460	0.501	0.815	0.833	1.071	0.396	−0.156
Mar	−0.592	−0.990	−1.011	−0.804	−1.014	0.602	1.154	1.471	1.130	0.743	−0.202	−0.660
Apr	−1.073	−0.365	−0.627	−0.666	−0.997	0.730	1.558	1.571	1.604	0.622	−0.556	−0.928
May	−0.986	−0.798	−0.803	−1.037	−0.919	0.465	1.102	1.459	1.522	0.444	−0.689	−0.941
Jun	−0.434	−0.718	−0.695	−0.836	−0.583	0.539	1.443	1.472	1.111	0.441	−0.444	−0.693
Mean	−0.910	−1.151	−1.184	−1.235	−1.321	0.424	1.293	1.261	1.363	0.942	0.109	−0.528
1933–4												
Jul	−1.168	−1.188	−0.927	−1.270	−0.281	0.859	1.540	1.693	1.506	0.899	−0.216	−0.787
Aug	−1.045	−1.012	−0.724	−0.944	−0.601	0.847	1.470	1.868	1.875	0.969	−0.063	−0.898
Sep	−0.713	−0.806	−0.788	−0.891	−0.344	0.996	1.608	1.640	1.841	0.898	−0.046	−1.152
Oct	−1.053	−1.286	−1.155	−1.473	−0.971	0.703	1.302	1.462	1.986	1.630	0.416	−0.426
Nov[a]	−1.410	−1.461	−1.919	−1.707	−1.868	0.338	1.006	0.548	1.815	1.472	0.907	0.277
Dec[a]	−1.032	−1.642	−1.346	−1.500	−1.739	0.098	0.764	0.097	0.513	0.968	1.233	0.945
Jan	−1.085	−1.077	−1.417	−1.234	−1.294	−2.626	−2.219	−1.318	1.321	4.090	4.720	1.808
Feb	−0.419	−0.567	−0.875	−0.834	−0.517	−1.903	−1.563	−1.107	1.352	1.253	2.344	1.620
Mar	−0.528	−1.462	−1.770	−1.029	−1.048	−2.117	−2.390	−0.602	2.422	3.687	2.688	1.273
Apr	−0.978	−1.132	−1.536	−1.371	−1.735	−2.309	−0.640	1.174	3.781	2.976	0.598	0.147
May	−0.855	−1.052	−1.529	−1.926	−1.969	−2.032	0.038	1.992	3.770	2.955	0.429	−0.648
Jun	−1.161	−1.083	−1.575	−1.775	−2.657	−2.436	−0.970	1.220	3.208	3.467	1.137	0.256
Mean	−0.954	−1.147	−1.297	−1.330	−1.252	−0.799	−0.005	0.519	2.116	2.105	1.179	0.202
1934–5												
Jul	−1.820	−1.787	−1.935	−2.048	−2.643	−2.238	−0.187	2.263	4.343	4.191	1.338	−0.795
Aug	−0.952	−1.222	−1.904	−1.292	−2.542	−2.845	−0.321	2.391	5.230	4.776	0.959	−0.251
Sep	−1.457	−1.648	−2.182	−1.673	−1.885	−2.076	0.152	3.109	4.970	4.012	1.282	−0.886
Oct	−1.441	−1.538	−1.941	−1.625	−1.807	−1.788	−0.311	1.523	3.420	3.012	1.779	0.642
Nov	−1.458	−1.198	−1.683	−1.757	−1.750	−2.090	−0.690	0.832	0.376	4.084	2.603	2.021
Dec	−1.022	−0.858	−1.290	−1.518	−1.545	−2.784	−1.692	0.063	1.925	2.892	3.193	2.275
Jan	−0.664	−0.916	−1.290	−1.088	−1.740	−2.743	−0.943	−0.882	0.792	2.886	3.322	1.897
Feb	−1.029	−1.727	−1.509	−1.552	−1.607	−3.010	−1.679	−0.882	1.673	2.797	3.466	1.857
Mar	−1.550	−1.932	−1.470	−2.149	−2.074	−2.883	−1.078	0.424	2.830	3.775	2.524	1.076
Apr	−1.928	−1.694	−1.985	−1.633	−2.098	−2.733	−0.672	1.163	3.615	3.779	1.336	−0.457
May	−1.809	−2.206	−2.036	−2.089	−3.373	−2.437	−0.531	1.134	3.181	3.517	0.889	−0.438
Jun	−1.267	−1.691	−1.782	−1.809	−2.795	−2.513	−0.755	1.012	3.768	5.063	2.334	−1.606
Mean	−1.366	−1.535	−1.751	−1.686	−1.947	−2.512	−1.845	1.012	3.010	3.732	2.085	0.445

[a]Values for November and December 1932.

related to the time of sunrise, whereas those of Figures 3 and 5 leave this less obvious. Such discrepancies are, of course, typical of attempts at correlation when there are insufficient data.

Harmonic content of potential-gradient characteristics

In an attempt to further disentangle the various effects which go to make up the complex phenomenon here involved, the composite Polar-Year data for Wyanet, New York, and Houlton have been analyzed by the Fourier method. The results are shown graphically in Figures 8, 9 and 10 and are given numerically in Table 3.

A comparison of Figures 8, 9, and 10 shows essential differences not readily explicable. The fundamentals for Wyanet and New York are similar in that they show a definite preferred direction along a northwest-southeast line. However, they also show a phase-difference corresponding to about five hours time. The difference in local time is only

TABLE 2—*Diurnal variation in millivolts per kilometer of earth-potential for selected quiet days* Eastward component at Wyanet, Illinois—Concluded

Month	Mean values for 90° west meridian hours												No. days
	12	13	14	15	16	17	18	19	20	21	22	23	
1932–3													
Jul	−1.462	−0.862	−0.308	1.003	1.028	0.700	0.603	0.527	0.089	0.304	0.018	0.005	25
Aug	−1.518	−0.796	−0.463	0.702	1.178	0.699	0.772	0.470	0.140	0.017	0.291	−0.040	28
Sep	−1.440	−0.785	−0.559	0.787	0.900	0.479	0.915	0.571	0.178	0.392	0.004	−0.459	23
Oct	−0.608	−0.062	−0.025	0.613	1.094	0.281	0.707	0.627	0.264	0.351	0.190	−0.622	27
Nov	−0.335	−0.319	0.071	0.857	0.618	0.365	0.773	1.041	0.365	0.254	−0.584	−1.106	23
Dec	0.005	0.098	−0.088	0.290	0.180	0.764	0.754	1.362	0.515	0.261	−0.639	−0.865	24
Jan	−0.334	−0.607	−0.070	0.642	0.399	0.441	0.585	0.904	0.298	0.127	−0.015	−0.435	20
Feb	−0.398	−0.777	−0.444	−0.193	0.278	0.296	−0.253	0.510	0.324	0.023	0.134	−0.167	19
Mar	−0.897	−0.901	−0.421	0.171	0.912	0.654	0.284	0.871	0.765	−0.264	−0.322	−0.667	23
Apr	−0.922	−0.763	−0.662	−0.136	0.733	0.966	0.622	0.799	0.529	−0.614	−0.463	−0.948	17
May	−0.874	−0.996	−0.567	0.399	1.468	0.998	0.923	0.969	0.517	−0.691	−0.241	−0.718	22
Jun	−0.712	−0.625	−0.633	0.094	1.147	0.605	0.430	0.338	0.207	−0.790	−0.301	−0.359	21
Mean	−0.792	−0.616	−0.347	0.436	0.827	0.604	0.593	0.749	0.349	−0.052	−0.160	−0.532	..
1933–4													
Jul	−0.770	−0.533	−0.368	0.806	1.340	0.718	0.287	0.333	0.427	−1.056	−0.980	−0.887	18
Aug	−1.120	−0.753	−0.334	1.134	0.826	0.765	0.152	0.367	0.465	−0.765	−1.090	−1.384	22
Sep	−0.942	−0.562	−0.128	0.628	0.556	0.123	−0.052	0.238	0.516	−0.733	−0.807	−1.078	14
Oct	−0.620	−0.173	−0.293	0.389	0.470	0.581	0.238	0.158	0.582	−0.562	−0.950	−0.950	8
Nov[a]	−0.335	−0.319	0.071	0.857	0.618	0.365	0.773	1.041	0.366	0.254	−0.584	−1.106	23
Dec[a]	0.005	0.098	−0.088	0.290	0.180	0.764	0.754	1.362	0.515	0.261	−0.639	−0.865	24
Jan	−0.787	−1.715	−1.733	−1.285	0.900	0.844	2.181	0.393	0.266	1.905	0.304	−0.922	22
Feb	−0.090	−0.556	−0.481	0.521	0.452	0.171	1.761	−0.087	−0.407	0.948	−0.310	−0.701	18
Mar	−0.457	−1.449	−0.994	−0.863	1.265	0.733	2.014	−0.260	0.381	1.479	0.364	−1.317	18
Apr	−0.384	−1.312	−0.583	0.097	0.306	0.574	1.921	0.300	0.120	1.241	0.181	−1.414	24
May	−1.406	−1.390	0.937	0.466	1.242	0.576	2.243	0.364	−0.546	0.569	−0.561	−1.704	20
Jun	−1.487	−0.493	0.998	1.118	1.235	1.217	2.282	−0.131	−0.581	0.654	−0.654	−1.863	19
Mean	−0.700	−0.763	−0.249	0.347	0.782	0.619	1.213	0.340	0.175	0.350	−0.477	−1.182	..
1934–5													
Jul	−1.368	−0.801	−0.230	0.930	1.189	0.638	2.806	0.118	−0.216	0.670	−0.492	−1.904	19
Aug	−1.981	−1.209	−0.565	−0.429	0.470	0.845	2.003	−0.136	−0.278	0.485	−0.251	−1.021	16
Sep	−2.340	−2.466	−1.313	0.382	3.514	0.400	1.280	0.219	0.016	0.593	0.050	−2.045	14
Oct	−1.328	−1.754	−0.320	0.379	1.620	0.441	1.545	0.314	0.394	0.945	0.161	−2.319	25
Nov	−0.581	−1.396	−0.759	−0.822	0.834	0.932	2.088	0.958	0.155	0.906	−0.120	−1.493	24
Dec	−0.509	−1.621	−1.369	−1.106	0.156	0.786	2.598	0.953	0.421	1.552	−0.139	−1.354	21
Jan	0.219	−0.283	−0.091	−0.635	0.471	1.260	1.749	0.264	−0.117	0.234	−0.183	−1.538	19
Feb	0.594	−1.262	−0.265	−0.984	−0.028	−0.433	2.648	0.809	0.752	2.430	0.674	−1.750	19
Mar	−0.440	−1.335	−0.750	−0.091	1.292	0.410	2.654	0.496	0.315	1.595	0.017	−1.658	21
Apr	−1.137	−1.618	−0.169	0.084	1.142	0.429	1.951	0.462	0.957	2.082	0.800	−1.690	22
May	−0.015	−1.126	−0.074	1.223	1.993	0.246	1.879	0.773	0.466	1.830	0.316	−1.325	23
Jun	−2.553	−2.662	−0.679	1.389	1.622	1.631	1.835	0.598	0.670	1.536	0.266	−1.564	20
Mean	−0.953	−1.461	−0.549	−0.807	1.190	0.632	2.086	0.486	0.292	1.238	0.092	−1.639	.

[a]Values for November and December 1932.

about one hour. The fundamental for Houlton on the other hand shows no definite preferred direction but corresponds to a vector of essentially constant magnitude rotating at constant angular-velocity.

At Wyanet the fundamental, and to a lesser degree the fifth harmonic also, shows a preferred northwest-southeast direction, whereas other harmonics are essentially rotary. At New York all components follow the preferred northwest-southeast direction except the fourth. At Houlton all components rotate except possibly the fifth. It is interesting that the latter preferred direction is northeast-southwest or roughly perpendicular to that so generally found at other points in the east. It will also be noted that all components rotate clockwise except the fourth and fifth harmonics at Wyanet. One cannot be certain, however, from the number of data and also from the approximate analysis used, that this effect is real. The relatively strong second harmonic manifest in both the New York and Houlton data and also reported by other observers, is of only moderate magnitude at Wyanet.

TABLE 2—*Diurnal variation in millivolts per kilometer of earth-potential for selected quiet days*
Northward component at Houlton, Maine

Month	Mean values for 75° west meridian hours											
	0	1	2	3	4	5	6	7	8	9	10	11
1932–3												
Jul	0.672	0.432	0.125	0.828	0.771	1.177	1.992	1.747	0.034	−1.151	−2.541	−3.174
Aug	1.461	0.225	0.295	0.186	0.536	0.593	2.516	1.479	0.404	−1.469	−2.736	−3.644
Sep	1.094	0.350	−0.363	−0.296	0.338	0.053	2.407	1.443	0.176	−2.184	−1.986	−3.205
Oct	0.144	0.619	0.301	−0.263	−0.525	−0.307	0.474	1.617	1.401	0.040	−1.957	−2.546
Nov	0.312	0.104	0.176	−0.166	−0.197	−0.358	1.049	1.339	0.878	0.076	−0.788	−1.478
Dec	0.509	0.137	−0.087	0.091	−0.197	−0.673	−0.222	0.678	0.489	−0.311	−0.536	−1.512
Jan	0.753	0.604	−0.102	−0.216	−0.787	−0.550	0.055	0.769	0.835	0.849	−1.213	−1.982
Feb	0.150	0.346	0.281	−0.278	0.066	−0.425	0.227	1.641	1.456	0.292	−0.638	−1.472
Mar	0.120	0.381	0.201	0.346	0.382	0.194	1.108	1.825	2.026	0.523	−1.048	−2.394
Apr	0.202	0.038	0.675	0.078	0.543	0.954	2.323	1.777	0.877	−0.722	−1.780	−2.877
May	0.789	−0.267	0.054	0.116	0.591	1.063	1.715	1.701	0.817	−1.114	−2.526	−3.649
Jun	0.320	0.771	0.325	0.337	0.744	0.952	1.392	1.269	0.652	−0.608	−1.528	−2.824
Jul	0.913	0.283	0.164	0.868	0.685	1.362	1.378	2.055	0.984	−0.864	−2.105	−3.038
Mean[a]	0.544	0.311	0.157	0.064	0.189	0.223	1.253	1.440	0.837	−0.482	−1.607	−2.563

Eastward component at Houlton, Maine

Month	0	1	2	3	4	5	6	7	8	9	10	11
1932–3												
Jul	1.437	−1.505	−0.422	−0.322	−0.497	−0.768	2.108	5.116	5.693	5.118	0.666	−5.423
Aug	−0.198	1.631	1.317	−3.255	0.198	−0.963	2.863	4.648	6.770	4.053	−0.472	−6.219
Sep	1.196	2.266	−5.951	3.912	−0.179	1.035	3.122	7.008	8.458	2.169	0.300	−6.664
Oct	0.145	−1.107	−2.460	−1.956	−0.307	0.042	1.001	4.742	6.210	5.633	0.294	−3.336
Nov	−1.151	−1.217	−2.104	−1.098	−1.525	−2.388	−0.592	1.985	4.390	5.461	3.704	0.866
Dec	−1.083	−0.502	−0.776	−0.693	−1.002	−1.172	−1.919	−0.478	3.248	3.672	3.479	1.123
Jan	−1.122	−0.818	−0.677	0.044	−1.073	−1.870	−0.470	0.955	3.814	6.426	4.153	−0.245
Feb	−1.793	−1.687	−1.338	−1.183	−0.234	−0.225	0.208	2.353	4.684	4.984	3.739	0.809
Mar	−2.178	−1.754	−0.905	1.152	−0.519	−0.664	0.578	3.130	6.393	6.534	4.352	−0.104
Apr	−1.657	−1.655	0.482	−1.018	−0.450	1.016	2.779	5.284	5.745	4.427	0.462	−3.069
May	0.551	−1.208	−0.689	−0.675	−0.324	1.436	3.671	5.671	6.885	3.933	−0.938	5.491
Jun	0.114	0.137	−1.180	−0.348	0.049	1.182	3.213	3.569	4.298	3.138	1.119	−1.780
Jul	−0.394	−0.316	−0.404	−0.485	−0.507	0.560	2.744	4.943	5.263	2.553	−0.742	−3.712
Mean[a]	−0.478	−0.618	−1.225	−1.105	−0.488	−0.278	1.380	3.665	5.549	4.629	1.738	−2.461

[a]Excluding values for July 1933.

FIG. 3—DIURNAL VARIATION OF EARTH-POTENTIAL GRADIENT, WYANET, ILLINOIS, JULY, 1932 TO JUNE, 1933

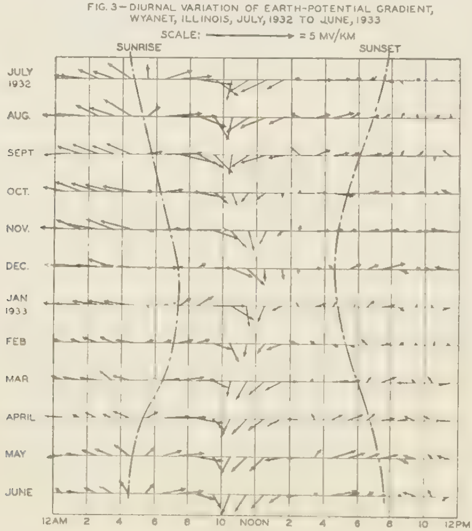


TABLE 2—*Diurnal variation in millivolts per kilometer of earth-potential for selected quiet days*

Northward component at Houlton, Maine—Concluded

Month	Mean values for 75° west meridian hours												No. days
	12	13	14	15	16	17	18	19	20	21	22	23	
1932–3													
Jul	−2.281	−2.039	−0.958	−0.261	0.967	0.955	0.731	0.320	0.331	0.759	0.610	−0.010	25
Aug	−2.952	−2.728	−1.048	−0.376	0.902	0.963	1.139	1.127	1.130	0.633	0.444	0.949	25
Sep	−1.919	−1.779	−0.806	0.120	1.055	0.340	0.874	1.606	0.509	0.723	1.122	0.019	22
Oct	−1.609	−1.421	−0.503	0.094	0.297	0.630	1.099	1.113	0.470	0.371	0.622	−0.184	22
Nov	−1.518	−1.037	−0.599	−0.287	−0.005	0.116	0.428	0.778	0.196	0.374	−0.032	0.652	16
Dec	−0.762	−0.720	−0.259	−0.016	−0.058	−0.127	1.054	1.301	0.737	0.430	0.179	−0.112	23
Jan	−1.105	−0.654	−0.282	−0.004	−0.321	−0.160	0.448	1.426	0.635	0.482	0.050	0.462	19
Feb	−1.417	−0.985	−0.488	−0.115	−0.356	−0.485	0.309	0.625	0.637	0.891	−0.170	−0.075	19
Mar	−2.341	−1.842	−1.621	−0.663	−0.248	0.022	0.549	0.370	1.076	0.557	−0.207	0.694	16
Apr	−2.397	−1.334	−1.121	−0.961	−0.326	−0.260	1.459	1.406	0.624	0.032	0.028	0.265	15
May	−1.636	−1.513	−0.504	0.488	0.727	0.732	0.814	1.246	0.588	−0.064	−0.245	0.090	16
Jun	−1.821	−0.943	−0.737	−0.472	0.119	0.562	0.504	0.144	0.295	0.167	0.119	0.519	15
Jul	−1.930	−1.677	−0.896	−0.578	0.284	−0.002	0.776	0.456	0.204	0.479	−0.101	0.316	18
Mean[a]	−1.813	−1.416	−0.743	−0.204	0.210	0.274	0.784	0.955	0.602	0.446	0.210	0.272	

Eastward component at Houlton, Maine—Concluded

Month	12	13	14	15	16	17	18	19	20	21	22	23	No. days
1932–3													
Jul	−7.160	−6.760	−5.480	−2.488	1.976	3.031	2.187	0.056	0.982	1.987	0.763	−0.231	24
Aug	−8.750	−8.779	−6.432	−3.020	1.092	3.377	0.882	2.990	1.401	2.135	1.599	3.192	28
Sep	−8.897	−8.077	−6.477	−1.747	3.702	1.636	0.097	2.452	0.528	3.585	4.689	−0.212	22
Oct	−8.583	−6.356	2.341	−1.592	0.832	1.331	0.328	1.548	1.172	1.296	2.510	0.902	20
Nov	−4.254	−6.410	−3.718	−2.920	−1.796	0.668	0.428	2.282	0.126	2.149	3.594	3.549	16
Dec	−2.400	−2.613	−1.583	−0.799	−0.970	0.005	−0.088	1.569	1.221	1.074	0.720	0.009	23
Jan	−4.017	−4.845	−4.293	−2.490	−1.025	0.040	−0.087	0.975	1.220	1.383	1.444	2.554	18
Feb	−3.279	−5.567	−3.653	−1.570	−0.917	0.247	0.450	−0.106	1.010	1.959	0.927	0.200	19
Mar	−3.370	−5.602	−6.664	−4.482	1.473	0.612	1.016	0.449	1.036	0.876	0.965	0.604	16
Apr	−7.976	−7.691	−5.963	−4.571	0.398	2.123	3.086	2.467	2.061	1.529	1.598	0.642	14
May	−7.417	−9.290	−8.390	−2.013	1.581	4.073	1.379	2.592	1.738	1.305	0.254	1.340	16
Jun	−5.781	−4.667	−3.831	−2.253	−1.807	1.364	0.903	0.134	0.048	0.281	0.452	1.657	16
Jul	−6.088	−5.625	−4.273	−2.823	−0.396	1.582	1.962	0.311	0.800	1.591	1.619	1.797	18
Mean[a]	−5.990	−6.388	−4.902	−2.495	0.378	1.542	0.866	1.451	1.045	1.630	1.626	1.184	..

[a]Excluding values for July 1933.

FIG. 4—DIURNAL VARIATION OF EARTH-POTENTIAL GRADIENT, HOULTON, MAINE, JULY, 1932 TO JUNE, 1933

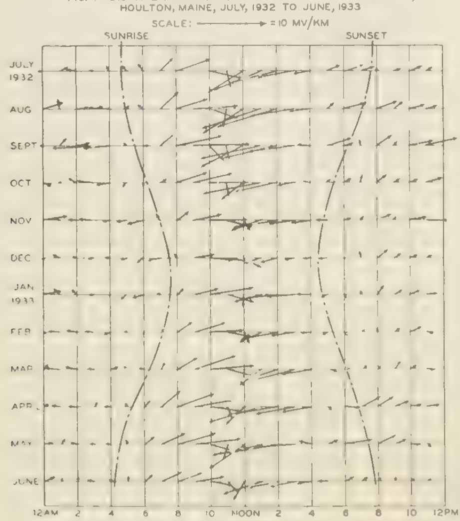


TABLE 2—*Diurnal variation in millivolts per kilometer of earth-potential for selected quiet days*

Northward component at New York, New York

Month	Mean values for 75° west meridian hours											
	0	1	2	3	4	5	6	7	8	9	10	11
1932–3												
Oct	−1.095	−0.099	0.368	0.729	0.175	0.078	0.509	−0.531	−1.069	−0.464	−1.484	−0.36[illegible]
Nov	−1.110	−0.469	0.130	0.364	0.559	0.860	0.553	0.071	−0.505	−1.247	−2.430	−1.866
Dec	−0.854	−0.080	0.493	0.107	0.514	0.137	0.871	0.192	−0.810	−0.557	−1.885	−2.059
Jan	−0.769	0.040	0.398	0.117	0.183	0.790	1.196	0.629	0.342	−1.472	−3.168	−2.486
Feb	0.162	0.554	0.550	−0.055	−0.778	1.153	1.355	0.355	−0.864	−1.666	−2.699	−2.491
Mar	−0.689	−0.036	0.365	0.573	0.439	0.261	0.884	0.484	−0.197	−1.200	−2.646	−1.741
Apr	−0.280	0.144	0.429	0.916	0.952	1.345	1.176	−1.317	−2.144	−2.425	−2.292	−0.660
May	0.060	0.333	0.434	0.340	1.378	1.448	−0.194	−1.300	−1.929	−1.930	−1.676	−0.639
Jun	−0.667	0.103	0.125	0.498	1.379	1.666	0.082	−1.020	−1.136	−1.679	−1.192	−0.214
Jul	0.164	0.118	0.485	0.704	1.125	1.234	0.350	−0.345	−0.951	−1.166	−1.148	−0.419
Aug	0.256	0.375	0.862	1.247	1.035	1.252	−0.311	−1.117	−2.944	−2.766	−1.638	−0.220
Sep	−0.039	0.501	−0.219	0.383	0.803	0.725	0.397	−0.614	−1.906	−2.603	−1.687	−0.711
Mean	−0.405	0 122	0 368	0.493	0 647	0.912	0 572	−0.376	−1.176	−1.598	−1.995	−1.156

Eastward component at New York, New York

Month	0	1	2	3	4	5	6	7	8	9	10	11
1932–3												
Oct	1.815	0.441	−0.550	−1.630	−0.576	−0.005	−0.939	1.032	1.800	1.275	2.203	0.282
Nov	2.230	0.381	−0.442	−0.832	−1.187	−1.429	−1.023	−0.579	0.449	2.158	4.125	3.412
Dec	1.571	−0.334	−0.840	−0.864	−1.170	−0.844	−1.275	−0.720	0.612	1.174	3.287	3.651
Jan	1.531	−0.600	−0.819	−0.698	−0.752	−1.442	−1.910	−1.807	−1.171	2.830	6.130	5.295
Feb	0.911	−0.871	−1.209	−0.060	−0.918	−2.358	−2.742	−0.150	1.136	3.007	2.554	2.848
Mar	1.450	−0.456	−1.290	−1.030	−1.322	−0.589	−1.674	−0.353	1.274	2.477	3.770	2.358
Apr	1.377	−0.373	−0.989	−0.932	−1.673	−2.261	−2.563	1.821	3.603	4.206	3.242	0.535
May	−0.292	−0.534	−0.909	−0.943	−2.559	−2.915	0.205	2.265	4.064	3.962	2.720	0.601
Jun	1.041	−0.471	−0.753	−1.071	−2.824	−2.947	−0.507	1.560	2.500	3.304	2.275	−0.143
Jul	−0.600	−0.961	−1.490	−2.432	−2.335	−2.928	−0.762	0.670	2.866	3.069	2.583	1.128
Aug	−0.389	−1.194	−1.612	−2.517	−2.254	−2.431	0.705	2.333	5.207	4.925	2.786	0.346
Sep	−0.463	−1.560	−1.090	−2.462	−1.876	−1.416	−0.527	2.421	4.168	5.202	3.618	1.872
Mean	0.848	−0.544	−0.999	−1.290	−1.621	−1.797	−1.084	0.708	2.209	3.132	3.274	1.849

FIG. 5—DIURNAL VARIATION OF EARTH-POTENTIAL GRADIENT, NEW YORK, N.Y., OCTOBER, 1932 TO SEPTEMBER, 1933

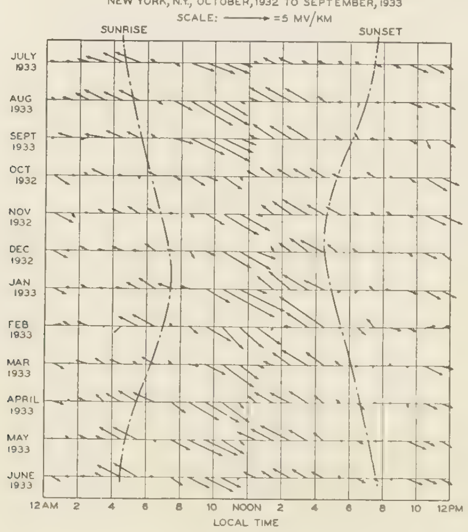


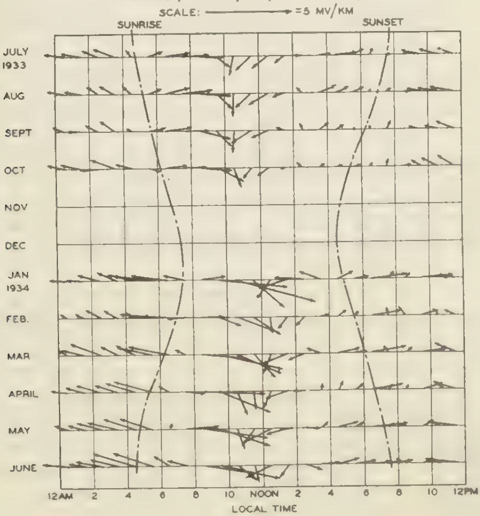
TABLE 2—*Diurnal variation in millivolts per kilometer of earth-potential for selected quiet days*
Northward component at New York, New York—Concluded

Month	Mean values for 75° west meridian hours												No. days
	12	13	14	15	16	17	18	19	20	21	22	23	
1932–3													
Oct	0.707	1.133	0.946	1.055	1.040	0.403	−0.032	0.627	0.088	−0.511	−1.392	−0.841	21
Nov	−0.194	1.101	1.550	1.806	1.196	0.962	0.176	0.368	0.429	−0.206	−1.108	−0.987	24
Dec	−0.807	0.606	1.148	1.614	1.367	0.912	0.453	0.220	0.154	−0.327	−0.607	−0.790	14
Jan	−0.439	1.680	2.260	1.820	1.140	−0.321	−0.294	0.470	0.170	−0.397	−1.063	−0.832	15
Feb	−2.076	−0.470	0.143	1.769	3.588	−0.059	−0.883	1.170	−0.316	−0.454	0.236	0.228	2
Mar	−0.352	0.447	1.454	1.670	1.455	0.459	−0.234	−0.159	0.218	−0.053	−0.990	−0.413	12
Apr	0.882	1.337	1.195	1.388	0.913	0.434	−0.783	0.158	0.069	−0.320	−0.627	−0.479	19
May	0.979	1.343	1.745	1.235	0.892	0.005	−0.341	−0.137	−0.051	−0.865	−0.440	−0.708	21
Jun	0.698	0.808	1.121	1.071	0.789	0.185	−0.453	−0.318	−0.125	−0.659	−0.486	−0.595	23
Jul	0.264	0.704	0.674	0.382	0.292	0.314	0.272	−0.227	−0.455	−0.902	−0.923	−0.559	16
Aug	0.516	1.669	1.668	1.579	0.640	0.095	−0.446	0.015	−0.101	−0.501	−0.542	−0.617	17
Sep	0.523	1.774	1.567	1.535	−0.083	0.526	0.838	0.218	0.323	−0.551	−0.867	−0.817	12
Mean	0.058	1.011	1.289	1.410	1.102	0.326	−0.144	0.200	0.034	−0.479	−0.734	−0.618	..

Eastward component at New York, New York—Concluded

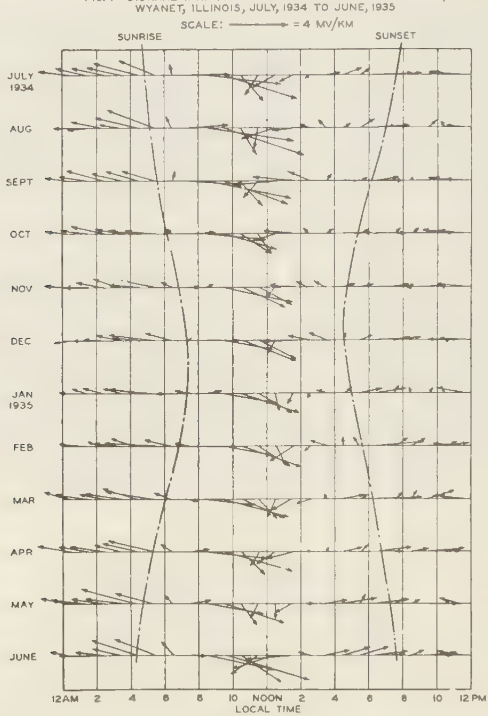
Month	12	13	14	15	16	17	18	19	20	21	22	23	No. days
1932–3													
Oct	−1.207	−1.706	−1.598	−1.730	−1.678	−1.119	−0.645	−0.992	−0.170	0.755	2.630	2.082	21
Nov	0.700	−1.578	−2.278	−2.958	−2.255	−1.912	−0.791	−0.495	−0.650	0.296	2.380	2.270	24
Dec	2.103	−0.349	−1.445	−2.008	−2.876	−1.583	−0.723	−0.320	−0.794	0.695	1.625	1.421	25
Jan	1.215	−2.056	−2.705	−3.103	−2.405	−0.521	−0.399	−0.776	−0.797	1.179	1.975	1.816	16
Feb	3.096	−0.307	−0.781	−2.493	−5.264	−0.397	0.912	−0.890	1.238	1.046	1.349	0.672	2
Mar	1.017	−1.206	−2.172	−2.864	−2.122	−0.612	−0.141	−0.056	−0.207	0.192	2.127	1.424	21
Apr	−0.939	−2.198	−2.384	−2.107	−1.162	−0.526	0.986	0.105	−0.039	0.047	1.602	1.312	19
May	−2.207	−2.755	−3.535	−2.101	−1.199	0.332	0.569	0.436	0.106	2.149	1.195	1.355	21
Jun	−1.528	−1.302	−2.035	−1.685	−1.515	−0.102	1.049	0.551	0.095	1.744	1.476	1.303	23
Jul	−0.385	−1.361	−1.469	−0.844	−0.857	−0.294	−0.386	0.502	0.934	2.116	2.094	1.159	24
Aug	−1.028	−3.023	−3.138	−2.624	−0.838	−0.537	−0.160	0.437	0.414	1.520	1.630	1.451	14
Sep	−1.001	−2.533	−2.544	−2.552	0.032	−0.799	−0.437	−0.617	−0.440	1.112	0.458	1.645	12
Mean	−0.014	−1.698	−2.174	−2.256	−1.850	−0.672	−0.014	−0.176	−0.026	1.071	1.712	1.493	..

FIG. 6—DIURNAL VARIATION OF EARTH-POTENTIAL GRADIENT, WYANET, ILLINOIS, JULY, 1933 TO JUNE, 1934



There is a lack of consistency amongst the various data discussed above which precludes very definite conclusions beyond that of a profuseness of harmonics. Such effects, of course, might be produced by a rather intricate combination of unrelated sources or they might be due

FIG. 7—DIURNAL VARIATION OF EARTH-POTENTIAL GRADIENT, WYANET, ILLINOIS, JULY, 1934 TO JUNE, 1935



to a single external inducing force, sinusoidal in nature and intimately related to the rotation of the Earth, together with a medium possessing non-linear characteristics such as, for instance, the crust of the Earth or possibly the Kennelly-Heaviside layer or both.

Earth-potentials as related to location

The essential differences between the gradients prevailing at the three points mentioned above, suggested a more complete study of how they may vary over the country. To this end, measurements have been made over a month or so at each of several different points. The data so obtained have for convenience been plotted as hodographs on a map of the United States, Figure 11. The plots, of course, indicate magnitude and direction only. The circles shown in Figure 1 represent more closely the areas to which the data apply. The data have not been corrected for the slight seasonal effects shown in Figures 3 to 7. The region of

FIG. 8—HARMONIC CONTENT OF EARTH-POTENTIAL GRADIENT CHARACTERISTICS, WYANET, ILLINOIS, POLAR YEAR ENDING AUGUST 1, 1933

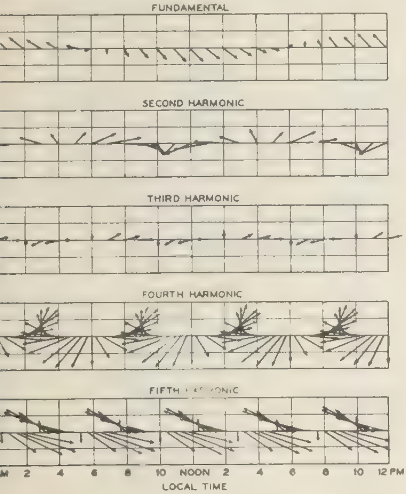


FIG. 9—HARMONIC CONTENT OF EARTH-POTENTIAL GRADIENT CHARACTERISTICS, HOULTON, MAINE, POLAR YEAR ENDING AUGUST 1, 1933

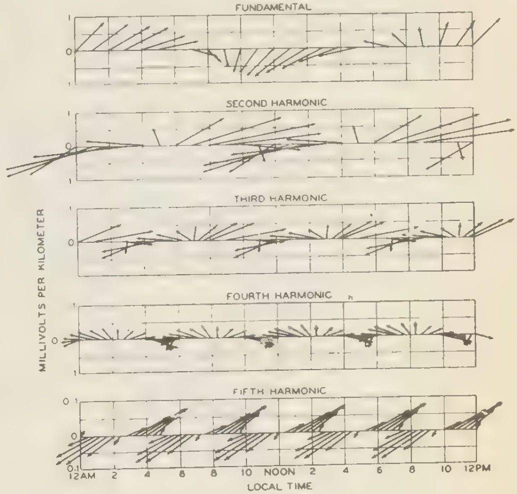


FIG. 10—HARMONIC CONTENT OF EARTH-POTENTIAL GRADIENT CHARACTERISTICS, NEW YORK, N.Y., OCTOBER, 1932 TO SEPTEMBER, 1933

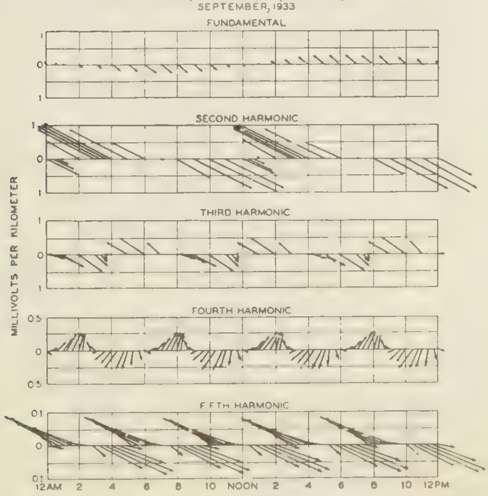


Table 3—*Constants obtained by harmonic analysis (mechanical method)*

$$A = a_0 + a_1 \sin \phi + a_2 \sin 2\phi + a_3 \sin 3\phi + \ldots\ldots$$
$$+ b_1 \cos \phi + b_2 \cos 2\phi + b_3 \cos 3\phi + \ldots\ldots$$

Observing point and epoch	Component	Harmonic constants										
		a_0	a_1	a_2	a_3	a_4	a_5	b_1	b_2	b_3	b_4	b_5
Wyanet, Ill. Mean Aug. 1, 1932 to July 31, 1933	Northward	−0.007	−0.037	0.201	−0.105	0.034	−0.070	0.401	−0.290	0.100	−0.081	0.000
	Eastward	−0.028	−0.115	−0.630	−0.033	0.125	+0.145	−0.441	−0 746	0.436	0.031	−0.080
Houlton, Me. Mean Aug. 1, 1932 to July 31, 1933	Northward	0.034	0.018	−0.280	−0.084	0.306	−0.080	0.808	−0.779	0.494	−0.175	0.020
	Eastward	0.031	1.001	−2.854	1.243	−0.625	−0.138	0.859	−1.459	1.312	−0.531	0.013
New York, N. Y. Mean Oct. 1, 1932 to Sept. 30, 1933	Northward	−0.018	−0.286	0 925	−0.489	0 283	0.085	0.081	−0.516	0.009	−0.056	−0.010
	Eastward	−0.047	0.375	−1 792	0.752	0.144	−0.185	−0.053	1.033	0.189	0.094	0.055

high resistivity reported by Card [10] is indicated by the stippled area.

It is to be noted that the directions along which the potential-gradients are greatest appear to be approximately at right-angles to the major axis of this region of high resistivity. It is convenient to regard the voltage measured as being due to a resistance-current drop between points in the Earth's crust and the directivity noted to be due to the resistivity along a northwest-southeast direction being larger than at right-angles thereto. But it is possible to place a slightly different interpretation on these results. It is to be noted that the directions of the potential-gradient are also perpendicular to the coast-line east of which there are iarge quantities of water of high conductivity. It is

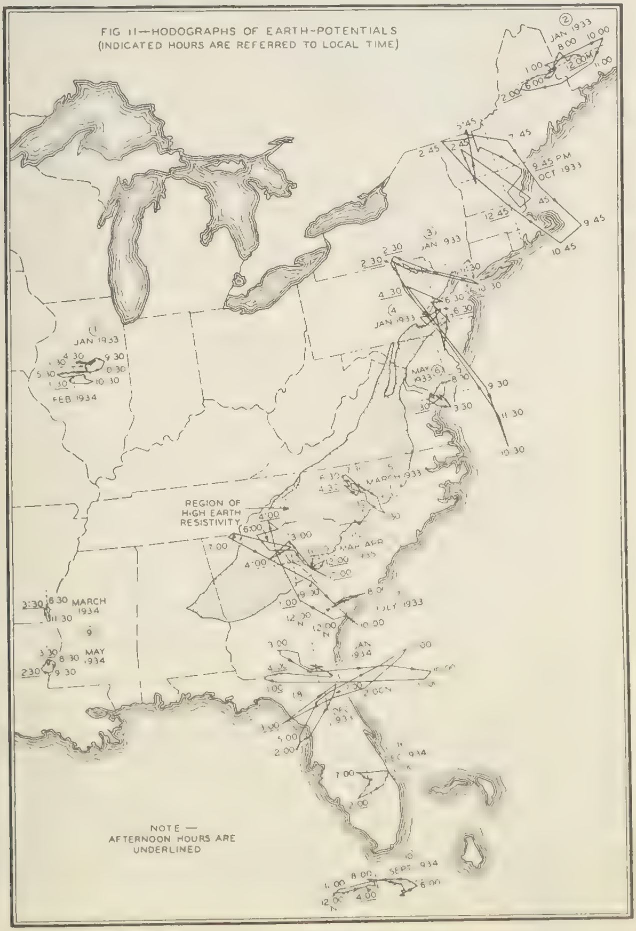


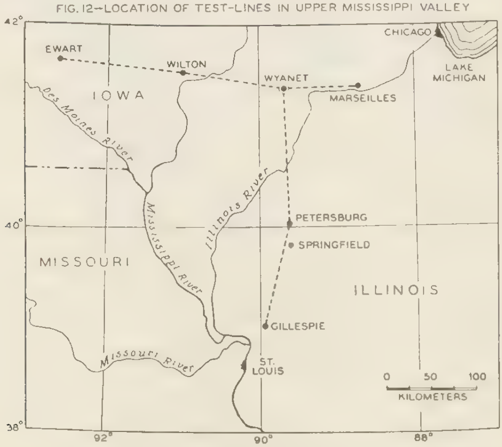
FIG 11—HODOGRAPHS OF EARTH-POTENTIALS (INDICATED HOURS ARE REFERRED TO LOCAL TIME)

to be expected that even though no region of especially high resistivity prevailed in eastern America, potential-gradients might approach the ocean perpendicular to the shore-line.

The data taken at Key West (Florida) on lines extending to Havana (Cuba) and to Miami (Florida), however, show a relatively high potential-gradient having a preferred direction. This seems rather surprising since the region under test is almost completely covered by water, in some places more than a mile deep. It might be expected that such a region would possess substantially uniform conductivity in all directions and that the potential-gradient pattern would not only be small but would show no definite direction. It is interesting also that the data taken at West Palm Beach (Florida), where conditions are by no means homogeneous, show little preferred direction while those taken at Jacksonville (Florida), under very similar conditions, indicate a direction perpendicular to the axis of the Peninsula. It is true that the former point is located near a large swampy area. However, this swamp is relatively shallow so that the overall resistivity after all should be more like that at Jacksonville than like the region near Key West.

A somewhat more detailed study was made at Wyanet of the potential-gradients measured between various points located to the east, west, and south of that station. The relative locations of these points as well as certain other physical characteristics of the neighborhood are shown in Figure 12. These regions at first sight at least appear to be

FIG. 12—LOCATION OF TEST-LINES IN UPPER MISSISSIPPI VALLEY



extremely uniform geologically. However the earth-potential data indicate definite differences particularly along the east-west line. Also these differences seem to change with the time of day.

The comparative data taken over these areas are shown in Figures 13, 14 and 15. They show that the prevailing north-south gradient is substantially uniform over the distances measured, whereas that to the east and west varies over rather wide limits, in general, indicating higher potentials to the west than to the east.

A somewhat similar study was made of potentials prevailing between points to the north and south of Jackson (Mississippi). In this case, the north-south gradients were approximately equal and substantially the same as those measured south of Wyanet. This suggests that the

FIG. 13—DIURNAL VARIATION OF EARTH-POTENTIAL GRADIENT NORTH-SOUTH, WYANET, ILLINOIS

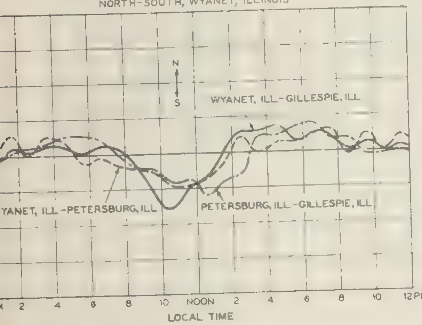
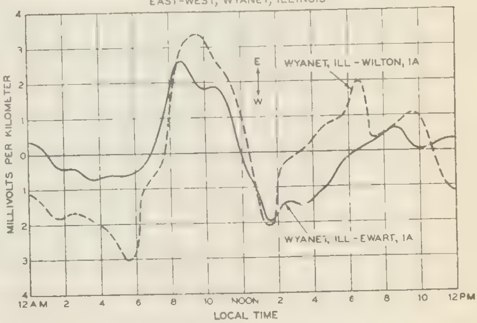


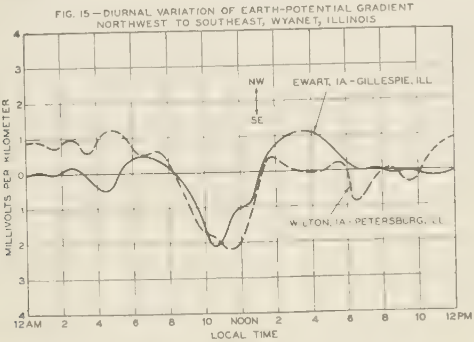
FIG. 14—DIURNAL VARIATION OF EARTH-POTENTIAL GRADIENT EAST-WEST, WYANET, ILLINOIS



north-south component of this gradient remains substantially constant along this section of the Mississippi Valley, whereas the east-west component decreases gradually as we pass from north to south.

The author wishes to express his appreciation to the many Bell System employees who have cooperated in taking the data here presented.

FIG. 15—DIURNAL VARIATION OF EARTH-POTENTIAL GRADIENT NORTHWEST TO SOUTHEAST, WYANET, ILLINOIS



Their number obviously precludes the recognition that they may individually deserve. Particular recognition is due to Miss M. K. Corr for coordinating the analysis and to the Misses Goeltz and Routhier for the extremely laborious task of reading the records and translating the same into hourly means.

Summary

The normal diurnal-variation of earth-potential varies rather widely from place to place over the eastern part of United States. In certain localities, particularly where the earth-resistivity is high, these potentials are relatively high and are frequently directed along some preferred line. In other localities where the terrain is relatively homogeneous, there is little or no indication of a preferred direction. A harmonic analysis of certain of the records shows that in some cases this preferred direction prevails in all components. In others, it is confined to certain components only. Measurements made on lines extending from the island of Key West to Cuba and also to the mainland indicate directional effects even where great depths of water prevail.

Condensed bibliography of Bell System papers relating to solar and terrestrial phenomena

(Arranged chronologically)

[1] H. W. Nichols and J. C. Schelleng, Propagation of electric waves over the Earth, Bell Sys. Tech. Jour. **4**, 215-235, 1925.
[2] L. Espenschied, C. N. Anderson, and A. Bailey, Trans-Atlantic radio telephone transmission, Proc. I. R. E., **14**, 7-56, 1926.
[3] R. A. Heising, J. C. Schelleng, and G. C. Southworth, Some measurements of short-wave transmission, Proc. I. R. E., **14**, 613-647, 1926.
[4] R. A. Heising, Experiments and observations concerning the ionized regions of the atmosphere, Proc. I. R. E., **16**, 75-99, 1928.
[5] C. N. Anderson, Correlation of long-wave trans-Atlantic radio transmission with other factors affected by solar activity, Proc. I. R. E., **16**, 297-347, 1928.
[6] J. C. Schelleng, Note on the determination of the ionization in the upper atmosphere, Proc., I. R. E., **16**, 1471-1476, 1928.
[7] C. N. Anderson, Notes on the effect of solar disturbances on trans-Atlantic radio transmission, Proc. I. R. E., **17**, 1528-1535, 1929.
[8] Isabel S. Bemis, Some observations of the behavior of earth-currents and their correlation with magnetic disturbances and radio transmission, Proc. I. R. E. **19**, 1931-1947, 1931.
[9] J. P. Schafer and W. M. Goodall, Kennelly-Heaviside Layer studies employing a rapid method of virtual-height determination, Proc. I. R. E., **20**, 1131-1147, 1932.
[10] R. H. Card, Some recent earth-resistivity measurements in the United States, Trans. Amer. Geophys. Union, published by National Research Council, pp. 111-115, 1933.
[11] G. C. Southworth, Some earth-potential measurements being made in connection with the International Polar Year, Proc. I. R. E., **21**, 1740-1748, 1933.
[12] E. T. Burton and E. M. Boardman, Audio frequency atmospherics, Proc. I. R. E., **21**, 1476-1494, 1933; Bell Sys. Tech. Jour., **12**, 498-516, 1933.

Bell Telephone Laboratories,
New York, New York

NUMERICAL CHARACTER-FIGURES OF MAGNETIC DISTURBANCE IN RELATION TO GEOMAGNETIC LATITUDE

By J. M. Stagg

§1. In recommending the publication of daily values of the quantity $C_n = (HR_H + ZR_Z) \times 10^{-4}$ at a number of selected observatories the primary intention of the responsible sub-committee of the International Union of Geodesy and Geophysics was to provide a numerical criterion for differentiating between days of greater or less magnetic disturbance at any one station. At best C_n has only a facial resemblance to one of the constituents in the expression for the total energy of the Earth's magnetic field. Without complex modifications to take account of the contributions to the quantity made by the regular quiet- and disturbed-day variations C_n can hardly be regarded even as a rigorous comparative measure of the major features of disturbance at various observatories. To serve this purpose it would require to be shown that not only the same type of perturbation dominated disturbance at all observatories, so that the extreme daily range R and not, say, the mean hourly range, could be regarded as a universal index of disturbance, but that the dominant perturbations occurred at least within the same Greenwich day at all observatories.

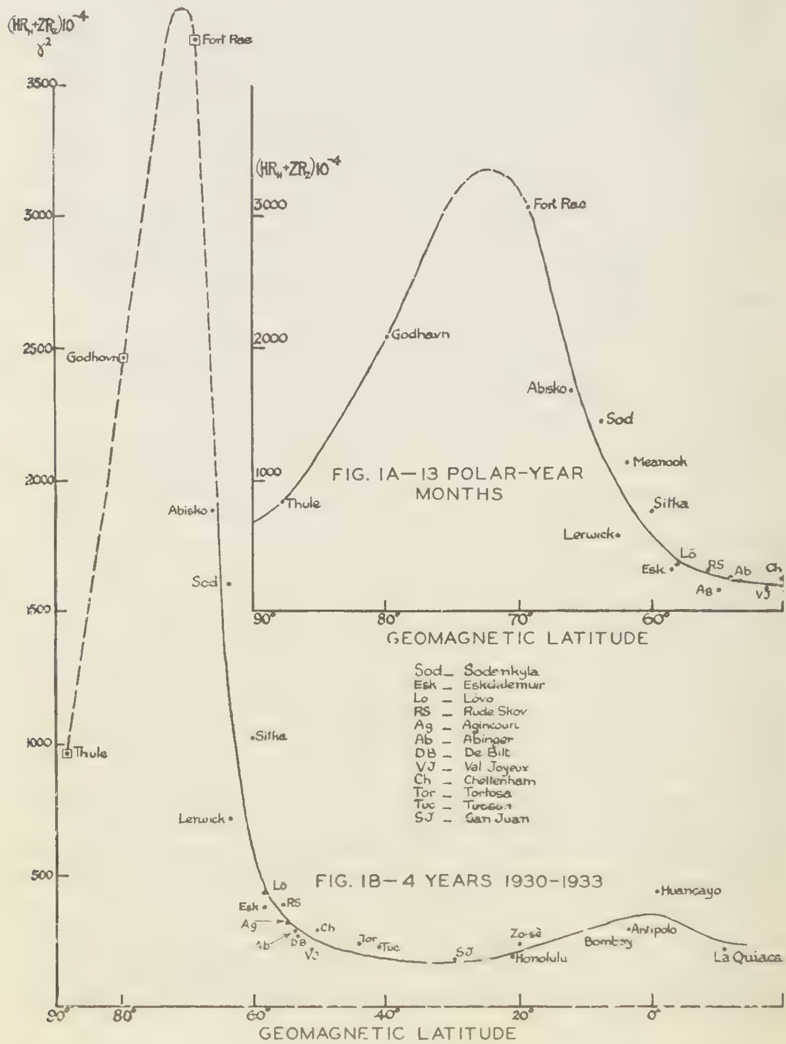
In spite of these limitations the values of C_n published by the Royal Netherlands Meteorological Institute supply the only available numerical measure of one aspect of magnetic disturbance, and, so long as the local-time effect is avoided by comparing average values covering many days, they may be used as a basis for comparison of the scales of the largest oscillations in disturbance at observatories in different localities.

§2. With this in view and without prejudice to their intended use mean values of C_n were formed for the 15 stations listed in descending order of geomagnetic latitude (ϕ_m) in Table 1. Unpublished values for the two very instructive Danish stations, Godhaven and Thule, have been made available by the courtesy of Dr. la Cour and for Fort Rae by permission of the British National Polar Year Committee. At all stations except Thule the values of C_n in the column y refer to the same 13 months of the Polar Year, namely, August 1932 to August 1933; data for August 1933 are not available for Thule. The column w in Table 1 contains the means for the four winter months, namely November and December, 1932, January and February, 1933, and column e the four months September and October, 1932, and April and May, 1933; August 1933 is accounted for in the column s by weighting each of the two Augusts equally with May, June, and July. Judging from the two August values for the other stations, the effect of omitting August 1933 from the Thule values is to make the y- and s-means at that Station relatively (but only slightly) higher than the corresponding means for the other 14 stations.

Figure 1A shows C_n plotted against ϕ_m. In spite of anomalies to be considered in a later paragraph the general trend of the curve is unmistakable. If C_n is to be regarded as an index the Figure shows that in the neighbourhood of $\phi_m = 72°$, inside the Fritz line of maximum auroral

activity, there is a zone characterised by intense magnetic disturbance. In this zone the scale of disturbance is probably about 12 times as great as at Eskdalemuir in a quiet year. Disturbance increases towards this zone, steadily from 50° to 60°, then steeply between 60° and 70°, the fall away towards the magnetic-axis pole is less steep. At the pole disturbance on the average is about the same as at 60°.

By a simple numerical grafting process Figure 1*A* has been extended southwards to just beyond the magnetic equator (Fig. 1*B*). The basis



of this curve was kindly provided by Dr. A. Crichton Mitchell. From the De Bilt publications Dr. Crichton Mitchell formed annual means of C_n covering the four years 1930-33 for twenty stations which have contributed complete C_n-values during those first years of operation of the scheme of numerical characterisation. The stations ranged in latitude from Abisko, $\phi_m = 66°$, to La Quiaca, 11° south of the magnetic equator. Twelve of these stations were already included in Table 1.

TABLE 1—*Mean Values* $C_n = (HR_H + ZR_Z)\ 10^{-4}$, *August 1932 to August 1933*

Observatory	ϕ	ϕm	y	w	e	s
	°	°				
Thule	76.5	88.0	809	421	705	1302
Godhavn	69.2	79.8	2070	1791	2112	2259
Fort Rae	62.8	69.1	3088	2807	3410	3055
Abisko	68.3	66.0	>1670	1505	>1945	1582
Sodankylä	67.4	63.8	1427	1251	1723	1332
Lerwick	60.1	62.6	586	471	666	615
Meanook	54.6	61.8	1136	659	1316	1373
Sitka	57.1	60.0	785	624	889	829
Eskdalemuir	55.3	58.5	306	233	331	345
Lövo	59.3	58.1	370	294	419	392
Rude Skov	55.9	55.8	335	273	369	358
Agincourt	43.8	55.0	196	141	205	234
Abinger	51.2	54.0	266	211	284	295
Val-Joyeux	48.8	51.3	193	146	208	217
Cheltenham	38.7	50.1	248	198	239	296

For these the ratio of the four-year mean to the y-mean of Table 1 ranged between 1.31 to 1.09 except at Agincourt (1.64). These narrow limits justified the computing of four-year mean values for the three stations, Thule, Godhavn, and Fort Rae, by using the average ratio 1.19. In this way the C_n-curve for 1930-33 was extended by broken line, northward of Abisko.

Figure 1*B* makes it seem likely that, after the scale of disturbance has fallen away steeply to 50°, it continues to fall less slowly to a minimum about 30°, after which it rises in a zone 20° broad centred on the magnetic equator.

§3. Both curves of Figure 1 show the average state of affairs for the year as a whole. One way of inquiring into the seasonal change in the relation of disturbance (as indicated by C_n) to geomagnetic latitude is to express the seasonal values of C_n in terms of the corresponding value of C_n for any one of them. This is done for the 15 stations of Table 1 in Table 2*a*, taking Eskdalemuir as the station of reference for the denominator of the ratio ρ.

Overlooking Meanook temporarily, the seasonal changes in the value of ρ suggest that, though the belt of maximum disturbance remains throughout the year concentrated between ϕ_m 70° and 75°, the concentration is greatest in winter. In summer it diffuses both to north and south, so that in this season the scale of disturbance at stations on the polar and equatorial sides of the belt more closely approximates that in the immediate vicinity of the belt. In particular at Thule, 2° from the magnetic-axis pole, summer disturbance is more than double that of

TABLE 2—*Ratios* (ρ) *of seasonal values of* C_n *and constituent products* HR_H *and* ZR_Z *to corresponding seasonal values at Eskdalemuir*

Observatory	(a) $C_n = HR_H + ZR_Z$			(b) HR_H			(c) ZR_Z		
	w	*e*	*s*	*w*	*e*	*s*	*w*	*e*	*s*
Thule	1.8	2.1	3.8	0.8	0.8	0.9	2.5	3.0	5.2
Godhavn	7.7	6.4	6.5	2.7	1.7	1.9	11.5	9.5	9.6
Fort Rae	12.0	10.3	8.8	3.9	3.1	2.9	17.6	15.1	12.7
Abisko	6.5	5.9	4.6	4.1	4.2	3.2	8.1	7.0	5.5
Sodankylä	5.4	5.2	3.9	3.0	2.9	2.5	7.0	6.7	4.8
Lerwick	2.0	2.0	1.8	1.1	1.1	1.2	2.7	2.6	2.2
Meanook	2.8	4.0	4.0	2.3	2.5	2.1	3.2	5.1	5.2
Sitka	2.7	2.7	2.4	1.2	1.3	1.2	3.7	3.6	3.2
Eskdalemuir	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Lövo	1.3	1.3	1.1	0.8	0.8	0.9	1.6	1.6	1.3
Rude Skov	1.2	1.1	1.0	0.9	0.9	0.9	1.3	1.3	1.1
Agincourt	0.6	0.6	0.7	0.8	0.7	0.8	0.5	0.7	0.6
Abinger	0.9	0.9	0.9	1.0	1.0	0.9	0.8	0.8	0.8
Val-Joyeux	0.6	0.6	0.6	0.9	0.8	0.8	0.4	0.5	0.6
Cheltenham	0.8	0.7	0.9	0.9	0.8	0.8	0.8	0.8	0.9

winter and even at Godhavn summer disturbance exceeds that of the equinoctial months.

Tables 2*b* and 2*c* do for the constituent products $HR_H \times 10^{-4}$ and $ZR_Z \times 10^{-4}$ separately what 2*a* does for the combined products forming C_n. In Table 2*b*, for example, the seasonal mean values $HR_H \times 10^{-4}$ for all stations are given as ratios of the corresponding values $HR_H \times 10^{-4}$ at Eskdalemuir. For both components the zonal diffusion of disturbance in summer as compared with its greater concentration within a narrow belt in winter is clear. But Table 2*c* shows that the seasonal

TABLE 3

Observatory	(a) Seasonal values C_n expressed as percentage of *y*				(b) *y*-means of constituent-range products, mean values of components and ranges					
	w	*e*	*s*	(*s*−*w*)	HR_H	ZR_Z	*H*	*Z*	R_H	R_Z
					$10^2\gamma$	$10^2\gamma$			γ	γ
Thule	52	87	161	109	99	710	46	557	22	128
Godhavn	87	102	109	22	224	1846		554		333
Fort Rae	91	110	99	8	391	2697	77	600	505	450
Abisko	90	116	95	5	456	1213	119	498	383	243
Sodankylä	88	121	93	5	335	1092	121	492	277	222
Lerwick	80	114	105	25	139	448	145	466	96	96
Meanook	58	116	121	63	278	865	127	594	219	146
Sitka	80	113	106	26	150	634	154	551	97	115
Eskdalemuir	76	108	113	37	122	184	166	449	74	41
Lövo	79	113	106	27	103	267	155	465	67	57
Rude Skov	82	110	107	25	109	226	168	448	65	50
Agincourt	72	105	119	47	94	110	154	569	61	20
Abinger	79	107	111	32	117	149	185	429	63	35
Val-Joyeux	76	108	112	36	99	94	196	416	50	23
Cheltenham	80	96	119	39	101	156	185	542	55	29

change is much greater in the vertical component of the field than in the horizontal meridian component.

The phenomenon may be illustrated in another way. If the seasonal mean values of C_n in Table 1 are expressed as percentages of the y-values for each station, as is done in Table 3*a*, it is seen that disturbance in the winter months is relatively greatest in the immediate vicinity of the zone of maximum disturbance, and falls off, more steeply to the north than the south, on both sides of the zone. Disturbance in the equinoctial months, above the average of the year at all stations from Godhavn to Val Joyeux (except Meanook), increases relatively to the year's average with decreasing latitude from the pole to latitude 64°, then tends to decrease. And anti-parallel with this latitude-change in equinoctial disturbance, disturbance in summer is relatively greatest at the pole, falls off to $\phi_m = 64°$ and then rises steadily to 50°.

In connection with these seasonal changes in the concentration of disturbance in high latitudes, it may be noted that a similar change has been found in the diurnal distribution of irregular disturbance[1] and also in the mean position of the current-system responsible for the regular diurnal variation of disturbance[2, 3] in moderately high latitudes. Whereas, however, the zone of maximum irregular disturbance indicated by the present inquiry lies northward of Fort Rae, the position of the zone of greatest current-concentration indicated by the disturbing vectors in the diurnally varying field is probably south of Fort Rae.

§4. The mean values of C_n in Table 1 refer to the same months for all stations except Thule. If C_n were an adequate numerical measure of the major features of disturbance, free from spurious contributions of instrumental origin, all the stations should lie on a smoothed curve if the scale of disturbance were purely a function of geomagnetic latitude. The closeness with which most of the stations lie on the curve of Figure 1*a* indicates that C_n in the main is a function of ϕ_m but a minority of the 15 stations clearly falls out of alignment. Judged by the standards of Cheltenham, Abinger, Rude Skov, Lövo, and Sitka, C_n at Val Joyeux, Agincourt, Eskdalemuir, and Lerwick is low. Suspicions might also fall on Fort Rae as being inordinately high, but the upward slopes of the C_n, ϕ_m curve from Lövo to Sodankylä and Abisko on the equatorial side and from Thule to Godhavn on the polar side point unquestionably to a belt of very high values somewhere in the region 70° to 75°. Meanook is also anomalous, not so much in its position in Figure 1 as in the seasonal ratios and percentages of Tables 2 and 3. Table 2*c* indeed suggests that Meanook's irregularity may be traceable primarily to the Z-component and probably became large early in 1933. That the Z-component is also the chief contributing factor to the low values for Agincourt, Eskdalemuir, Lerwick, and perhaps Val Joyeux is seen from Table 3*b*, which gives the means of the separate constituent products of C_n, $HR_H \times 10^{-4}$ and $ZR_Z \times 10^{-4}$, for the same thirteen months of the Polar Year (again except Thule). This Table gives also the mean values of the force components H and Z used in forming the products, and, in addition, the mean ranges R_H and R_Z. The values of these latter show that, even when the effect of the known large regional anomalies in the

[1]Proc. R. Soc., A, **149**, pp. 298-311, 1935.

[2]A. H. R. Goldie Trans. R. Soc., Edinburgh, **57**, p. 161, 1931.

[3]Paper by present author already submitted for publication.

surface vertical field is eliminated, Val Joyeux, Agincourt, Eskdalemuir, and Lerwick remain together in a class of R_Z low compared with other stations of comparable ϕ_m. At the last two stations the values R_Z would require to be increased by at least 40 per cent to bring them into alignment with similarly situated localities.

§5. It is of more than prying interest to enquire how such disparities may arise. They may be due (1) to real regional peculiarities in the disturbance-field, or (2) to fictitious (for example, instrumental) causes. If locality can impose abnormalities in the C_n-measure of disturbance, it might be expected (though, remembering the nature of C_n, it would not be a necessary corollary) that other measures of the scale of disturbance would be similarly affected. In particular, the behaviour of the range of the regular daily disturbance-variation with increasing ϕ_m should give alternative information about the reality of regional anomalies. Unfortunately this is not readily tested. Magnetic publications differ widely in mode of presentation of data and time of appearance. But the following figures for the range of the average diurnal variation in H and Z on the internationally selected disturbed days in 1926 show that on this criterion the scale of disturbance at Lerwick is slightly higher than at Sitka as its ϕ_m would indicate it should be.

Observatory	Range average diurnal variation	
	H	Z
Lerwick	198γ	132γ
Sitka	142γ	130γ
Eskdalemuir	79γ	95γ

This is admittedly an inadequate test and probably represents the relationship between geomagnetic latitude and regular disturbance as being more simple than it really is. At the same time it shows that at least for one of the anomalous stations (Lerwick) the replacement of a disturbance-index based on instantaneous extreme values by one based on hourly means brings it into better relations with stations in comparable ϕ_m.

It will be noticed that this result only lessens the probability that the regular diurnally varying aspect of disturbance is affected anomalously to the same extent as that aspect which is primarily measured by C_n, it does not dispose of the possibility of localised peculiarities in the short-period perturbations which, superposed on the regular variation, make such large contributions to the extreme range R at moderate- and high-latitude stations. If, as seems likely, these short-period oscillations are largely controlled by earth-currents, regional characteristics of geology and topography will allow them to reach greater proportions at some stations than at others in the same latitude, so that measures of disturbance based on the large scale regular diurnal variations might be quite comparable, while those based on extreme ranges might be wholly different.

The second possible cause of the disparities discussed in §4, that attributable to the technique of registration, though unlikely by itself to be held responsible for such large anomalies as, for example, those found for Lerwick and Eskdalemuir, will now be considered. Dismissing,

as improbable, systematic defects in scale-value, there remain the constructional characteristics of the variometers by which the field-changes are recorded. To see how these can contribute to the measures of disturbance, it is only necessary to consider two observatories A and B in the same geomagnetic latitude and with similar mean values of the surface force-elements of the field, but differing in the form of the magnets and damping systems used, especially in the variometers recording Z. If A's variometer has a large magnet of the older type magnetograph with massive plates placed in close proximity so that the moving system is very efficiently damped, while at B the variometer incorporates a modern-type small magnet mounted, as is the tendency in such instruments, in a partially evacuated chamber with no specific damping accessories, it is to be expected that the measures of disturbance based on momentary extreme values in perturbations at A will on the average be less than those at B. It might further be expected that the ratio of the measures A to B will be less on quiet days than on days of disturbance, especially if the latter be of a short-period oscillatory character. What the contribution from such instrumental causes may be is not readily estimated, but is probably not likely to exceed 10 per cent even in the sharpest oscillations. If this estimate be correct, the largest part of the anomalies of §4 must be ascribed to regional peculiarities in the disturbance-field, more especially that part of the field made up by induced earth-currents. But at the same time until there is a greater common measure of similarity in magnetograph-construction, particularly in regard to the vertical-force variometer, uncertainty will remain about the interpretation of C_n as a measure of disturbance.

§6. Another feature of the irregular disturbance primarily catered for by C_n requires attention, especially if, as was suggested in the proposals for publishing C_n for a number of selected stations, the daily values of C_n from all cooperating stations are to be combined to give a composite numerical character-representation of the whole Earth for correlation with solar or meteorological phenomena. It has been demonstrated[4] that irregular disturbance is controlled by local time and is very largely concentrated into the period within four or five hours of local midnight at stations below $\phi_m = 70°$. As illustration, this means that those large irregular oscillations which in large measure decide the magnitude of C_n and occur in the late evening hours of day n at stations in western Europe, will on the average occur six to eight hours later at places in western America and therefore will fall to be tabulated on Greenwich day $(n+1)$. This has been demonstrated decisively from the Polar Year records from Fort Rae. On frequent occasions at this Station days which would have been selected as most disturbed locally followed a day later than those days selected by criteria mainly derived from west European observatories. The same feature can be demonstrated by correlating the daily values of C_n at a western European station with the corresponding values at other stations distributed over the Earth. This has been done with Eskdalemuir as representative station. As was to be expected, the correlation falls off with increasing latitude-difference from Eskdalemuir, confirming what has been long known that disturbance may be in progress far to the north of Eskdalemuir which is

[4]Proc. R. Soc., A, No. 867, **149**, 1935.

not recorded at this station and similarly between Eskdalemuir and stations farther south. But, in addition, the closeness of correlation decreases with increasing distance east and west of Eskdalemuir.

The necessary inference from these latter considerations is that before the C_n-measure of disturbance can be utilised for characterising numerically individual days for the Earth as a whole on the lines of J. Bartels' u_1-measure of activity for months, a more detailed examination will be necessary of the local time- and latitude-effects in disturbance-distribution over the Earth's surface.

Meteorological Office,
Edinburgh, Scotland,
April 12, 1935

SUR LA VARIATION ANNUELLE DU CHAMP MAGNÉTIQUE TERRESTRE À PARIS

PAR LOUIS EBLÉ

La variation séculaire du champ magnétique, même lorsqu'elle est rapide, n'est pas linéaire, ni représentable par une fonction algébrique du temps; si l'on s'en tient aux moyennes mensuelles, les résidus sont tantôt en excès, tantôt en défaut. Cela ne doit pas étonner, car l'effet des perturbations suffit à l'expliquer: Après une forte perturbation, par exemple, la valeur de H reste relativement faible pendant plusieurs jours, ce qui doit abaisser légèrement la moyenne mensuelle. Les écarts présentent-ils une répartition annuelle? c'est ce qui a été controversé. On possède aujourd'hui des séries d'enregistrements assez longues pour essayer de déterminer les marches annuelles superposées aux variations séculaires des éléments magnétiques. Celle de Paris s'étend aujourd'hui à 52 ans, 18 au Parc Saint-Maur et 34 au Val-Joyeux. A. Angot[1] l'a utilisée en étudiant séparément ces deux groupes dont le dernier ne comprenait alors que 20 ans; il a trouvé deux marches annuelles faibles, mais assez semblables pour lui permettre de conclure à leur réalité.

TABLEAU 1—*Ecarts moyens mensuels*

Obs.	Janv.	Févr.	Mars	Avril	Mai	Juin	Juil.	Août	Sept.	Oct.	Nov.	Déc.
						Déclinaison						
(I)	−0.08	−0.06	+0.11	−0.01	−0.13	−0.08	+0.03	+0.13	+0.14	+0.01	0.00	−0.05
(II)	+0.01	−0.01	−0.01	−0.08	−0.11	−0.03	+0.05	+0.09	+0.04	+0.05	−0.01	−0.04
(III)	0.00	−0.07	−0.02	−0.19	−0.22	−0.03	+0.02	+0.16	+0.10	+0.19	+0.03	+0.03
yenne	−0.02	−0.05	+0.03	−0.09	−0.15	−0.05	+0.03	+0.13	+0.09	+0.08	+0.01	−0.02
Maur	+0.04	+0.04	+0.10	+0.04	−0.11	−0.17	−0.08	−0.01	+0.07	+0.07	−0.06	+0.07
						Inclinaison						
(I)	+0.04	+0.18	+0.08	+0.05	−0.18	−0.32	−0.39	−0.17	+0.16	+0.29	+0.30	+0.07
(II)	+0.11	−0.13	+0.12	−0.09	−0.16	−0.50	−0.45	+0.07	+0.09	+0.30	+0.20	+0.29
(III)	−0.16	+0.01	+0.06	−0.05	−0.16	−0.28	−0.18	+0.03	+0.37	+0.43	−0.01	−0.06
yenne	0.00	+0.02	+0.09	−0.03	−0.14	−0.37	−0.34	−0.02	+0.21	+0.34	+0.16	+0.10
Maur	+0.16	+0.23	+0.22	−0.06	−0.34	−0.53	−0.32	−0.13	+0.03	+0.19	+0.39	+0.16
						Composante horizontale						
J (I)	−5.9	−5.5	−2.8	+1.8	+4.9	+9.1	+9.4	+7.0	+0.1	−5.7	−7.8	−4.9
J (II)	−3.4	−1.5	−3.8	+1.1	+5.2	+10.3	+9.4	+1.8	−2.3	−5.8	−5.5	−5.0
J (III)	−0.5	−1.6	−0.8	+3.2	+4.1	+6.3	+5.3	+2.2	−4.1	−8.0	−3.7	−2.2
yenne	−3.3	−2.9	−2.5	+2.0	+4.7	+8.6	+8.0	+3.7	−2.1	−6.5	−5.7	−4.0
Maur	−3.6	−5.6	−3.9	+1.6	+5.8	+10.1	+7.4	+3.1	−1.4	−3.9	−6.5	−3.1
						Composante verticale						
J (I)	−10.1	−6.9	−2.4	+2.8	+5.7	+10.0	+9.4	+9.2	+2.9	−4.0	−8.5	−9.1
J (II)	−3.9	−7.5	−4.2	−0.6	+9.7	+6.4	+7.4	+6.2	−1.2	−4.6	−6.7	−1.3
J (III)	−7.4	−4.6	−0.5	+4.3	+4.0	+4.2	+5.2	+5.4	+5.5	−0.7	−7.8	−8.0
yenne	−7.1	−6.3	−2.4	+2.6	+6.5	+6.9	+7.3	+6.9	+2.4	−3.1	−7.7	−6.1
Maur	−1.9	−4.1	−1.5	+1.5	+1.4	+5.2	+5.9	+2.0	−2.1	−2.4	−1.5	−2.4

[1] A. Angot, Les variations périodiques du magnétisme terrestre à Paris, Paris, Ann. Inst. Physique du Globe, 1, 276-286 (1923).

Nous avons divisé la série du Val-Joyeux en trois périodes. (I) 1901-1912, 12 années; (II) 1913-1922, 10 années; (III) 1923-1934, 12 années, qui coincident à peu près avec les périodes d'activité solaire, il n'est pas important qu'elles aient des durées strictement égales. La série du Parc Saint-Maur, calculée par A. Angot constituera un quatrième groupe. Nous avons sur chaque année éliminé la variation séculaire supposée linéaire, puis établi les écarts moyens mensuels pour les quatre éléments D, I, H, et Z (en minutes et en γ).

Les marches annuelles pour les trois périodes sont assez semblables entre elles, et assez semblables à celle du Parc Saint-Maur pour qu'on puisse admettre qu'elles ont une existence permanente, et même indépendante de la valeur absolue de la variation séculaire; les différences peuvent s'expliquer par le nombre encore trop petit d'années utilisées. Toutefois, il nous semble que la périodicité annuelle est seulement approchée et ne présente pas la régularité d'autres variations rencontrées dans l'étude du magnétisme terrestre, celle de l'amplitude diurne par exemple. Ceci n'est pas étonnant si l'on admet que la cause de la variation annuelle se trouve dans les perturbations dont la répartition moyenne, en effet, n'est pas uniforme au cours de l'année. Comme celles-ci ont une fréquence plus grande au moment des maxima d'activité solaire, il est fort probable que la variation annuelle doit être aussi plus marquée dans ces années. A. Angot, dans le mémoire cité, estime ne pas posséder une assez longue période d'observations pour faire cette recherche, mais on peut l'essayer aujourd'hui. Nous grouperons donc les années de minimum de taches solaires: 1888, 1889, 1890, 1900, 1901, 1902, 1912, 1913, 1914, 1922, 1923, 1924, 1932, 1933, 1934, et les années de maximum, 1883, 1884, 1892, 1893, 1894, 1905, 1906, 1907, 1916, 1917, 1918, 1927, 1928, 1929, soit 15 années d'une part et 14 de l'autre, et nous établirons les marches annuelles moyennes pour chaque groupe. Les résultats du calcul se trouvent dans le Tableau 2.

TABLEAU 2—*Ecarts mensuels aux époques de minimum d'activité solaire*

Elément	Janv.	Févr.	Mars	Avril	Mai	Juin	Juil.	Août	Sept.	Oct.	Nov.	Dé
Déclin., D	−0.06	−0.05	+0.10	+0.01	*−0.17*	−0.05	−0.06	+0.08	+0.06	**+0.13**	+0.10	−0
Inclin., I	+0.06	+0.01	0.00	−0.11	−0.24	*−0.30*	−0.25	−0.01	+0.19	**+0.27**	+0.24	+0
Comp.hor., H	−4.2	−2.2	−0.6	+1.0	+3.8	**+7.2**	+7.1	+3.5	−2.1	−4.9	*−5.2*	−4
Comp.vert., Z	−3.3	−3.1	−1.5	−1.5	+0.6	+5.1	**+6.6**	+4.9	+1.5	−2.1	*−3.8*	−3

Ecarts mensuels aux époques de maximum d'activité solaire

Elément	Janv.	Févr.	Mars	Avril	Mai	Juin	Juil.	Août	Sept.	Oct.	Nov.	Dé
Déclin., D	−0.13	−0.19	−0.05	−0.10	*−0.20*	−0.10	−0.02	+0.10	**+0.32**	+0.25	+0.09	+0
Inclin., I	−0.21	+0.15	+0.17	−0.06	−0.24	*−0.37*	+0.05	+0.11	+0.07	+0.21	**+0.22**	−0
Comp.hor., H	−0.4	−6.0	−4.2	+3.6	+8.6	**+10.4**	+4.5	+1.9	−1.5	−6.6	*−7.7*	−2
Comp.vert., Z	−2.2	−4.8	−4.8	+6.8	+9.4	+9.6	**+10.0**	+5.7	−0.7	−7.0	*−11.3*	−1(

Pour tous les éléments, à l'exception de l'inclinaison, l'amplitude de la variation annuelle est nettement plus grande aux époques de maximum de taches solaires. Cela confirme l'existence de la variation annuelle et sa liaison avec les perturbations.

INSTITUT DE PHYSIQUE DU GLOBE,
UNIVERSITÉ DE PARIS,
Paris, France

TERRESTRIAL-MAGNETIC ACTIVITY IN THE YEAR 1933 AND AT HUANCAYO

By J. Bartels

The measures u and u_1 of magnetic activity in the year 1933 were computed from the daily means of horizontal intensity at six stations, with the following conversion-factors k: Watheroo, 1.30, Honolulu, 1.14, San Juan, 1.24, Tucson, 1.44, Cheltenham, 1.55, and De Bilt, 1.75. To insure statistical conformity with the former series, in which always only one station with values of k over 1.50 was included, half weight was given to the results of Cheltenham and De Bilt. For Cheltenham, which appears for the first time in this series, $k=1.55$ was obtained by direct comparison of the interdiurnal variability of H at Cheltenham and Seddin for the eight years 1919 to 1926. The data for the four American stations were kindly supplied, in advance of publication, by the Director of the United States Coast and Geodetic Survey.

Table 1 continues tables for 1872 to 1930 of u-measure previously published in the Journal (**37**, p. 9 where the table for u is, by misprint, headed erroneously as u_1), and of u_1-measure (**37**, p. 15), and for 1931 and 1932 (**39**, p. 1), where short definitions of u and u_1 are repeated also.

Table 1—*Monthly means, u- and u₁-measure, 1933*

Measure	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.	Mean
u	0.68	0.63	0.68	0.48	0.96	0.61	0.56	0.81	0.88	0.61	0.51	0.64	0.67
u_1	38	33	38	18	62	31	26	50	56	31	21	34	36

The average u_1 for the Second International Polar Year, August 1932 to July 1933, is 37, indicating quiet conditions as compared with the first Polar Year, 1882 to 1883, when $u_1=64$, including the highly disturbed months October ($u_1=108$) and November 1882 ($u_1=136$). This compares with the relative sunspot-numbers, namely, $R=59$ for the first and $R=8$ for the second Polar Year.

In view of the unexpectedly large diurnal variations of horizontal intensity, H, observed at Huancayo Observatory, Peru, it was considered of interest to compute the interdiurnal variability also. From 73 months with complete observations at Huancayo in the years 1922, 1923, and 1927 to 1932, the average interdiurnal variability of the daily means of H, expressed in the unit $10\gamma=0.0001$ gauss, was found as 1.008, while, for the same months, the average of the monthly u-values (as calculated from three to five other observatories and published in the Journal) is 0.846. The conversion-factor for Huancayo becomes, therefore, $k=0.846/1.008=0.840$. This is the first case in which k has been found smaller than unity; it expresses the fact that the daily means of H at Huancayo vary more than the extrapolation to the magnetic equator from the results of other observatories, including Batavia, would in-

dicate; this seems to be in keeping with the results for the diurnal variation of H at Huancayo. The ten most disturbed months (judged from both nominator and denominator of k so as to avoid trivial statistical mistakes) yield $k=1.42/1.61=0.88$, and the ten least disturbed months yield $k=0.57/0.68=0.84$, values sufficiently close to the general average $k=0.84$ to indicate linear relations between the interdiurnal variability at Huancayo and other observatories. The exceptional size of the interdiurnal variability of the daily means at Huancayo may be partly due to the large diurnal variation changing from day to day, high noon values raising the daily mean level of H more above the comparatively steady night values than small values; this will be tested later.

Department of Terrestrial Magnetism,
Carnegie Institution of Washington,
Washington, D. C.

CORRELATION BETWEEN AURORAL AND MAGNETIC ACTIVITIES AT CHESTERFIELD, CANADA, 1932-33

By F. T. Davies

Fifty-six clear nights of the winter of 1932-33 are chosen for this comparison of (*a*) inter-diurnal and (*b*) diurnal auroral and magnetic activity. This eliminates the effect of cloudiness on auroral observations. The progressive effect, as the season changes, of increasing daylight on auroral observations, has been minimized by considering only the night hours from 18^h to 6^h. The intensity of aurora seen within a few minutes of the exact hour is expressed on a scale of 0 to 4 for increasing intensity. The intensity of magnetic disturbance is estimated from inspection of the magnetograms for this period. Ordinarily a scale of 0 to 2 is used for increasing intensity of magnetic disturbance. For the present comparison a scale of 0 to 3 is used. As only eleven of more than 700 observations of aurora in this period are recorded as intensity 4, auroral and magnetic intensities are thus estimated on practically the same scale.

Inter-diurnal relation between auroral and magnetic activity—Table 1 gives $A = \Sigma$ auroral intensities for each night and $M = \Sigma$ magnetic intensities for the same hours for each night. The correlation-coefficient between A and M is $+ 0.63$. This is more than eleven times the probable error and represents very good evidence of a relation between auroral and magnetic activity from one day to another.

Table 1—*Inter-diurnal relation between auroral and magnetic activities*

Date	A	M	Date	A	M	Date	A	M	Date	A	M
1932											
Oct. 9	7	0	Dec. 14	0	17	Jan. 24	10	14	Mar. 8	3	1
Oct. 28	4	2	Dec. 16	18	26	Jan. 25	23	18	Mar. 9	0	7
Nov. 2	10	18	Dec. 19	18	21	Jan. 26	30	22	Mar. 10	8	9
Nov. 19	32	20	Dec. 20	19	18	Jan. 27	25	25	Mar. 12	2	6
Nov. 20	19	14	Dec. 29	16	4	Feb. 3	10	0	Mar. 15	11	9
Nov. 22	24	16	Dec. 30	23	15	Feb. 4	6	3	Mar. 21	18	20
Nov. 23	14	8	Dec. 31	19	17	Feb. 14	14	17	Mar. 26	18	8
Nov. 24	13	0	*1933*			Feb. 15	18	18	Mar. 27	12	13
Nov. 25	3	2	Jan. 5	11	3	Feb. 17	5	0	Mar. 28	4	11
Nov. 26	16	7	Jan. 7	13	17	Feb. 20	23	17	Apr. 6	6	10
Dec. 6	2	1	Jan. 9	3	9	Feb. 27	15	16	Apr. 8	6	7
Dec. 8	15	11	Jan. 18	14	8	Feb. 28	18	11	Apr. 9	2	7
Dec. 9	8	17	Jan. 19	27	10	Mar. 1	8	6	Apr. 10	0	6
Dec. 10	8	6	Jan. 22	11	14	Mar. 7	0	3	Apr. 18	12	15
			Jan. 23	28	15						

Correlation coefficient $(r) = +0.63$; probable error $(e) = 0.055$; ratio $r/e = 11.5$

Diurnal relation between auroral and magnetic activity—Table 2 gives comparable values of A and M for the mean of the 56 nights considered. The magnetic intensities are mean values for whole hours and therefore

Table 2—*Diurnal relation between auroral and magnetic activities*

Activity	90° west meridian time											
	18.5	19.5	20.5	21.5	22.5	23.5	0.5	1.5	2.5	3.5	4.5	5.5
Auroral, A	36	46	51	60	62	62	76	74	60	55	40	25
Magnetic, M	25	39	48	60	68	77	79	58	47	30	14	14

Correlation-coefficient (r) $= +0.86$; probable error (e) $= 0.05$; ratio $r/e = 17$

correspond to half-hours. This is equivalent to a smoothing process as compared with the auroral intensities which are not hourly means but are values at exact hours. Each pair of hourly A-values has therefore been averaged and this value assigned to the half-hour for which an average magnetic intensity has already been found. In this case the correlation-coefficient is $+ 0.86$ and is 17 times the probable error.

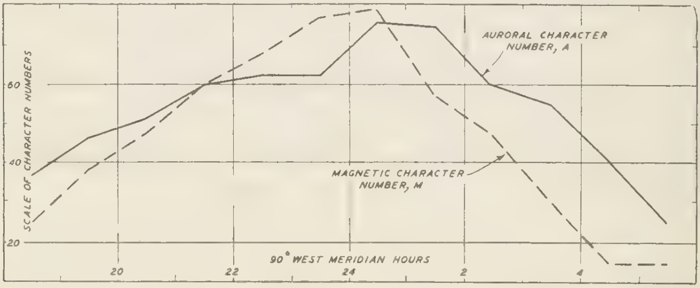


FIG. 1—VARIATION OF AURORAL AND MAGNETIC CHARACTER-NUMBERS DURING NIGHT AT CHESTERFIELD, CANADA, OCTOBER, 1932 TO APRIL, 1933

Figure 1 illustrating Table 2 indicates that the maximum of auroral activity lags behind the magnetic maximum by rather less than one hour.

Both inter-diurnal and diurnal comparisons indicate that a close relation exists between auroral and magnetic activity.

Department of Terrestrial Magnetism,
Carnegie Institution of Washington,
Washington, D. C.

A REPORT OF WORK ON THE AURORA BOREALIS FOR THE YEARS 1932–1934

By Veryl R. Fuller*

It is the purpose of this paper to give a summary of the visual observation of the aurora at the Alaska Agricultural College and School of Mines (to be designated University of Alaska from July 1, 1935) during the years beginning August 1932 and ending December 31, 1934. The data used represent practically a continuous watch during the hours of darkness for these years and it is felt that it is rather complete in detail. For the methods of observation and the type of data taken reference is made to an earlier publication.[1]

Since the auroral season begins in August and continues until the following May the data have been grouped by seasons rather than by years. For the two and one-half seasons covered, an idea of the number of displays relative to the total number of days in each group may be gained from Table 1.

Table 1—*Particulars on occurrence of aurorae at College, 1932-34*

Number days	Season			Percentage,[a] season		
	1932–33	1933–34	1934	1932–33	1933–34	1934
Aurorae observed	191	163	91	69.6	62.3	67.4
Clear but no aurorae observed	35	25	10	12.8	9.0	7.4
Cloudiness prevented observation	48	75	34	17.6	28.7	25.2

[a]Percentage of the total number of days.

It will be observed that these percentages of total number of days are of approximately the same magnitude for each group and it can therefore be assumed in the discussion which follows that "average years" are being studied.**

The graphs of Figures 1*A*, 1*B*, and 1*C* place the observations in such form that a rapid survey may be made. Days and months are given as abscissas and Greenwich mean times (G. M. T.) as ordinates. The time of duration of each display is represented by vertical lines showing the times of beginnings and endings of displays, the total time of the day's activity, and by means of a cross the times, marked by crosses on the vertical lines, when the displays appeared to have maximum intensity. The envelope-curves, which might be drawn connecting the ends of the lines which show the duration of displays would indicate roughly the hours of darkness.

*This report was received from Professor Fuller shortly before his sudden death May 30, 1935. It is hoped the work of reduction and discussion, which he had well under way, on the large number of parallactic auroral photographs he had made may be completed by his successor, Dr E. H. Bramhall.

[1]Veryl R. Fuller, Auroral observations at the Alaska Agricultural College and School of Mines for the year 1931-1932, Terr. Mag., **38**, 207-238 (1933); see also Terr. Mag., **36**, 297-308 (1931), and **37**, 159-166 (1932).

**Perhaps the percentages of *clear* nights on which aurorae were observed are of greater significance than the percentages indicated in Table 1; on this basis the percentages would be for 1932-33 84.5, for 1933-34, 86.6, and for 1934, 90. It will be noted these show a slight increase—a condition to be expected because of increasing solar activity from the minimum of 1932-33.—*Ed.*

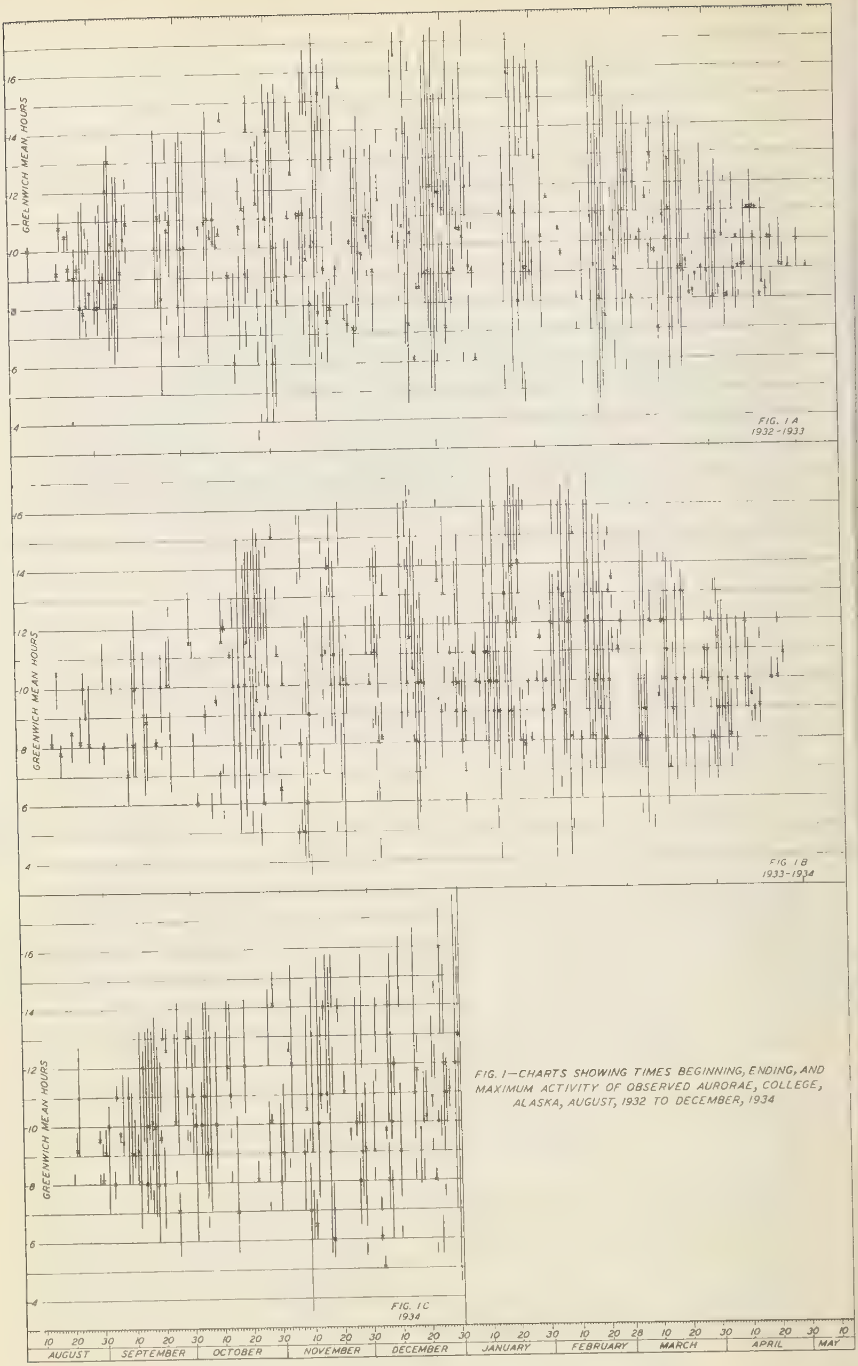


FIG. 1—CHARTS SHOWING TIMES BEGINNING, ENDING, AND MAXIMUM ACTIVITY OF OBSERVED AURORAE, COLLEGE, ALASKA, AUGUST, 1932 TO DECEMBER, 1934

At first glance it is seen that certain groupings occur which might be of interest even though it is realized that in part, at least, these are influenced by nights when cloudiness prevented observation.

To obtain a more detailed survey of these periods the curves of Figures 2*A*, 2*B* and 2*C* were drawn. These show the day-by-day variation of the aurorae by a "character-number" which was devised to express the activity of displays in terms of the duration, area covered, and form. Total cloudiness is shown by dotted portions of the graphs. These dotted portions are not intended to represent the auroral conditions prevailing for the days covered by them but only to give continuity to the graphs.

An examination of Figure 1*A* for August 1932 to May 1933, indicates that the apparent grouping of graphs is real, and that a more or less regular sequence of times of maximum and of minimum displays occurs. Examining the graphs to determine the frequencies of these periods of activity one of short duration seemed to be indicated. No single period of only a few days was found. A longer period of about twenty-seven days and a weaker one of about fourteen days are indicated. Figure 3*A* was drawn from data obtained by dividing the entire period of nearly nine months into groups of 27-day intervals and taking the average character-number for each day of the interval. Inspection of the Figure indicates a period of 27 days and also other periods ot shorter time; these, however, were not found by applying the same procedure of groupings.

As a result of the apparent 27-day period, a correlation was attempted between magnetic activity and auroral displays. This is not easy since few magnetic data were available and because of the complexity of the auroral activity and the rregularity of observation because of cloudiness. As a preliminary the auroral character-number was compared with the international magnetic character-number for each day of the season 1932-33. This comparison, of course, would represent only the activity for the day and would not take into account the type of magnetic activity, the type of auroral activity, nor weather-conditions. Nevertheless, a correlation of better than 60 per cent was found. Probably it would require a minute comparison of the magnetogram with continuous auroral observation to arrive at a satisfactory knowledge of the s multaneous activity of the two phenomena.

It is of interest to note that a similar study has been made by W. J. Rooney[2] in which the relation of earth-current at this Station to auroral observations showed a correlation approximately the same.

Figures 3*B* and 3*C* show this 27-day period to occur for the seasons 1933-4 and for the fall of 1934.

To determine the distribution of displays throughout the year, the curves of Figures 4*A*, 4*B*, and 4*C* were drawn; these must be considered separately since they are not similar for the different years. Graph *a* of Figure 4*A* shows that the maximum number of displays occurs in December with a smaller maximum in March. The corresponding curve of Figure 4*B* indicates a maximum of occurrences in February, and that of Figure 4*C*, which shows occurrences for only the months from August through December, indicates a maximum for December with a smaller

[2]Aurorae and earth-currents, Terr. Mag., 39, 103-109 (1934).

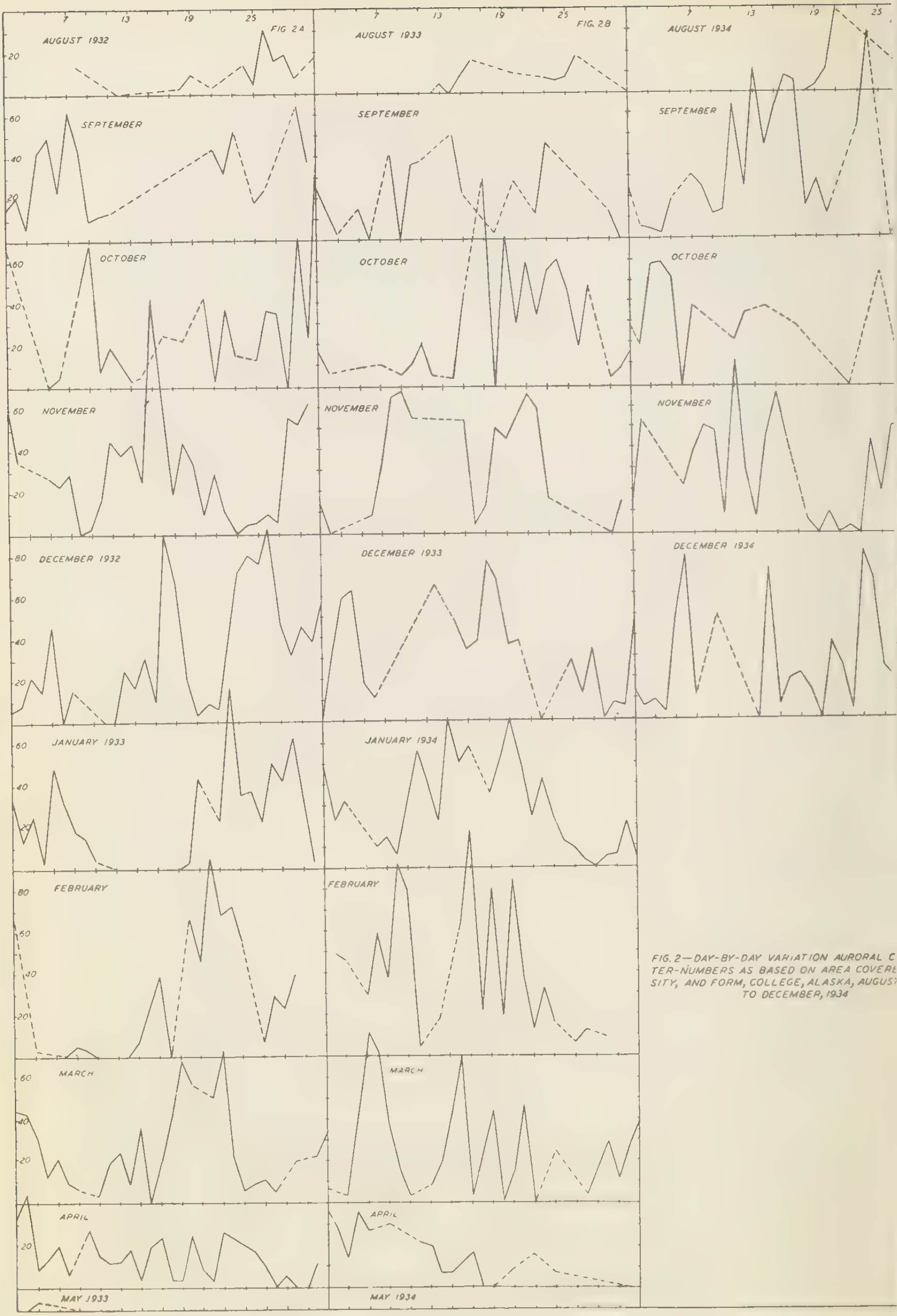


FIG. 2—DAY-BY-DAY VARIATION AURORAL C TER-NUMBERS AS BASED ON AREA COVERE SITY, AND FORM, COLLEGE, ALASKA, AUGUS TO DECEMBER, 1934

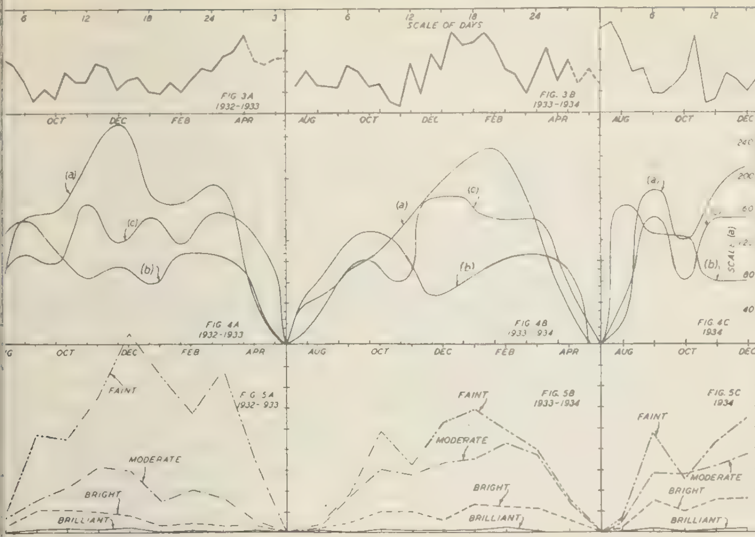


FIG. 3—AVERAGE DAILY AURORAL CHARACTER-NUMBER FOR EACH DAY OF 27-DAY INTERVALS, COLLEGE, ALASKA, AUGUST, 1932 TO DECEMBER, 1934

FIG. 4—(a) TOTAL NUMBER TIMES AURORAE OBSERVED DURING EACH HALF-HOUR OF WATCH, (b) PERCENTAGE NUMBER OBSERVED AURORAE TO POSSIBLE NUMBER TAKING ACCOUNT TOTAL HOURS DARKNESS AND CLOUDINESS, AND (c) VISIBILITY AS PERCENTAGE OF HOURS OF DARKNESS AND CLOUDINESS, COLLEGE, ALASKA, AUGUST 1932, TO DECEMBER, 1934

FIG. 5—MONTHLY TOTAL NUMBER OBSERVED AURORAE OF DIFFERENT INTENSITY, COLLEGE, ALASKA, AUGUST, 1932 TO DECEMBER, 1934

one in September. Graph *b* of this figure shows the percentage of displays observed to those which could have been seen taking into consideration cloudiness and hours of darkness. Graphs *c* of Figures 4*A*, 4*B* and 4*C* show the visibility for each month in terms of hours of darkness and cloudiness.

To show graphically the distribution of intensity according to months throughout the auroral season the curves of Figures 5*A*, 5*B*, and 5*C* were drawn. For the sake of simplicity the intensity of displays are considered as faint, moderate, bright, and brilliant, although of course graduations of each actually occur. Figure 5*A* shows two maxima, for faint displays, one in December, the other a smaller one in March. Moderate displays are at a maximum in November and December. Bright displays are shown to be most numerous during the months of September, October, November, and December. Brilliant displays seem to be fairly evenly distributed through the season except for the months of August, January, and March when none was observed. Figure 5*B* shows maxima for faint displays in October and January. Moderate displays are quite evenly distributed showing a greater number in October and February. Bright displays are more numerous in October, November, and January. Brilliant displays are evenly distributed except for September and April. Figure 5*C* shows a maximum for displays of all intensities for September and December.

To study the variation of auroral occurrence throughout the night, the curves of Figures 6*A*, 6*B*, and 6*C* were drawn. These show the total number of times that auroræ of all forms were seen during each half-hour of the night for each month. An inspection of these curves indicates no particular one time during the night when aurora is observed to occur more often than another. There is, however, a predominatingly

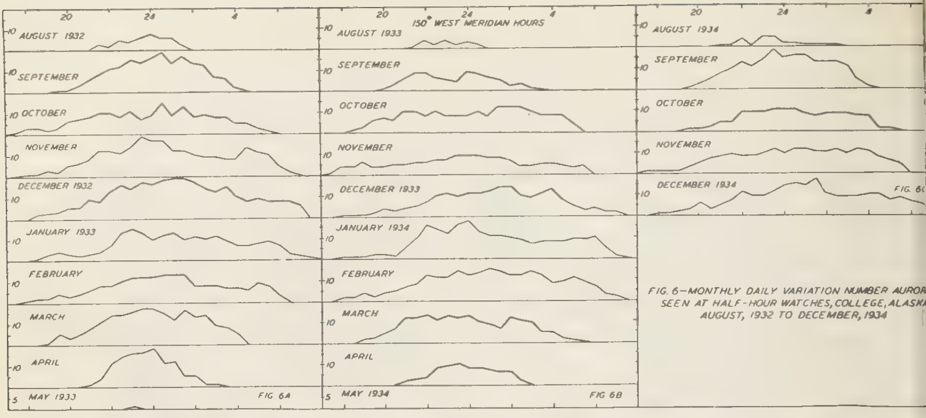


FIG. 6—MONTHLY DAILY VARIATION NUMBER AURORAE SEEN AT HALF-HOUR WATCHES, COLLEGE, ALASKA, AUGUST, 1932 TO DECEMBER, 1934

large number during the hours between 22^h and 2^h local time. This holds true for all of the curves. The hour of greatest number of displays does not appear to follow any particular sequence of change throughout the year nor does it occur at the same hour for the same months for different years. If the ordinates of each figure are added for the different hours the greatest number of observed displays is found to be between 24^h and 1^h, as shown by the graphs for "totals" of Figures 7*A*, 7*B*, and 7*C*.

The distribution of the different forms through the night for the three seasons is shown by Figures 7*A*, 7*B*, and 7*C*. Figure 7*A* indicates a predominating maximum of occurrence at 1^h with the exception of the graph of distribution for arcs, which shows two maxima—one at 23^h and the other at 1^h.

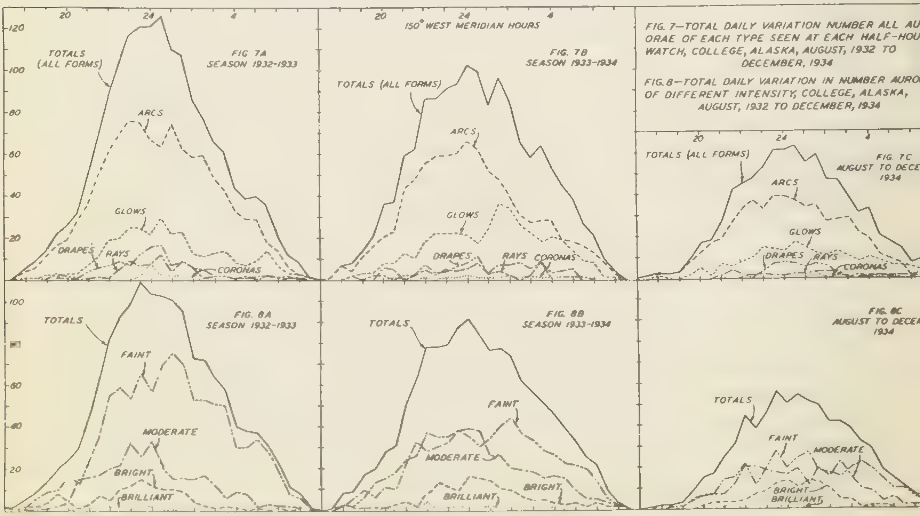


FIG. 7—TOTAL DAILY VARIATION NUMBER ALL AURORAE OF EACH TYPE SEEN AT EACH HALF-HOUR WATCH, COLLEGE, ALASKA, AUGUST, 1932 TO DECEMBER, 1934

FIG. 8—TOTAL DAILY VARIATION IN NUMBER AURORAE OF DIFFERENT INTENSITY, COLLEGE, ALASKA, AUGUST, 1932 TO DECEMBER, 1934

In the curves of Figures 8*A*, 8*B*, and 8*C* is seen the diurnal variation throughout the night of displays of different intensity. Moderate, bright, and brilliant displays are found to be most numerous between 23^h and 24^h. Again, however, we find that these maxima occur at a different time for the different years.

These curves for diurnal variation show little agreement with observations made by B. W. Currie of the Canadian Meteorological Service of Canada at Chesterfield Inlet during the Polar-Year Expedition of 1932-33.[3]

This would lead to the conclusion that perhaps local conditions are a factor influencing the occurrence of displays at a given place.

An examination of the data here presented will show, that while it may represent the average year it covers far too short a period of years to yield sufficient material for a thorough comparison with other apparently related phenomena. So many complexities enter into the observation of the aurora which is itself of so varying a nature that continuous data should be taken over an extended period of years. It is unfortunate that economic conditions are such as to prevent the appropriation of funds for the continuance of the observations.

[3]Summary of some auroral height-measurements and observations at Chesterfield, Canada, Terr. Mag., 39, 293-297 (1934).

UNIVERSITY OF ALASKA,
College, Alaska

SCRASE, F. J.: *The charged and uncharged nuclei in the atmosphere and their part in atmospheric ionization.* London, Met. Office, Geophys. Mem., No. 64, 1935 (15 with 1 pl.).

An Aitken nuclei-counter, fitted with a cylindrical condenser for removing charged condensation-nuclei, was employed in this investigation to secure the number of charged (N) and uncharged (N_0) nuclei in the atmosphere at the Kew Observatory. Simultaneous observations of the positive conductivity of the air were made, employing the Wilson method. The ratio of uncharged to charged nuclei was plotted with visibility-values made in a horizontal direction, for relative humidity of the air, and for the nuclei-content (Z) of the air.

Below 80 per cent relative humidity the value of the ratio N_0/N is practically independent of relative humidity. The two lowest values of N_0/N are associated with the highest humidities. This fact was interpreted as an indication that the nuclei increase in size at the higher relative humidities owing, presumably, to condensation of water around the nuclei. The nuclei-content of the air was found to increase more or less linearly with increase in relative humidity. Poor visibility appears to be associated with low values of N_0/N and, hence, with the presence of larger nuclei, the growth of which is a consequence of the high humidities. The values of N_0/N when grouped according to the values of Z, show no systematic change until the nuclei-content exceeds 50 per mm^3, at which point a drop in the ratio occurs. This result is contrary to evidence recently published by O. W. Torreson and G. R. Wait. This evidence indicated that increase in the ratio occurs with increase in the nuclei-content of the air when results from various stations over the world are considered as may be seen on the accompanying graph.

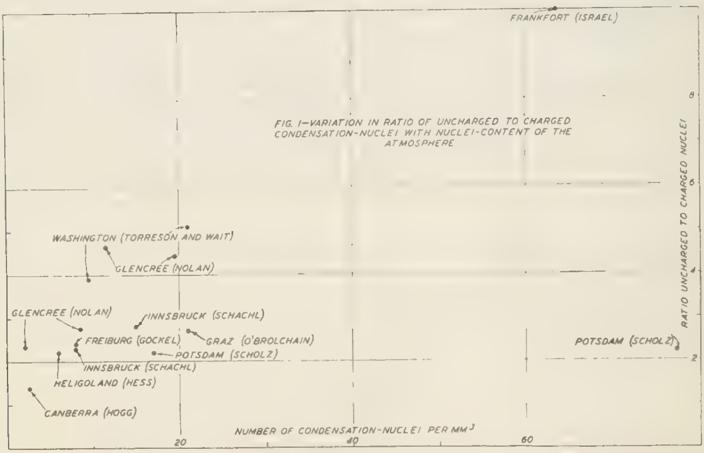


FIG. 1—VARIATION IN RATIO OF UNCHARGED TO CHARGED CONDENSATION-NUCLEI WITH NUCLEI-CONTENT OF THE ATMOSPHERE

Combination-coefficients between small ions and charged and uncharged nuclei have been computed using formulas recently developed by Whipple. The combination-coefficients remain practically constant when the nuclei-content is below 50 per mm^3 but increase above this value. The absolute values of the coefficients are much smaller than those obtained by Nolan and de Sachy, but the author calls attention to the fact that the latter values were obtained for room-air. Whipple's formulas were also used to calculate the rate of small-ion formation in open air, the average rate being about 4.5 pairs per cc per second.

The influence of smoke-particles on the resistivity of the air has been reexamined. It is concluded that the effect is much smaller than Wright's original estimates appear to indicate.

Several points raised by Scrase can be discussed to advantage but any adequate discussion could hardly be kept within the limits of a review.

G. R. WAIT

ON THE ORIGIN OF THE SPACE-CHARGE IN THE ATMOSPHERE

By W. W. Hansen

The purposes of the present note are to point out some of the errors in a paper by F. J. W. Whipple[1] and to discuss qualitatively the space charge near the Earth's surface. This latter problem has been solved more exactly by W. F. G. Swann,[2] E. R. von Schweidler,[3] and others and the discussion to follow adds nothing essentially new to their results but is perhaps of interest because the main results are obtained by extremely simple considerations.

As Whipple says, the current at the surface of the Earth is carried entirely by positive ions. This is because the negative ions are removed by the field and none comes from the Earth to replace them. Thus a positive space-charge exists at the surface of the Earth; and this conclusion holds irrespective of whether there is any diffusion or not. In fact if i is the air-earth current, F is the field strength (normally positive), and w is the mobility of the positive ions, the space-charge will be $+(i/F w)$, also $dF/dx = +(4\pi\, i\ /Fw)$. These conclusions depend on the assumptions that all the current at the ground is carried by positive ions, that at $x = 0$ there is no space-charge except that of the positive ions, that Poisson's equation holds, and that a steady state has been reached. Now Whipple's equations [either (9) or (11)] indicate that the space-charge at the surface depends on his diffusion constant k, in fact, if $k = 0$ they lead to an infinite value for the space-charge. The cause of this trouble is that Whipple's equation (2) is incorrect as is seen if we make k approach zero when it leads to

$$i = (\lambda_1 + \lambda_2)\ F$$

which, as Whipple states, is incorrect at the Earth's surface. Thus Whipple's statement, "The accumulation of positive electricity near the ground on account of the interplay of conduction and diffusion is known as the electrode-effect," is entirely misleading. There is a positive space-charge at the ground because the field carries the negative ions away and the conclusion is independent of whether there is or is not diffusion.

It is perhaps worth while to state the above reasoning in another way. Let i and F be considered as vectors and let a conductivity λ be defined such that $i = \lambda F$. Then when equilibrium is reached, we have

$$\nabla \cdot i = 0$$

$$\nabla \cdot \lambda F = 0 = \lambda \nabla \cdot F + F \cdot \nabla \lambda$$

Thus

$$4\pi\rho = \nabla \cdot F = -\frac{F \cdot \nabla \lambda}{\lambda}$$

[1] Terr. Mag., 37, 355-359 (1932).
[2] Terr. Mag., 18, 163-184 (1913).
[3] Wien. SitzBer. Ak. Wiss., 117, IIa, 653-664 (1908).

We conclude that if Ohm's law holds (corresponding to the assumption that the mobility is independent of F), and if the laws of electrostatics apply, then the necessary and sufficient condition for the existence of a steady space-charge is that λ shall change going along the lines of F. In the above the change of conductivity considered was due to the decrease in the number of negative ions near the ground because of their being swept out by the field but variation of λ from any cause whatever will give rise to a space-charge. Thus if λ is known as a function of height and also i, or, what is equivalent, F at some height, then the space-charge is readily computed as

$$\rho = -(i/4\pi\lambda^2)\, d\lambda/dx$$

If we neglect diffusion it is easy to get a semi-quantitative treatment of the phenomenon. Thus let us assume as a rough starting approximation that both F and n_1, the number of positive ions per cm^3, are independent of height. Of course this cannot possibly be correct as the product n_1F is twice as great at $x = 0$ as at $x \rightarrow \infty$ (we assume the mobilities of the two signs of ion to be the same) but we will show presently that because of the fortunate values of certain constants this assumption will not lead to any serious error.

Since we assume n_1 constant the loss of negative ions by recombination is proportional to their number, which we denote by n_2. Also since F and hence the ion-velocity is constant there is a linear relation between the time and the distance a negative ion has gone. This leads to an exponential behaviour.

$$n_2 = n_1\left[1 - e^{-(\alpha n_1/Fw)x}\right]$$

Finding the space-charge from this and integrating to find the field at the ground, we get

$$F_0/F_\infty = [1 + 4\pi\,(e\,w/\alpha)]$$

with α the recombination-coefficient, F_0 the field at the surface of the Earth, and F_∞ the field at a great height. If we take the value of α for young ions, the quantity $4\pi\,(e\,w/\alpha)$ is not far from unity, so that F_0 is about twice F_∞ and n_1 is practically constant, and so our approximate solution is quite good. Thus we find that the ratio of field-strength at ground to field-strength at a great height depends only on the quantity $(e\,w/\alpha)$ and that the distance in which the "electrode-effect" is important is of the order $n_1\,F\,w/q = i/2\,q\,e$, though the exact value of this distance may also be a function of $(e\,w/\alpha)$. Here q is the rate of formation of ions. How good the above approximation is depends on the value of $(4\pi\,e\,w/\alpha)$ and this is rather uncertain because α is a function of the age of the ion[4]. If we take a value of α corresponding to ions several seconds old, as seems reasonable, this quantity is about ten and our calculation is not of much use quantitatively, though it is still instructive from a qualitative point of view.

We note that n_1 and hence the conductivity due to positive ions is practically independent of height, as is assumed by Whipple. But this is entirely accidental, for it is possible to change $(e\,w/\alpha)$ without changing $(\sqrt{q}\,e\,w/\sqrt{\alpha})$. In this way we could change the value of n_1 at the ground

[4] O. Luhr, Phys. Rev., 35, 1394-1404 (1930); 36, 24-34 (1930).

without changing its value at a great height and so alter the ratio of current to voltage.

These conclusions can be reached more exactly and without appreciably greater complication by examination of the differential equation for F, which is

$$F^3(\mathrm{d}^2 F/\mathrm{d}\, x^2) + (1 - \alpha/8\pi\, e\, w)\, F^2\, (\mathrm{d}F/\mathrm{d}x)^2 + [8\pi\, q\, e/w]\, [F_\infty^2 - F^2] = 0$$

This equation is to be solved subject to the boundary conditions that F shall approach F_∞ at large values of x and that $F\,(\mathrm{d}\, F/\mathrm{d}\, x) = (8\pi \sqrt{q\, e/\alpha})\, F_\infty$ at $x = 0$. Now we note that if $F\,(x)$ is a proper solution, so also is $2F\,(2x)$. This proves rigorously, what we previously surmised, that the distance in which the electrode-effect is important is a linear function of F_∞. The above observation together with the knowledge that $(i/q\, e)$ is a fundamental length suggests the following change of variables. We let

$$y = 2\sqrt{q/\alpha}\,(e\, w/i)\, F$$

$$z = (2q\, e/i)\, 2\sqrt{2\pi}\, \sqrt{w\, e/\alpha}\, x$$

and so find

$$y^3\,(\mathrm{d}^2 y/\mathrm{d}z^2) + (1 - \alpha/8\pi\, w\, e)\, y^2\, (\mathrm{d}\, y/\mathrm{d}\, z)^2 - y^2 + 1 = 0$$

The boundary-conditions on y are that as $z \to \infty$, y shall approach one, and that at $z = 0$, $y\,(\mathrm{d}\, y/\mathrm{d}\, z) = -\, 8\pi\, w\, e/\alpha$. We now see clearly that the ratio of the values of y at $z \to \infty$ and at $z = 0$, and hence the ratio F_∞/F_0, can depend only on the quantity $8\pi\, w\, e\, /\, \alpha$.

Finally we may state that it is not the object of this paper to contend that transportation of the ions by turbulence of the air is unimportant. In fact, we believe the contrary is probably true as can be seen by comparing the velocity of an ion in the Earth's field with likely velocities of air-currents. We merely state that a steady space-charge arises only when the conductivity of the atmosphere is not constant while on the other hand if the conductivity were constant no amount of turbulence could produce a space-charge.

I wish to express my thanks to Professor J. G. Brown for calling my attention to this problem and for valuable aid, especially in pointing out the papers by Swann and von Schweidler.

STANFORD UNIVERSITY,
Stanford University, California

NOTES

(See also page 342)

20. *Meteorological Institute of the University of Berlin*—As a consequence of the reorganization of the meteorological service in Germany, the Prussian Meteorological Institute was discontinued July 1, 1934. At the same time there was established at the Friedrich Wilhelms-Universität in Berlin, a Meteorological Institute, the chief functions of which will be instruction and research in meteorology. Professor H. von Ficker, who since 1923 had been Director of the former Prussian Meteorological Institute, has been chosen the first Director of the new University-Institute which at present is located at Schinkelplatz 6, Berlin, W. 8, Germany.

21. *Geophysical survey of northern Australia*—The Governments of the Commonwealth, Queensland, and Western Australia are undertaking an aerial, geological, and geophysical survey of selected areas north of approximately the twenty-second parallel of south latitude. It is expected that the survey will occupy a period of about three years and that about 30,000 square miles of territory will be covered. This survey is of outstanding national importance and its principal object is the development of the northern parts of Australia through the medium of mineral discoveries.

22. *Loss of the Dana*—It is reported in *The Times* of June 24, 1935, that the Danish Government's scientific research ship *Dana* sank on June 23 in the North Sea, sixty miles west of Ringkjöbing, Jutland, after a collision with a German trawler. The Director of the vessel's scientific work, Dr. A. V. Tåning, and the crew were saved. The *Dana* was well known to men of science and others through the work of the late Professor Johannes Schmidt, Director of the Carlsberg Laboratory, Copenhagen, on the migration of eels. It may be remembered that the oceanographical expedition of the *Dana* in 1928-30 was described in an article by Professor Schmidt in *Nature* of March 21, 1931, and March 28, 1931.

23. *Errata*—In the June number of the JOURNAL the following correction is to be made in the article by Victor F. Hess and Rudolf Steinmaurer: Page 201, in last sentence of third paragraph for "711 mm for Innsbruck" read "710 mm for Innsbruck"; page 202, in footnote 4 the order of authors named should be "V. F. Hess, H. Graziadei, und R. Steinmaurer."

24. *Magnetic survey of Portuguese colonies*—It was decreed by the Portuguese Government July 17, 1935, that a magnetic survey of the colonies of Angola and Moçambique should be carried out within the next two years and the necessary funds were appropriated for the purpose. This decision on the part of Portugal will contribute greatly to our knowledge of the secular variation of the magnetic elements in portions of Africa where such information is greatly needed for investigations of the Earth's general magnetic field.

25. *Intercomparisons of observatory standards*—R. H. Mansfield of the Department of Terrestrial Magnetism, Carnegie Institution of Washington, engaged since April 1934 in reoccupying magnetic stations in Africa primarily to determine secular variations, returned to Washington August 2, 1935. En route he compared his instruments with the standards of the Niemegk Magnetic Observatory in Germany and of the Abinger Observatory in England. Early in August he compared them with the standards of the Cheltenham Magnetic Observatory.

Now that facilities of electric current and additional piers and housing for absolute observations are available at the Cheltenham Magnetic Observatory of the United States Coast and Geodetic Survey it has been possible in cooperation with the Department of Terrestrial Magnetism to install, as long planned, C. I. W. sine-galvanometer No. 1 and appurtenant current-measuring equipment. During July 1935 comparisons were obtained by observers of the Survey and of the Department between the sine-galvanometer, the Observatory's magnetometer, and the standard instrument of the Department upon which the latter's provisional international magnetic standards were based in 1914 from many intercomparisons by observers of the Department at the principal magnetic observatories of the world. Through the comparisons previously made at the Agincourt Observatory and those above referred to it will be possible to effect a comparison and control of the electromagnetic standards in use at Cheltenham, Abinger, and Agincourt as well as the C. I. W. provisional international magnetic standards. The C. I. W. sine-galvanometer will be used hereafter as the standard at Cheltenham for horizontal intensity.

UEBER DIE NACHLIEFERUNG VON RADIUMEMANATION AUS DEM ERDBODEN

Von P. Robert Zeilinger

Abstract—The amount of radon given off from the soil was measured by the method of P. Zupancic with certain improvements. During December 1933 to October 1934, 150 measurements were made at two points in a meadow near Hötting; observations were made alternately at these. The average value is 43.5×10^{-18} curie per cm² per sec. Considering the fact that very few measurements were made in the autumn, it must be concluded that the annual average is nearly 50×10^{-18} curie per cm² per sec. This amount of exhalation agrees well with the results of Smyth (Dublin) and Wright and Smith (Manila), who obtained 73×10^{-18} and 20.5×10^{-18} curie, respectively. It is interesting to note that P. Zupancic in the garden of the Institute in Innsbruck obtained an average exhalation of 23×10^{-18} curie per cm² per sec (December-July), that is, about half as much as the author on the meadow in Hötting where the soil was never mixed with fragments of brick, etc. It was found that the meteorological factors governing the exhalation of radon are not the same in different seasons of the year. In winter the influence of snow or ice on the ground prevails. The exhalation increases rapidly when the snow is melting. In spring rather large variations in the amount of exhalation were found corresponding to large differences of temperature, etc. A lawn-covered area gives off less radon than an area of the same soil without lawn. The highest values of exhalation were found in July and August. The relationship of exhalation to different meteorological factors is discussed. Wind decreases the amount of exhalation measured. This may be due to the removal of radon within the collecting vessel by the small rapid changes of barometic pressure during high winds. The daily variation of exhalation was studied and found in good agreement with the curve of P. Zupancic. Simultaneous measurements of the radon-content of open air showed a daily variation similar to that of the exhalation.

§ 1—*Einleitung und Messverfahren*

In der Zeit von Dezember 1932 bis Juni 1933 wurde im Institut für Strahlenforschung an der Universität Innsbruck die Nachlieferung an Radiumenmanation aus dem Erdboden untersucht. Methode und Ergebnisse dieser von P. R. Zupancic gemachten Beobachtungen sind in Terrestrial Magnetism and Atmospheric Electricity[1] veröffentlicht worden.

Die vorliegende Arbeit ist eine Fortsetzung und Erweiterung der genannten Untersuchung. Die Versuchsanordnung, die P. Zupancic bei seinen Messungen verwendet und die sich bewährt hatte, wurde —abgesehen von geringen Abänderungen— beibehalten. Da sie in der obenangegebenen Veröffentlichung eingehend beschrieben ist, seien hier nur die Änderungen erwähnt, die bei meinen Untersuchungen eingeführt wurden. Bei den Messungen 1932–33 blieb das hermetisch abgeschlossene Ansammlungsgefäss während der ganzen Versuchsdauer eines halben Jahres auf demselben Platz. Dadurch waren aber Niederschläge, Sonnenstrahlung, und überhaupt sämtliche atmosphärischen Einflüsse von der zu untersuchenden Bodenfläche abgeschirmt und die betreffende Rasenfläche einer unnatürlichen Umbildung ausgesetzt.

Für die vorliegenden Untersuchungen wurden zwei Plätze durch kreisförmige, schmale Rinnen abgegrenzt. In diese mit Wasser gefüllten Rinnen konnte nun das Ansammlungsgefäss abwechselnd eingesetzt werden und so war jeweils ein Platz der Freiluft ausgesetzt. Dem Wasser musste natürlich im Winter eine Flüssigkeit beigemengt werden, die ein Gefrieren verhinderte. Die Rinnen reichten etwa 15 cm in den Boden hinunter, sodass eine Ventilation nach aussen durch den Erdboden hindurch praktisch hintangehalten war.

[1]Terr. Mag., 39, 33-46 (1934).

Da nun während der Entnahme durch einen zweiten offenen Hahn Freiluft zugeleitet werden muss—um nicht durch die rasche Druckverminderung Emanation aus dem Boden zu saugen—darf der Ansammlungsraum nicht zu klein sein, sondern muss in einem derartigen Verhältnis zur abgeführten Luftprobe stehen, dass die Menge der eintretenden Freiluft den Emanationsgehalt nicht merklich fälscht. Entsprechend den Dimensionen des von mir verwendeten Ansammlungsgefässes (60-cm Durchmesser) und der Ionisationskammer (9 Liter) musste der Ansammlungsraum ungefähr 45 Liter betragen und somit konnte das Gefäss nur so tief eingegraben werden, dass es mindestens 15 cm über dem Erdboden herausragte.

Bei den oben angegebenen Dimensionen ist es immerhin möglich, bereits innerhalb 1 bis 2 Stunden eine bequem feststellbare Menge Emanation anzusammeln. Das war insbesondere wertvoll bei den Untersuchungden des täglichen Ganges der Exhalation. (Unter Exhalatien verstehen wir die pro qcm und sec aus dem Boden entströmende Emanationsmenge, ausgedrückt in curie qcm sec). Dadurch, dass nach jeder Ansammlung das Gefäss von der Bodenfläche abgehoben wird, ist erreicht, dass die zu untersuchende Bodenfläche immer wieder den atmosphärischen Einflüssen ausgesetzt wird, und ausserdem kann das Ansammlungsgefäss in Freiluft ausgelüftet werden, wodurch gewährleistet ist, dass zu Beginn jeder neuen Ansammlung die Exhalation der vorausgegangenen spurlos verschwunden ist. Ausserdem wurde das Gefäss jeweils nach Einsetzen in die Rinne mit emanationsfreier Pressluft durchlüftet. Es war also zu Beginn jeder Ansammlung der Emanationsgehalt im Gefäss Null.

Die theoretischen Grundlagen der Methode und der Berechnungen der Exhalation (curie/cm^2/sec) aus den gemessenen Daten ist bereits in der genannten Arbeit von P. Zupancic beschrieben. Die Messung der angesammelten Radiumemanation erfolgte nach der Auflademethode. Es wurden zwei Ionisationszylinder in Differentialschaltung verwendet. Der eine davon wurde mit emanationsfreier Luft gefüllt und blieb die ganze Versuchszeit hindurch in diesem Zustand. Der andere war ganz gleich gebaut, nur war seine Wandspannung zwar gleich gross wie beim ersten aber von entgegengesetztem Vorzeichen. Diese zweite Ionisationskammer wird mit der zu untersuchenden Luftprobe gefüllt. In dieser Schaltung sind die beiden Innenelektroden der Kammern untereinander und mit dem Schlingensystem des Elektrometers verbunden. Diese Differentialschaltung, die sich bereits bei mehreren Untersuchungen ausgezeichnet bewährt hatte, ist in der eingangs erwähnten Arbeit von P. Zupancic eingehend beschrieben worden.

Die Kapazität des ganzen Systems (Kolhörster-Einschlingen-Elektrometer+Kondensatoren) wurde für diese Untersuchung neuerdings bestimmt u. zw. nach der von Harms angegebenen Methode. Es ergab sich der Wert 18.65 cm. Dieser wurde allen hier behandelten Messungen zugrunde gelegt. Eine eingehende Studie zeigte auch, dass innerhalb der verwendeten Spannungsempfindlichkeit des Elektrometers (5 bis 100 Partes pro Volt) die Kapazität praktisch konstant bleibt, dass sie jedoch bei höherer Empfindlichkeit sehr merklich ansteigt.[2, 3, 4]

[2] L. Dees, Physik. Zs., **33**, 131-134 (1932).
[3] E. Rumpf, Physik. Zs., **34**, 735-742 (1933).
[4] H. Benndorf, Wiener Ber., II*a*, **142** (1933).

Von Zeit zu Zeit wurde auch im Laufe des Jahres die Eichung der Apparatur in Curie wiederholt und in all den zehn Fällen eine befriedigende Übereinstimmung mit den Werten von P. Zupancic erhalten.

Messvorgänge

Sobald das Ansammlungsgefäss in eine der Rinnen eingesetzt und mit emanationsfreier Pressluft hinreichend durchgelüftet war (es wurde durch drei Minuten ungefähr dreimal soviel Luft hindurch getrieben, als dem Inhalt des Gefässes entsprach), wurden bis auf einen Hahn alle Öffnungen geschlossen. Der eine Hahn aber stellte durch ein Filter die Verbindung mit der umgebenden Freiluft her, um auch im Innern des Gefässes die natürlichen Luftdruck- und Temperaturschwankungen zu erhalten. Als Filter diente ein kurzes enges Glasrohr, gefüllt mit Kokusnusskohle, die ungefähr alle drei Wochen frisch ausgeglüht wurde und so etwa von aussen kommende Emanation der Freiluft am Eindringen ins Sammelgefäss hinderte. Nach der gewünschten Ansammlungsdauer wurde eine evakuierte Ionisationskammer durch ein Filter von Watte und $CaCl_2$ an Stelle des Kohlefilters angesetzt und dann eine Luftprobe angesaugt. Ein zweiter Hahn muss—wie im vorigen Abschnitt bereits erwähnt wurde—wegen des Luftausgleiches geöffnet werden. Das vorgeschaltete Filter hat dabei den Zweck, die eingesaugte Luft zu trocknen und Kondensationskerne abzufangen. Die Messung geschah regelmässig 2.5 Stunden nach dem Einsaugen. Spätestens eine Stunde vor der Messung wurde an beide Kammern die Spannung von $\pm$ 200 Volt gelegt u.zw. an die Probekammer, in der die emanationshältige Luft war, die negative Spannung. Die sogenannte "Natürliche" Aufladung, die durch verschiedene Resteffekte in der Ionisationskammer bedingt ist, erreichte in den meisten Messungen kaum 2 Prozent des Emanationseffektes. Bei der Verfolgung des täglichen Ganges bei der die Einzelmessungen rasch nacheinander gemacht wurden, mussten mehrere Ionisationskammern abwechselnd verwendet werden, um den aktiven Niederschlag in jedem Gefäss vor Wiederverwendung auf Null abklingen zu lassen. So konnte erreicht werden dass in diesen Fällen die natürliche Aufladung nur ungefähr 1/7 der effektiven ausmachte.

§ 2—*Beobachtungsplätze*

P. Zupancic hat für seine Messungen das Ansammlungsgefäss im Hofe des Physikalischen Instituts im südwestlichen Viertel der Stadt Innsbruck aufgestellt. Zuletzt machte er aber mehrere Untersuchungen ausserhalb der Stadt auf einer gegen Süden exponierten Wiese in Hötting (nördlich der Stadt). Man muss nämlich annehmen, dass der Boden im Institutshofe durch Bauschutt u. dgl. verunreinigt ist, sodass man da keine natürlichen Bodenverhältnisse vor sich hat. Ausserdem ist der Hof durch die umliegenden Gebäude vor verschiedenen Einflüssen der Witterung (Wind, Sonnenstrahlung, u. a.) abgeschirmt. Da nun aber der Zweck dieser vorliegenden Arbeit, gerade möglichst natürliche Verhältnisse des Bodens sowohl, wie der umgebenden Freiluft voraussetzt, so wurde für unsere Messungen ausschliesslich die Wiese in Hötting benützt, obwohl nach den vorläufigen Feststellungen P. Zupancic' kein bemerkenswerter Unterschied zu bestehen schien.

Die beiden kreisrunden Rinnen—durch welche die beiden Versuchsplätze umgrenzt waren—wurden unmittelbar nebeneinander einge-

setzt, sodass sie im allgemeinen als ganz gleichwertig gelten konnten. Zufällig entwickelte sich im Frühjahr ein verschieden starker Graswuchs, ein Umstand, der zu interessanten Feststellungen hinsichtlich der Exhalation führte (s. unter § 4).

Ergebnisse

§ 3—*Exhalation im Winter*

Die Zeit, welche in diesem Abschnitt betrachtet wird, beginnt mit 18. Dezember 1933 und dauert bis 19. Februar 1934. Das sind die zwei Wintermonate, in denen der Erdboden ununterbrochen mit Schnee bedeckt war. In dieser Zeit wird die Exhalation fast ausschliesslich durch die Schneeverhältnisse bestimmt und alle übrigen meteorologischen Faktoren treten stark zurück. Dadurch, dass abwechselnd bald an dem einen, bald an dem anderen Platz gemessen wurde, entsprach die Schneedecke ungefähr den natürlichen Verhältnissen der Umgebung. Tabelle 1 zeigt die Mittelwerte, ferner die grössten (max) und die kleinsten (min) Werte der Exhalation in diesen drei Monaten. Die in den folgenden Tabellen angegebenen Werte der Exhalation sind immer mit 10^{-18} zu multiplizieren. Die zwei Spalten rechts zeigen ausserdem die mittlere Temperatur im Boden [die Temperatur des Bodens (T_B) wurde in 5-cm Tiefe dreimal während jeder Ansammlung gemessen] bezw. im Freien (T_F). Die erste Spalte enthält in Klammern die Anzahl (n) der verwerteten Messungen.

Tabelle 1—*Exhalation im Winter*
[Exhalation, (curie/cm²/sec) $\times$ 10^{-18}]

Monat	n	Mittelwert	Max	Min	T_B	T_F
Dezember 1933	(8)	4.3	10.2	0	−3	−6.4
Jänner 1934	(18)	5.6	17.3	0	−1	−3.6
Februar 1934	(8)	6.6	12.4	0	−1	−4.1
....	(34)	5.5	17.3	0	.	.

Wenn in dieser Tabelle für das Minimum der Exhalation Null angegeben wird, so heisst das natürlich, dass ihr Wert unterhalb der Messgrenze lag, also beträchtlich weniger als 1×10^{-18} curie/qcm/sec betrug. Vergleicht man die zweite Spalte mit der letzten, so erscheint ein direkter Zusammenhang zwischen Exhalation und Freilufttemperatur. Diese Parallele wird noch besser ersichtlich, wenn man (s. Tabelle 1*a*) den ganzen Zeitraum in zwei Perioden teilt, von denen jede einen Monat umfasst.

Tabelle 1*a*—*Exhalation (curie/cm²/sec)* $\times$ 10^{-18}

Periode	n	Mittelwert	Max	Min	T_F
I. (18. Dezember, 1933—13. Jänner 1934)	(20)	3.2	10.2	0	−5.8
II. (17. Jänner 1934—19. Februar 1934)	(14)	6.9	17.3	0	−2.7
Verhältnis, (II/I)	....	2.1	1.7	..	...

Die Exhalationswerte dieser zwei Zeiträume sind beiläufig direkt proportional den Temperaturen (in Celsiusgraden). Dies erklärt sich

sehr einfach dadurch, dass mit zunehmender Temperatur *öfter* der Fall eintrat, dass die Schneedecke teilweise schmolz und dadurch die im Schnee zurückgehaltene Emanation frei wurde. Von den 34 Messungen ergaben 15 einen Wert der höher war, als der Mittelwert des betreffenden Monats und in allen diesen Fällen war Tauwetter. Der grösste Wert der Exhalation in dieser Zeit, nämlich 17.3×10^{-18} curie/qcm/sec ergab sich an dem Tag, an dem auch das stärkste Tauwetter zu verzeichnen war, und nicht an den Tagen mit dem höchsten Temperaturmittel. Die Bodentemperatur der obersten Bodenschichten war im Jänner und Februar recht konstant -1 und kommt daher nicht in Betracht, wenn man den Wechsel der Exhalation in dieser Zeit erklären will.

Wohl aber zeigte sich noch eine Abhängigkeit der Exhalation vom Luftdruck, wie aus Tabelle 2 zu ersehen ist. Die linke Hälfte (*a*) dieser Tabelle enthält die niedrigsten, die rechte Hälfte (*b*) die höchsten Werte, welche in diesem Monate gemessen wurden. Daneben stehen die mittleren Barometerstände an den entsprechenden Tagen.

TABELLE 2—*Exhalation* [$(curie/cm^2/sec)10^{-18}$] *und Luftdruck*

Datum	(*a*) Exhal	(*a*) Barom.	Datum	(*b*) Exhal.	(*b*) Barom.
		mm			*mm*
22. Dezember 1933	1.7	721.4	27. Dezember 1933	4.9	694.7
30. Dezember 1933	0	700.3	5. Jänner 1934	4.1	712.1
8. Jänner 1934	1.1	724.0	11. Jänner 1934	7·3	715.8
9. Jänner 1934	1.1	718.6	13. Jänner 1934	8.3	705.1
12. Jänner 1934	1.6	716.0	19. Jänner 1934	15.8	709.9
21. Jänner 1934	2.0	724.6	20. Jänner 1934	17.3	710.5
3. Februar 1934	3.4	718.0	1. Februar 1934	12.3	707.2
5. Februar 1934	4.6	713.9	2. Februar 1934	15.0	715.0
7. Februar 1934	4.8	717.3	9. Februar 1934	12.4	709.2
Mittelwerte	2.3	717.1	Mittelwerte	10.8	708.8

Die Exhalation dürfte aber wohl nicht direkt vom absoluten Luftdruck abhängen, sondern eher von Luftdrucksschwankungen, wobei im Winter dem Ansteigen des Barometers ein Abnehmen der Exhalation entspricht. Sehr deutlich veranschaulicht diese Tatsache ein Vergleich des Ganges der Exhalation mit dem Barogramm der betreffenden Ansammlungskeit. Am 28. Dezember z. B. war die Exhalation am grössten in diesem Monat, das Barogramm zeigt ein Fallen des Luftdruckes von 709.7 bis auf 694.7, nach dem 28. ds. steigt die Kurve rasch an auf 715.4 mm und die Exhalation sinkt am 30 ds. auf 0. Der Einfluss des Windes, der in einem späteren Abschnitt eingehend besprochen wird, machte sich auch im Winter, allerdings viel seltener als im Sommer bemerkbar.

Im allgemeinen jedoch werden im Winter beinahe sämtliche meteorologischen Faktoren durch den Einfluss der Schneeverhältnisse überdeckt. Sooft die Exhalation das Monatsmittel überstieg, war ein teilweises Schmelzen der Schneedecke festzustellen, eine Parallelität, die sich meist sogar graduell verfolgen liess. Bemerkenswert ist auch, dass sich bei Schneefall während der Sammelzeit immer eine geringe Exhalation zeigte, wie aus Tabelle 3 ersichtlich ist.

TABELLE 3—*Werte bei Schneefall (curie/cm²/sec) 10⁻¹⁸*

Datum	Exhalation	Monatsmittel
21. Dezember 1933	1.4	4.31
22. Dezember 1933	1.7	
22. Jänner 1934	2.0	5.63

Festgefrorener Schnee (Harsch) und die Eisbildung am Boden scheinen die Exhalation fast ganz zu unterbinden. So trat z. B. am 17. und 19. Februar bei starkem Tauwetter, als der Schnee auf den Beobachtungsstellen vollständig weggeschmolzen war, plötzlich Frost ein und so konnte über Glatteis gemessen werden. In beiden Fällen war die Exhalation praktisch 0. Zusammenfassend kann also zur Nachlieferung an Exhalation aus dem Erdboden im Winter gesagt werden: Hauptsächlich wird die Exhalation bestimmt durch die Dicke und Art der Schneedecke. Bei Schneefall, bei steigendem Luftdruck, und bei heftigem Wind ist ein deutliches Abnehmen der Exhalation festzustellen. Die Temperatur der Freiluft scheint mehr indirekt—durch ihren Einfluss auf die Schneehülle—zu wirken, die Bodentemperatur wie erwähnt, ist sehr konstant und hat jedenfalls auf die relativen Werte der Exhalation keinen Einfluss.

§ 4—*Die Exhalation in der Übergangszeit (Frühling)*

Die Zeit, welche jetzt behandelt wird, beginnt mit der ersten Messung auf schneefreiem Boden am 23. Februar und dauert bis zu dem Zeitpunkte, wo der Erdboden durch und durch aufgetaut und dessen Oberfläche vollständig mit frischem Rasen bedeckt ist, d. i. bis Ende März. In dieser Periode erscheinen sehr hohe Werte der Exhalation, was wohl mit dem allmählichen Auftauen der Erdkruste zusammenhängt und vielleicht auch damit, dass die Bodenoberfläche von Vegetation noch nicht überwuchert ist. Dies letztere scheint durch folgende Beobachtung gerechtfertigt zu sein:

Der eine der beiden Messungsplätze (Platz II) blieb in dieser ganzen Periode nur zu 2/3 mit Rasen bewachsen, während der andere Platz von Anfang an keinen rasenfreien Fleck aufwies. Wie nun die Tabelle 4 zeigt, wurde auf Platz II fast konstant ein grösserer Exhalationswert festgestellt. Diese Feststellung gilt aber nur für diese Periode des allmählichen Graswuchses. Im Winter und Sommer dagegen war

TABELLE 4—*Exhalation und Graswuchs am Boden*
[Exhalation (curie/cm²/sec) 10⁻¹⁸]

Datum	Platz I	Platz II
1934		
26. Februar	....	25.9
27. Februar	23.2	....
28. Februar	....	47.3
1. März	20.4	....
2. März	....	22.4
3. März	19.8	....
5. März	....	16.3[a]
6. März	32.1	....
7. März	....	68.2
8. März	40.2	....
10. März	....	70.7
27. März	53.7	....
28. März	....	74.5
Mittelwerte	31.5	46.4

[a]Witterungsrückfall: Schnee, Regen.

zwischen den beiden Versuchsplätzen kein Unterschied in der Exhalation. Zu diesen Zeiten waren beide Plätze auch hinsichtlich des Graswuchses ganz gleich.

Die Rasenbedeckung auf Platz I verhielt such zu der auf Platz II wie 3:2. Vergleicht man die zu unterst der Tabelle angegebenen Mittelwerte auf beiden Plätzen miteinander, so fällt mit überraschender Deutlichkeit auf, dass sie im umgekehrten Verhältnis stehen, wie der Graswuchs auf beiden Plätzen.

Tabelle 5 gibt eine Übersicht über sämtliche Messungen der Frühjahrsperiode.

TABELLE 5—*Übersicht der Frühjahrswerte*
[Exhalation, (curie/cm²/sec) 10^{-18}]

Periode	n	Mittel	Max	Min	Max/Min	T_B	T_F
1934							
Februar (ende)	(4)	31.5	47.3	23.2	2.2	1.5	4.1
März 1. Hälfte	(9)	39.8	70.7	16.3	4.3	2.8	4.4
März 2. Hälfte	(9)	56.9	74.5	22.6	3.3	8	7.1

Bezeichnend für diesen Zeitraum sind die enorm hohen maximalen Werte der Exhalation (über 70×10^{-18}) die sonst nur noch im Hochsommer vorkommen. Aber auch die Mittelwerte sind im Frühjahr sehr hoch. Das Verhältnis zwischen den höchsten und niedrigsten Werten max/min ist in dieser Periode auffallend gross.

Der niedrigste Exhalationswert in dieser Zeit, 16.3×10^{-18} (s. Tabelle 4), ist durch einen Witterungsrückfall bedingt, wie überhaupt ein geringer Witterungswechsel in dieser Zeit sofort den Wert der Exhalation ganz beträchtlich verändert. Die hohen Werte (über 60×10^{-18}) gehören zu den Tagen mit dem schönsten Wetter. Wiederholt aber zeigte sich, dass bei *Wind* (Föhn) trotz schöner Witterung die gemessene Exhalation fast nur halb so gross war, als an windstillen—aber schönen, sonnigen—Tagen. Diese Tatsache ist aus der Tabelle 6 ersichtlich und muss später noch einmal erwähnt werden, wenn auch keine vollständig befriedigende Erklärung gegeben werden kann.

TABELLE 6—*Exhalation bei Wind und bei Windstille*
[Exhalation, (curie/cm²/sec) 10^{-18}]

Wind		Windstille	
Datum	Exhalation	Datum	Exhalation
1934		*1934*	
26. Februar	25.9	7. März	68.2
27. Februar	23.2	10. März	70.7
8. März	40.2	20. März	72.8
15. März	53.1	28. März	74.5
17. März	61.0	26. März	62.0
21. März	22.6	27. März	53.7
....	37.6	....	70

Leicht begreiflich ist die Abhängigkeit der Exhalation von der in dieser Zeit allmählich ansteigenden Bodentemperatur. Zusammenfassend kann hervorgehoben werden, dass die hohen Werte an schönen, sonnigen Frühlingstagen oft ganz unvermittelt auftreten, dass die Schwankungen der Exhalation in dieser Zeit am lebhaftesten sind und schliesslich, dass eine ganz deutliche Parallele besteht zwischen Exhalation und Temperatur.

§ 5—*Die Exhalation im Frühsommer*

Die hier betrachtete Zeitspanne beginnt mit 1. April, weil von diesem Datum an der Boden vollständig aufgetaut und beide Beobachtungsplätze mit frischem Gras bedeckt waren. Im allgemeinen zeigten die folgenden drei Monate—bis Anfang Juli—sommerlichen Charakter, wenn auch sehr häufig Störungen durch Regen und besonders durch Wind vorkamen. Diese Störungen hatten zur Folge, dass die Monatsmittel der Exhalation verhältnismässig tief sind, was einem normalen jährlichen Gang nicht entsprechen dürfte, wenngleich vermutet werden darf, dass auch die normale Jahreskurve bereits im März—Zeit des Auftauens—ein Maximum erreicht. Tabelle 7 soll einen Überblick über den Gang der Exhalation vom 1. April bis 1. Juli geben.

TABELLE 7—*Exhalation im Frühsommer*
[Exhalation, (curie/cm²/sec) 10^{-18}]

Monat	n	Mittelwert	Max	Min	Max/Min
1934					
April	(17)	41.3	65.7	13.6	4.83
Mai	(4)	50.7	61.3	42.–	1.46
Juni	(11)	54.2	67.5	23.4	2.88
....	(32)	48.7	67.5	13.6	4.97

In Tabelle 7 fällt vor allem das Verhältnis zwischen Max und Min auf, das im April seinen höchsten Wert erreicht. Dies dürfte sich dadurch erklären, dass bei normalem, schönem Wetter im April sehr hohe Werte der Exhalation festgestellt wurden, die aber an Tagen mit heftigem Wind um mehr als die Hälfte niederer waren. An Tagen jedoch, wo ausser starkem Wind auch noch feuchtes kaltes Wetter war, sank die Exhalation fast bis auf ein Fünftel des maximalen Wertes. Solche regnerisch-windigen Tage waren vom 19. bis 28. April. Diese Regenperiode wird in der Tabelle 8 der übrigen schöneren Zeit des April gegenübergestellt.

TABELLE 8—*Einfluss der Temperatur und des Regens in deiser Periode*
[Exhalation, (curie/cm²/sec) 10^{-18}]

Periode	n	Mittelwert	Mittl. Temp.	Mittl. Temp.—Diff.	
				Freiluft	Boden
1. Hälfte April (schön)	(9)	49.1	12.2	17	21
2. Hälfte April (Regen)	(7)	28.4	11.6	8	14

In Tabelle 8 sind die beiden letzten Spalten beachtenswert, da sie klar zeigen, dass die Exhalation mehr von den Schwankungen der Temperatur, als von deren absolutem Wert abhängt. Die Zahlen der letzten Spalten bedeuten nämlich die Differenz zwischen der höchsten und tiefsten Temperatur während der Ansammlung u. zw. die vorletzte Spalte bezieht sich auf Freiluft, die letzte auf den Erdboden. Die

Temperaturdifferenz auf dem Boden wurde dadurch gemessen, dass unter das Ansammlungsgefäss ein Extremthermometer auf die Erde gelegt wurde.

In dieser Frühsommerperiode sind die Exhalationswerte auffallend klein, was seinen Grund darin hat, dass verhältnismässig viel Regen und Wind den normalen Jahresgang gestört haben. Ohne diese Störungen müsste man sicher den Juni der folgenden Hochsommerperiode zuteilen die wieder einen gewaltigen Anstieg der Exhalationskurve aufweist.

§ 6—*Die Exhalation im Hochsommer*

Hier werden die Messungen in den beiden Monaten Juli und August behandelt. Die Exhalationskurve, die P. Zupancic gefunden hat, steigt in dieser Periode zum Jahresmaximum (vermutlich im August) konstant an. Die bei unseren Messungen zu dieser Zeit (Frühsommer) auftretenden verhältnismässig tieferen Werte sind auf die ungünstige Witterung in diesen Monaten zurückzuführen. Im Hochsommer, wo meist nur an schönen, sonnigen Tagen gemessen wurde, war die Exhalation sehr hoch und die Kurve erreicht tatsächlich in dieser Zeit ihr Maximum (s. Tabelle 9).

TABELLE 9—*Exhalation im Sommer*
[Exhalation, (curie/cm²/sec) 10^{-18}]

Monat	n	Med	Max	Min	Max/Min
1934					
Juli	(6)	67.4	92	61.5	1.49
August	(9)	89.7	117	69.2	1.69
......	(15)	78.5	117	61.5	1.9

Der auffallend rasche Anstieg der Exhalationskurve in dieser Periode ist dadurch bedingt, dass sämtliche Messungen im Juli bei hervorragend schöner Witterung gemacht wurden. Im August aber konnten wenige Messungen gemacht werden und hiefür wurden Tage gewählt, die für den Hochsommer typisch waren. Aus der bisherigen Darstellung ist zu ersehen, dass es nicht leicht ist, zwischen der Nachlieferung an Emanation und den verschiedenen meteorologischen Faktoren eindeutige Beziehungen aufzustellen. Vielfach überdecken sich die meteorologischen Einflüsse und je nach Jahreszeit hat bald der eine bald der andere Faktor einen stärkeren Effekt. Am wenigsten kann hier gesagt werden über einen eventuellen Zusammenhang mit dem Feuchtigkeitsgehalt der Atmosphäre. Ein Vergleich des Exhalationsganges mit dem Hygrogramm des Jahres liess keinerlei Zusammenhang erkennen. Andererseits ist klar, dass doch eine indirekte Abhängigkeit bestehen muss, weil ja die Feuchtigkeit durch die übrigen meteorologischen Gegebenheiten der Atmosphäre mitbestimmt ist. Sehr auffallend aber ist der Zusammenhang zwischen Exhalation und Bodenfeuchtigkeit. Wegen der technischen Schwierigkeiten, graduelle Unterschiede der Bodenfeuchtigkeit bei den einzelnen Messungen festzustellen, musste leider von einer eingehenden Untersuchung dieses Zusammenhanges abgesehen werden. Es kann aber gesagt werden, dass bei feuchtem Boden stets weniger Emanation nach-

geliefert wird, als dem betreffenden Monatsmittel entspräche. Das erklärt sich dadurch, dass in diesem Falle die Poren teilweise verstopft sind; tritt zu der Feuchtigkeit noch Gefrieren des Bodens, dann wird (aus demselben Grund) die Exhalation noch geringer, ja sie hört überhaupt vollständig auf, sobald sich über der Untersuchungsstelle Glatteis gebildet hat. Gerade umgekehrt aber ist der Zusammenhang in der Frühjahrsperiode. Da ist die Bodenfeuchtigkeit eine Folge des allmählichen Auftauens der Erdkruste und dabei zeigt sich begreiflicherweise ein Ansteigen der Exhalation. Im folgenden seien noch einige Gruppen von Untersuchungen angeführt, die der Reihe nach den Einfluss des Regens, des Windes und der Sonnenstrahlung zeigen.

§ 7—*Einfluss des Regens*

Tabelle 10 zeigt die Werte der Exhalation an Tagen, die in eine längere Regenzeit fielen. Es handelt sich also um Messungen, bei denen es nicht bloss während der Ansammlungszeit regnete, sondern auch schon—was selbstverständlich für die Ergebnisse bedeutungsvoller ist—eine längere Zeit vorher geregnet hatte.

TABELLE 10—*Exhalation bei Regen*
[Exhalation, (curie/cm²/sec) 10^{-18}]

Datum	Exhalation (a)	Monatsmittel (b)	b/a
1934			
11. April	40.8	41.3 (59.4)	1.01
24. April	34.7	41.3	1.19
5. Juni	19.8	....	2.74
6. Juni	22.5	54.2 (81.1)	2.41
9. Juni	24.4	....	2.23

In Klammer stehen die Exhalationswerte des letztvorausgegangenen regenlosen Tages.

Im April beeinflusst also der Regen die Exhalation nicht merklich, wohl aber im Sommer. Dieser Unterschied dürfte dadurch begründet sein, dass die Regenperiode im Juni eine starke Temperaturerniedrigung (von ungefähr 30° auf 10°) mit sich brachte, was im April nicht der Fall war.

Es muss hervorgehoben werden, dass Regen während der Ansammlungszeit nicht ohne Einfluss ist auf die Emanationsnachlieferung, obwohl die Versuchsstelle dabei durch das Gefäss überdeckt ist. Es war nämlich jedesmal nach Abheben des Ansammlungsgefässes festzustellen, dass doch der Boden sogar nasser war, als der unbedeckte in der Umgebung, der durch den Luftzug rascher wieder austrocknen konnte. Sicher ist die Verminderung der Exhalation bei Regen (abgesehen von der Temperaturerniedrigung), so zu erklären, dass das Wasser die Poren verstopft und die Transpiration der Bodenluft sehr behindert.

Hier sei auch eine Bemerkung betreffs des Effektes bei Gewittern angebracht. In drei Fällen wurde festgestellt, dass ein Gewitter während der Ansammlung die Exhalation verringert, was wohl auf indirekte Ursachen wie z. B. Regenwirkung, Wind u. dgl. zurückzuführen sein dürfte.

§ 8—*Einfluss des Windes*

Sehr viele Messungen, die bei Wind gemacht wurden, zeigen einwandfrei (s. Tabelle 11), dass in diesen Fällen eine Exhalation gemessen wurde, die ungefähr halb so gross war, als in Anbetracht der sonstigen Witterungsverhältnisse zu erwarten war.

Es lag die Vermutung nahe, dass durch eine dynamische Wirkung Emanation aus dem Ansammlungsgefäss entfernt worden wäre. Es wäre nicht schwer, in der böigen Struktur des Windes diesen Faktor zu vermuten. Die raschen Druckschwankungen, die schliesslich mehr oder minder bei jedem Wind beobachtet werden, würden in unserem Falle während der Saugphase (Druckverminderung aussen) angesammelte Emanation in das Kokosnussrohr treiben, wo sie absorbiert wird und der Messung entgeht, während in der Phase der Drucksteigerung schwach emanationshältige Freiluft von aussen in das Kohlerohr eintritt, dort entemaniert wird und somit in diesem Zustand in das Sammelgefäss eintritt. Wenn dieser Effekt in merklichem Grade in Betracht kommt, so würde das heissen, dass die beobachtete Verringerung der Exhalation bei Wind ein Scheineffekt ist, der sich nur in der Messanordnung äussert. Die volle Erklärung des Windeffektes konnte in der gegenwärtigen Untersuchung nicht durchgeführt werden und muss künftigen Messungen vorbehalten bleiben.

Es sei nur erwähnt, dass zur provisorischen Prüfung der vorerwähnten Hypothese bei einigen Messungen der Hahn zum Kohlefilter geschlossen wurde, dass aber trotzdem jene Verringerung der Exhalation bei Wind festgestellt wurde, was gegen obengenannte Hypothese sprechen würde. Jedoch sind diese wenigen Versuche nicht hinreichend, um eine abschliessende Erklärung zu gestalten. Zur Illustration sind alle Messungen, die bei Wind durchgeführt wurden, zahlenmässig in Tabelle 11 erwähnt. Darin enthält die erste Spalte den unter sonst gleichen Wetterverhältnissen, aber ohne Wind, zu erwartenden Wert der Exhalation und die letzte Spalte das entsprechende Monatsmittel. In allen Fällen sieht man, dass die tatsächlich gemessene Exhalation hinter der erwarteten bezw. hinter dem Monatsmittel zurückbleibt.

TABELLE 11—*Windeffekt*

Datum	Wert der Exhalation, (curie/cm²/sec) 10^{-18}		(a/b)	Monatsm.
	(a) Erwarteter	(b) Gemessener		
1934				
12. Jänner	5	1.1	4.5	5.63
26. Februar	40	25.4	1.6	31.5
21. März	60	22	2.7	48.4
24. März	70	41.8	1.7	48.4
6. April	60	26.5	2.3	40.3
19. April	60	23.7	2.5	40.3
20. April	60	25.7	2.3	40.3
24. April	60	34.7	1.7	40.3
28. April	50	19.6	2.5	40.3
Mittelwert	..	....	2.4	..

Ein ähnliches Verhältnis wie in der 4. Spalte, Tabelle 11, konnte noch bei sehr vielen Messungen in den Monaten Mai und Juni festgestellt werden. Es zeigte sich auch—wie ja zu erwarten ist—eine Abhängigkeit von der Dauer des Windes. Die obige Verhältniszahl steigt mit zunehmender Dauer des Windes.

§ 9—*Einfluss der Sonnenstrahlung*

Um für die Untersuchungen des täglichen Ganges den Beobachtungsplatz näher beim Laboratorium zu haben, wurde das Ansammlungsgefäss zeitweise im Hofe des Instituts an einem Platze aufgestellt, der nur kurze Zeit (ungefähr zwischen 8 und 9 Uhr) der Sonne ausgesetzt war. In Hötting dagegen wurde absichtlich ein Platz gewählt, der den ganzen Tag von der Sonne beschienen war. So waren Vergleichsmessungen möglich, um den Einfluss der Sonnenstrahlung zu beobachten. Tabelle 12 enthält die Werte der Exhalation von je vier Messungen, die alle im Mai an vollständig sonnigen Tagen (mit möglichst gleichen Wetterverhältnissen) einerseits in Hötting (*II*), anderseits im Institutshof (*I*) gemacht wurden. In der dritten Spalte der Tabelle steht die mittels Extremthermometer gemessene Temperaturdifferenz während der Ansammlungszeit. Alle diese Werte stellen bereits das Mittel aus je vier Messungen dar.

TABELLE 12—*Besonnung*

Ort	Exhalation	Bodentemp.	Maximale Temp.-Diff.
H	50.6	16	22
I	13.6	14.3	7.3

Es scheint daher, dass eine Erhöhung der Exhalation dadurch bewirkt wird, dass durch die Besonnung die Temperaturschwankungen der Bodenfläche bedeutend grösser werden.

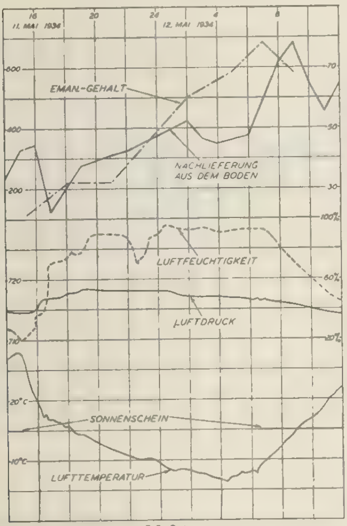
§ 10—*Der tägliche Gang der Exhalation*

Im Laufe des Jahres wurde wiederholt versucht, den täglichen Gang der Exhalation festzustellen. Zwei solcher Untersuchungen seien hier näher behandelt. Sie liegen beide in einem Zeitraum, in dem die Nachlieferung ziemlich stark war. Dies war auch deshalb günstig, da mit nur ein- bis zweistündigen Ansammlungszeiten gearbeitet werden musste. Das Ansammlungsgefäss wurde dabei jedesmal nach Entnahme der Probe abgehoben und wieder frisch eingesetzt und mit Pressluft durchgelüftet, also immer wie bei allen übrigen Messungen die Ansammlung mit dem Emanationswert Null begonnen.

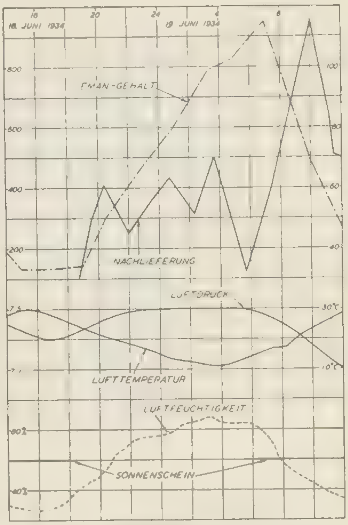
Die erste dieser beiden 24 stündigen Versuchsreihen begann am 11. Mai um 13 Uhr. Gegen Abend dieses Tages war ein ziemlich heftiges aber kurzes Gewitter, jedoch ohne Regen. Die stark gezeichnete Linie in Figur 1 bedeutet die Tageskurve der Exhalation. Sie sinkt um 18 Uhr sehr rasch auf ein Minimum, was wohl durch das Gewitter bedingt ist. Während der Nacht steigt sie allmählich an, erreicht um ungefähr 2 Uhr ein Maximum, fällt dann wieder auf ein Minimum um 6 Uhr und strebt schiesslich dem Tages-Maximum um 10 Uhr zu.

Parallel mit den Messungen der Nachlieferung machte Herr O. Macek Messungen des Emanationsgehaltes in Freiluft. Diese Kurve

läuft im allgemeinen parallel, nur scheint sich darin das Gewitter weniger stark auszuwirken und —was sehr auffallend ist —das Tages-Maximum ist ungefähr 2 Stunden vor dem der Exhalationskurve. Dieselbe Feststellung zeigt auch Figur 2. In dieser ist der Gang der Exhalation am



FIGUR 1



FIGUR 2

18. und 19. Juni dargestellt. Diese auffallende Verschiebung der Höchstwerte dürfte ihren Grund vielleicht darin haben, dass die Freiluft-Messung an einer anderen Stelle erfolgte als die Nachlieferung u. zw. an einem Platz, der bereits früher der unmittelbaren Sonnenstrahlung ausgesetzt war. Die Freiluftkurve stimmt mit den Ergebnissen von W. Messerschmidt,[5] K. W. Kohlrausch,[6] M. Aliverti und W. Illing[7] gut überein. Übrigens deckt sich auch die Exhalationskurve ziemlich gut mit den Angaben von P. Zupancic. Nur erscheint in der letztgenannten Arbeit das Tages-Maximum etwas später, was aber sicher mit der Sonnenstrahlung zusammenhängt, da P. Zupancic an einer Stelle gemessen hat, die erst von Mittag an direkt besonnt war. Überdies datiert die betreffende Messung des P. Zupancic vom Hochsommer, was auch die Verschiebung einigermassen erklären dürfte.

Die übrigen in beiden Figuren angegebenen Kurven, sollen den Zusammenhang mit den verschiedenen meteorologischen Faktoren zeigen. Es gelten dabei folgende Einheiten:

Emanationsnachlieferung in 10^{-18} curie/qcm/sec
Emanationsgehalt in 10^{-18} curie/ccm
Luftdruck in mm Hg
Relative Luftfeuchtigkeit in Prozent
Lufttemperatur in Celsiusgraden

[5] Zs. Physik, **81**, 84-100 (1933).
[6] Wiener Ber., IIa, **119**, 1577-1604 (1910).
[7] Dissert., Innsbruck, 1933.

Die Sonnenscheindauer ist durch einen dicken Strich ersichtlich gemacht. Es zeigt sich ganz im Einklang mit den bereits oben erwähnten Feststellungen kein direkter Einfluss der Feuchtigkeit, wohl aber des Luftdrucks und besonders der Temperatur und der Sonnenstrahlung.

§ 11—*Der jährliche Gang der Exhalation*

Dieses letzte Kapitel gibt zusammenfassend einen Überblick über den Gang der Exhalation im Laufe der Zeit, in der die hier behandelten Untersuchungen gemacht wurden.

Tabelle 13 enthält die Monatsmittel der Exhalation. Diese von

TABELLE 13—*Monatsmittel der Exhalation* (*curie/cm²/sec*) 10^{-18}

Monat	Exhalation
Dezember 1933	4.31
Jänner 1934	5.63
Februar 1934	31.5
März 1934	48.4
April 1934	40.3
Mai 1934	50.7
Juni 1934	54.2
Juli 1934	67.4
August 1934	89.7
September 1934	....
Oktober 1934	(41.3)*
November 1934	(29.0)

uns aufgestellte Jahreskurve der mittleren Exhalation stimmt in ihrem Verlauf gut überein mit der von P. Zupancic angegebenen, nur liegen alle Werte etwas höher. Das mag darin begründet sein, dass ein anderer Versuchsplatz gewählt wurde, der sich vor allem dadurch auszeichnete, dass er fortwährend der Sonnenstrahlung ausgesetzt war. Insbesondere decken sich Minimum (Jänner) und Maximum (August) ganz genau.

Das Mittel aus den hier angeführten Monaten ist 43.5×10^{-18} curie/qcm/sec, das Jahresmittel—wenn man die Exhalation im Herbst berücksichtigt*—etwas höher, also um 50×10^{-18} curie/qcm/sec Vergleichshalber sei hier erwähnt, dass L. B. Smyth[8] in Dublin den Wert 73.3×10^{-18} curie/qcm/sec, J. R. Wright und O. F. Smith[9] in Manila dagegen in den Sommermonaten bloss einen Mittelwert von 20.5×10^{-18} curie/qcm/sec fanden. P. Zupancic erhielt als Mittelwert für die Monate Dezember bis Juni in Innsbruck den Wert 23×10^{-18} curie/qcm/sec.

*Im Herbst 1934 wurden nachträglich noch einige Messungen gemacht, aus denen allerdings nicht zuverlässige Mittelwerte abgeleitet werden konnten; die Ergebnisse sind in Tabelle 13 in Klammer beigefügt.

[8]Phil. Mag., **24**, 632-637 (1912).

[9]Phy. Rev., **5**, 459-482 (1915).

INSTITUT FÜR STRAHLENFORSCHUNG,
UNIVERSITÄT INNSBRUCK,
Innsbruck, Österreich

THE ANNUAL AND DIURNAL VARIATION OF IONS IN AN URBAN COMMUNITY*

By A. P. Gagge and I. M. Moriyama

Abstract—Results of annual and diurnal ion-counts made at a station located in New Haven, Connecticut, are reported. The counter used was of the Zeleny type. For both positive and negative ions four mobility-thresholds were used, namely: 0.07, 0.017, 0.0014, and 0.0006 cm per second/volt per cm. The total ion-count was analyzed into three groups; the small-ion group with mobilities greater than 0.07, the intermediate-ion group with mobilities between 0.07 and 0.0014, and the large-ion group with mobilities between 0.0014 and 0.0006. There is a rise in positive light ions during the summer months while the negative ions, usually less in number than the positive, show a fall during the summer months. The heavy-ion group increases two-fold in number during the winter months over the summer months, while the intermediate group increases only slightly in number during the same period. The diurnal variation of the heavy-ion group shows two daily maxima at 8 a. m. and 6 p. m., eastern standard time (E. S. T.), during the heating season (October through April) while the 6 p. m. maximum nearly vanishes for the non-heating season (May through September); on the other hand the intermediate group shows an inverse relationship with the heavy-ion group for both seasons. From a characteristic curve based on assumptions justified partially by experiment the intermediate group is shown to be composed of ions with a mobility whose value varies seasonally between 0.01 and 0.0025 cm per second volt per cm. The highest mobility-value appears in the summer months.

Introduction

Recently the annual and diurnal variations of the large ions have been studied by G. R. Wait and O. W. Torreson[1] in Washington and A. R. Hogg[2] in Australia. Each of these observers has taken simultaneous counts of light and heavy ions. Torreson and Wait[3] have also reported simultaneous observations of condensation-nuclei and heavy ions. Wait,[4] too, has reported further ion-relationships by including the intermediate ions of mobility 0.07 cm per second volt per cm. All this work points, however, to the paucity of data available to show the various interrelationships extant in atmospheric ions. Therefore, since ion-counts, both seasonal and daily over various size groups, are particularly important, we present herewith the results and analysis of a year's observations at an urban station in New Haven, Connecticut.

The apparatus

The type of ion-counter used for these experiments was modelled after those of the Zeleny type and was built at the Dessauer Institute in Frankfurt, Germany. The counter is essentially a cylindrical electrical condenser, through which air is drawn. The inner cylinder of this condenser, however, is divided into four parts. The two end parts of the cylinder, *A* and *B* (see Fig. 1), serve to outline the path of the air-stream and are rounded to prevent turbulence. The two inner parts,

*Contribution No. 7, John B. Pierce Laboratory of Hygiene.

[1]Terr. Mag., **39**, 111-119 (1934).

[2]Beitr. Geophysik, **41**, 1-31 (1934).

[3]Terr. Mag., **39**, 47-64 (1934).

[4]Phys. Rev., **47**, 786 (1935), and **48**, 383 (1935).

C and *D*, are the actual measuring electrodes themselves. The shorter part *C* serves in the measurement of ions with mobilities within the range of 0.1 to 0.005 cm per second/volt per cm. With *C* and *D* joined together by switch *E*, the mobility-range is extended to 0.0003 cm per second/volt per cm. For this range-limit a potential of 280 volts is necessary. The measuring cylinders *C* and *D* are connected to a Lindemann electrometer, *F*. The needle of the electrometer is observed by a microscope with a magnification of 40 diameters. The grounding key, *G*, returns the needle to zero. When the cylinder *D* is not in use it is grounded by switch *E*. The cylinder-parts *C* and *D* are separated from part *B* by amber-insulators. All other insulators in the measuring circuit are also of amber. A potential is applied to the outer cylinder and to part *B* which is grounded. Part *A* is also at the same potential as the outer cylinder.

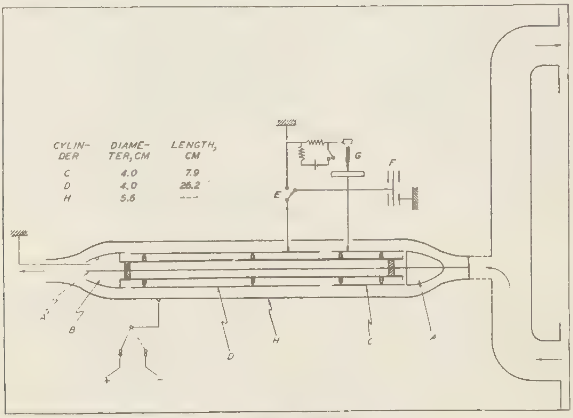


FIG. 1—DIAGRAM OF APPARATUS

By this arrangement the operating potential of the counter is not across the amber-insulators of the measuring circuit and polarization is avoided. The air-stream is formed by pulling the air through the counter by means of a vacuum-pump. A manometer placed in series with the line indicates the quantity of air-flow.

The method of measurement for the counter was as follows: First, the desired sensitivity of the electrometer was obtained by applying a known potential between the grounding key, *G*, and the ground. The usual sensitivity used was 200 scale-divisions per volt. The measuring circuit was then tested for insulator-leaks. The operating potential was next applied to *A* and *B*. With no air-flow the rate of charge would give a measure of radioactive contamination. In actual operation counts were disregarded whenever any insulator-leakage was detected. The radioactive background counts were kept at a minimum by occasionally cleaning the system. With the air-stream set for 100 cc per second the actual count was made by measuring the time for the needle of the electrometer to charge to known potential. This time was converted into actual numbers by the use of constants dependent upon the air-velocity and the number of condensers in use.

The counter was mounted on a board with the electrometer, microscope, and electrometer-controls to form a portable unit. For atmospheric counts the air was drawn through a tube four cm in diameter from a large *U*-shaped stove-pipe placed across the lower and upper parts of the window (see Fig. 1). The stove-pipe was 15 cm in diameter and 1.6 meters long. The window itself was 6.1 meters from the street-level and 3.0 meters from the roof-level of the Laboratory. Normally the air-velocity in the stove-pipe was about 60 cm per second. The outer electrode *A* of the counter was grounded to the window casing, while the ground of the electrometer acted as a secondary ground.

The Laboratory is located on a street used mainly by commercial vehicles and street-cars. The building is surrounded by apartment houses and stores which are sources of combustion-products during the heating season from October to April. The Yale Medical School power-plant (coal-burning) lies 100 meters to the north and has a single chimney 30 meters high. It operates the year around though on a reduced scale during the summer months. The city of New Haven itself is on the Long Island Sound and has many of the characteristics of a seaboard city.

Counting routine

Counts were made daily at 9 a. m., E. S. T., Sundays and holidays excepted. Each month a 24-hour series of counts was taken every hour on the hour. These diurnal counts were started only on days of normal weather-conditions, but when inclement weather arose after the count was once started, the series was continued to termination. When Daylight Saving Time was in effect, during the period between May and September, counts were made daily at 8 a. m., E. S. T.

In each series of observations there were eight counts made. Their order, the threshold-mobilities, and the average time of observation, are shown in Table A.

TABLE A—*Order of counts*

Count		Threshold-mobility	Sign	Average time of observation	Air-stream	Condenser
				minutes	*cc/sec*	
1	k_1	0.07	+	3	100	1
2	k_1	0.07	−	3	100	1
3	k_2	0.017	+	2	100	1,2
4	k_2	0.017	−	2	100	1,2
5	k_3	0.0014	+	1	100	1,2
6	k_3	0.0014	−	1	100	1,2
7	k_4	0.0006	+	1	100	1,2
8	k_4	0.0006	−	1	100	1,2

The whole series was carried through successfully during most seasons. During the months of July and August, when humidity was extremely high, counts 5 to 8 were by necessity occasionally omitted. When surface-leaks or other disturbing phenomena were present the results of observations were omitted and not included in the final results.

Method of analysis

In the case of aspiration-type counters, theory shows that for a volume of air ϕ cc per second, and an operating potential, V, the threshold mobility, k, is given by the expression

$$k = (\phi/2\pi\, l\, V) \log r_a/r_i{}^*$$

where l is the length of the inner electrode and r_a and r_i the radii of the outer tube and the axial inner electrode respectively. This means that for a given air-stream, ϕ, all ions of mobilities down to the value k are completely collected by the counter, when a voltage, V, is applied to the cylinders. Ions of mobility less than k are only partially collected. If one plots the number of ions on the ordinate versus the voltage or the reciprocal of the threshold-mobility on the abscissa, a diagram known as the characteristic plot is obtained. Such a plot is shown in Figure 2.

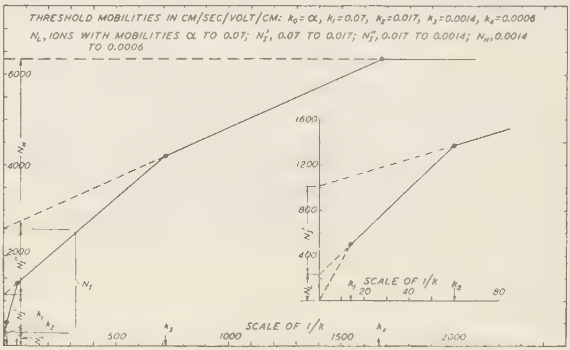


FIG. 2—COUNT GROUPING, MONTH OF JULY

Series of experiments show that the average saturation-threshold (where ions of all mobilities are collected) for both positive and negative ions was 0.0006 cm per second/volt per cm. This mobility-value corresponded to the mobility of the heavy Langevin ions collected by our apparatus. Therefore, in the analysis of our results, we have considered the number of ions collected at the threshold of 0.0006 cm per second volt per cm to be the total number of ions in the atmosphere at the time of counting, and further, we assume that no other ions of mobility less than 0.0006

*It has been pointed out to the authors by G. R. Wait of the Department of Terrestrial Magnetism of the Carnegie Institution of Washington that this equation is correct only if the capacity of the part of cylinder collecting ions is equal to $l/2 \log_e r_a/r_i$; that this is doubtful since it ignores end effects which are caused by the intense field at the first end (between A and C in Fig. 1) and the second end (between C and D or D and B); and further, that a measured capacity does not show the greater effectiveness of the first end in the removal of ions in comparison to the second. For purposes of calculation and discussion we shall use the value for the capacity given by the relation above as this will give threshold and mobility-values that are relatively accurate.

cm per second/volt per cm exist. We have no evidence to indicate such an existence at present.

In our routine counts the four threshold-values were 0.07, 0.017, 0.0014 and 0.0006 cm per second/volt per cm. The relation of ion-numbers of the reciprocal of these thresholds is shown in the characteristic plot in Figure 2. If we draw lines between the observed points as shown we notice that the total number of ions as represented by the saturation-point is divided into four groups, namely: N_L, the number of ions with mobility values equal to or greater than 0.07; N_I', the number of ions with mobility-values between 0.07 and 0.017. Similarly, N_I'' is the group with mobility between 0.017 and 0.0014; and N_H, those with mobilities in range 0.0014 to 0.0006. The group N_H is composed mainly of what we know as the Langevin ions. For purposes of analysis we have combined the groups N_I' and N_I'' into an intermediate group called N_I. The group N_L corresponds very closely to the number of light ions in the air.

In presenting our results we divided our total number into three groups. N_L is the intercept of the line through the first two points. N_H is the projection of the line through the last two points to the ordinate. N_I is the balance of the total. Another interpretation of the characteristic plot will be presented later in the discussion.

Results

The daily ion-counts were analyzed over a period of thirteen months. The corresponding counts for the eight routine counts observed daily were averaged over each monthly period. The arithmetic averages for the eight routine counts were then analyzed into groups as shown in Figure 2. The average values of N_L, N_I, and N_H, for the year beginning February 1934 through February 1935, were thus found and are given in Figure 3 and Table *B*.

In regard to positive and negative light ions, the numbers have been found to be generally small throughout the year. The number of positive ions is at a minimum during the winter months with a value of N_L of 90. During the summer months, however, the number of positive ions rises to a value of about 270. The negative ions, on the other hand, decrease from a value of 70 for February 1934 to a value of 54 for July. There is a rise toward the fall season with a drop again toward the winter months. This annual variation in the number of positive light ions is in agreement with those reported by Wait[1] and Torreson at Washington. The magnitude of the counts in the two localities is approximately the same. C. P. Yaglou and L. C. Benjamin[5] in Boston, also report an increase in the number of positive ions during the summer months. In regard to negative light ions, the decrease in the numbers during the summer months does not appear to coincide with the small increase found by Yaglou. It is to be noted, however, that Yaglou's negative light ion-counts were much higher than ours. Wait does not report his results for negative light ions.

The intermediate group shows no distinct annual change. There is a

[5]Heating, Piping, and Air Conditioning, January 1934, pp. 25-32.

slight decrease in number toward the summer months and a corresponding rise toward the winter months. The order of magnitude for the intermediate group, both positive and negative ions, is about the same throughout the year, and neither polarity-group seems to be significantly predominant. Annual changes in the intermediate-group number have not been reported before. The intermediate ion reported by A. Pollock[6] and Wait[4] does not have a mobility-value lying in the range represented

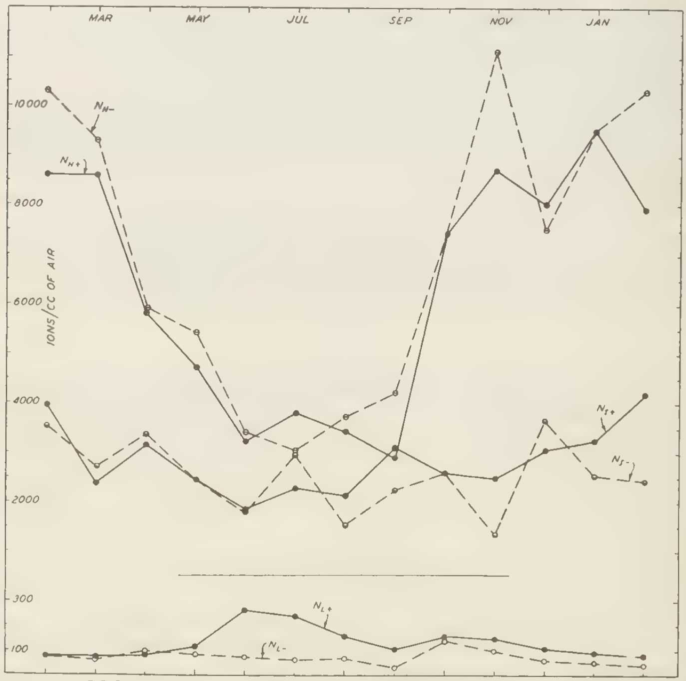


FIG. 3—ANNUAL LIGHT-, INTERMEDIATE-, AND HEAVY-ION COUNTS, 1934-1935

by our intermediate group. The significance of this intermediate group will be discussed later.

The heavy group, N_H, shows a sharp rise during the winter season and a minimum during the summer season. Generally the number of negative heavy ions exceeds those of the positive. The peak in the winter months (or heating season) is no doubt connected with the increase in the products of combustion from furnaces, etc. The annual changes in the heavy-ion count found by us agree very well with those reported by Wait and Torreson[1] for the positive heavy Langevin ion. They report a minimum value of about 3000 for the month of July and a maximum value of 9000 for December. The range of variation for these two months is found to be exactly the same in our case.

[6] Phil. Mag., **29**, 514-526 and 636-646 (1915).

TABLE B—*Monthly averages of daily ion-counts*

Month	Positive			Negative				
	N_L	N_I	N_H	N_L	N_I	N_H	N_I^+ / N_I^-	N_H^+ / N_H^-
1934								
Feb.	73	3980	8600	71	3530	10300	1.13	0.82
Mar.	70	2360	8600	64	2700	9300	0.87	0.92
Apr.	75	3140	5750	90	3350	5850	0.94	0.98
May	107	2450	4700	77	2470	5400	0.99	0.87
June	260	1840	3150	66	1780	3350	1.03	0.94
July	231	2270	3780	54	2950	3000	0.77	1.26
Aug.	152	2110	3390	58	1510	3650	1.40	0.93
Sep.	100	3100	2880	24	2225	4180	1.38	0.69
Oct.	152	2590	7400	118	2580	7350	1.00	1.01
Nov.	141	2460	8700	94	1310	11100	1.88	0.78
Dec.	101	3030	8040	55	3650	7500	0.83	1.07
1935								
Jan.	85	3240	9480	43	2510	9500	1.36	0.99
Feb.	73	4160	7850	33	2400	10300	1.73	0.76

In Table B is also given the ratio of the positive count to the negative count for both the intermediate- and the heavy-ion groups for each month of the year. In general the ratio for the intermediate group is greater than unity with a yearly average of 1.18. The heavy-ion group has a ratio less than unity with an annual average of 0.92. The ratio of the positive and negative ion-numbers is of importance in the derivation of equations for the production-rate of ions.

In the diurnal counts made once a month, the year was arbitrarily divided into two seasons—the heating, extending from October to April inclusive, and the non-heating, extending from May through September. All diurnal observations are presented in eastern standard time (E. S. T.)

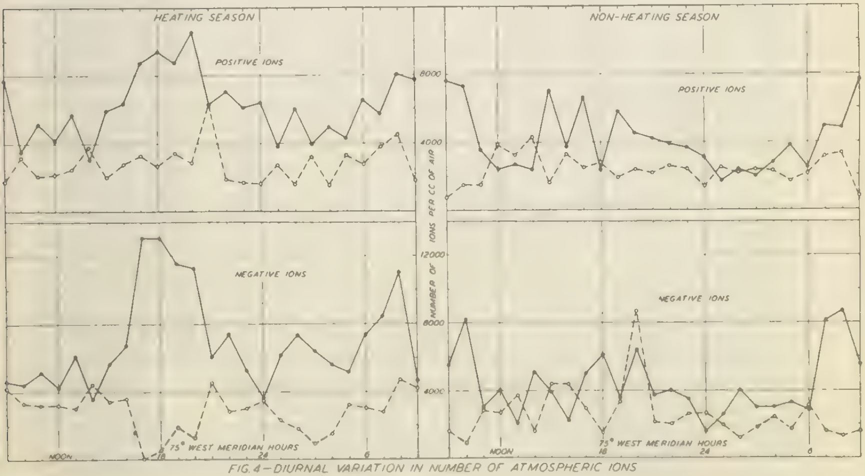


FIG. 4—DIURNAL VARIATION IN NUMBER OF ATMOSPHERIC IONS
LARGE IONS
INTERMEDIATE IONS

which is the 75° west meridian mean time. The typical diurnal variations for the heating season and also for the non-heating season were found by averaging the corresponding observation of the routine for each hour over the months contained in the season. The diurnal counts extend over a twelve-month period from February 1934 through January 1935.

The data contained in Figure 4 and Table C for the heating season represent an average of seven readings while those of the non-heating, five readings. Since the design of our counter is not the most suitable for small ions, and due to the small number of measurements for each average, only the diurnal variations for the large-ion group and the intermediate-ion plus light-ion group are given. Calculations show that the light-ion numbers are small compared to the number of intermediate ions, so our

TABLE C—*Results of diurnal counts*

Time E.S.T.	Heating season				Non-heating season			
	Positive		Negative		Positive		Negative	
	(N_I+N_L)	N_H	(N_I+N_L)	N_H	(N_I+N_L)	N_H	(N_I+N_L)	N_H
h								
9	1770	7700	4210	4600	800	7600	1580	5500
10	3150	3500	3270	4300	1450	7300	920	8200
11	2080	5100	3200	5050	1510	3600	2800	3020
12	2210	4150	3190	4150	3900	2400	2730	4080
13	2450	5680	2000	7050	3290	2680	3670	2030
14	3740	3000	4430	3500	4280	2640	1590	5050
15	1970	5900	3400	5550	1570	6950	4400	3920
16	2770	6300	3590	6700	3290	3700	4370	2150
17	3260	8720	0	13000	2470	6550	2960	5000
18	2670	9400	550	13000	2810	2380	1480	6120
19	3430	8700	1910	11500	1910	5800	3330	3600
20	2830	10500	1290	11200	2420	4500	870	6400
21	6290	6300	4470	6000	2150	4150	2100	3720
22	1880	7700	2760	7350	2640	3920	2010	4000
23	1690	6100	3040	5200	2360	3680	2600	3450
24	1580	6400	3480	3600	1380	3060	2730	1590
1	2700	3750	2250	6100	2470	1650	1990	2640
2	1580	6000	1750	7320	2210	2400	1230	3960
3	3190	3900	900	6400	2380	1960	1810	3040
4	1540	4950	1530	5600	2250	2830	2380	3020
5	3290	4260	3170	5100	1740	3800	1670	3250
6	2760	6500	2950	7300	2070	2500	3070	2820
7	3780	5700	2820	8400	3120	4900	1590	8100
8	4470	8000	4660	11000	3340	4760	1300	8700

curve for the intermediate positive light-ions gives for the most part the intermediate-ion numbers. It will be noted, therefore, that for such groupings the observations at mobility-thresholds, 0.0014 and 0.0006 cm per second/volt per cm, are all that are needed for the analysis. Since these observations are the most accurate of the routine observations we feel that our results are more significant in this form.

In regard to the diurnal variation in the number of positive and negative heavy-ions, during the heating season, there seems to be a rise in ion-content at 6 p. m. and at 8 a. m. In the non-heating season the rise

at 6 p. m. is very much reduced, while the maximum at 8 a. m. still remains clearly defined. These diurnal variations for the heavy ion agree, as a whole, with those reported by Wait and Torreson[1] in Washington. They found, however, that the evening peak remained throughout the year while the morning maximum disappeared with the summer season. This disagreement is probably due to the fact that their diurnal results are based on six to ten series of observations a month while we had but one each month. The essential features of the Washington and New Haven ion-surveys, however, are about the same.

In the intermediate group we find a distinct inverse relation when compared with the heavy-ion group. This is noticed especially in the negative ions during the heating season, and is generally true for all four curves. The actual daily maxima and minima for the intermediate groups are not as distinctive as those of the heavy ions and, in general, seem to bear no definite relationship to those of the heavy-ion group.

Discussion

The highest mobility threshold chosen for our routine counts was 0.07 cm per second/volt per cm. This is the highest value practical for use with the present type of counter for regular observations. Careful analysis of the characteristic curve through this region reveals no presence of an intermediate ion of the type (0.1 k 0.01) reported by Wait[4] and Pollock.[6] It must be noted, however, that the dimensions of our counter are not the most suitable for this particular region of the mobility-spectrum.

The lowest threshold used for our routine counts is 0.0006 cm per second/volt per cm. It corresponds to the critical point for the complete collection of the heavy Langevin ion and hence represents its mobility. Our value (0.0006) for the average Langevin ion mobility is high compared to those obtained by other workers. Wait and Torreson[1] reported a value of 0.00046, while P. Langevin[7] himself obtained a value of 0.00033. This discrepancy may be due to turbulence of air-flow and structural differences in the counting apparatus. It has been shown by Wait that turbulence tends to lower the value of the Langevin ion mobility. For his determination he used an air-stream of 700 cc per second, which reduces to a linear air-velocity of 82 cm per second in the counter. In our case (air-stream of 100 cc per second) the linear air-velocity in the counter was 8.3 cm per second. It therefore seems logical to assume that little turbulence occurred in our counter. Our value for mobility was calculated from the formula given previously in this paper (see footnote*). This formula does not take into account ions which are collected by the leads to the electrometer. This error is believed to be small since the leads to C and D (see Fig. 1) are one mm in diameter and make a simple push-fit on C and D. The lead holes in the outer cylinder are 5 mm in diameter. The projected area of these leads is 0.08 sq cm compared to a cross-section area of the ion path of 12.0 sq cm. In conclusion it may be pointed out that our work agrees with Wait in that it points to a higher mobility for the heavy-ion group than has been previously reported by other observers.

A modification of the characteristic curve for the ion-spectrum, as

[7]C. R. Acad. sci., **140**, 232-234 (1905).

given in Figure 2, is presented in Figure 5. If we assume that this is the true characteristic and draw straight lines through points 1 and 2, and through points 3 and 4, the curve reveals the presence of an intermediate ion of mobility k_1 (in his particular case for July of 0.0064 cm per second / volt per cm). Our previous group-distribution now takes a new form where N_L, N_I, and N_H represent the actual numbers of small, intermediate, and large ions (of mobility-values for this case of 1.5, 0.0064, and 0.0006, respectively).

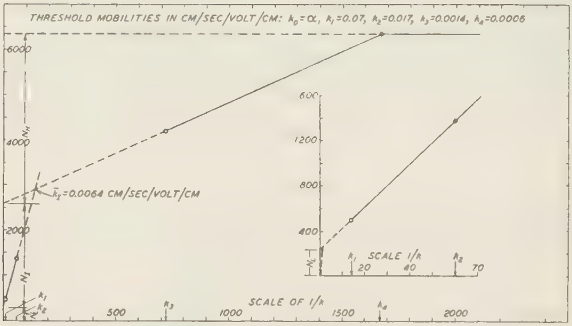


FIG. 5—METHOD OF ANALYSIS, MONTH OF JULY

For such a characteristic curve to be accurate, the following assumptions must be true. First, the mobility of the heavy ion does not vary during the course of the year. We believe such an assumption to be justified since Wait and Torreson[1] (in Table 1 of their paper) find approximately the same mobility-value for this ion at three different periods of the year. Secondly, there is only one critical mobility between the first point and the origin; this will have to remain a pure assumption although our data, perhaps not conclusively so, due to limitations of the apparatus mentioned before, fail to reveal the presence of any new ions in that high mobility-region. The third assumption is that there is no critical mobility between k_1 and k_2. We have analyzed this region thoroughly and have failed to locate any breaks in the normal characteristic curve. Finally, the fourth assumption is that there are no critical mobilities between points 3 and 4. We have checked this region very thoroughly and can say conclusively that there are no ions with mobilities between 0.0014 and 0.0006. In addition, if a critical mobility lies between k_1 (0.07) and k_2 (.017), the point at k_2 should be colinear with those of k_3 and k_4. Therefore, in view of the logic of these assumptions our data point to the existence of an intermediate ion of low mobility.

When we calculate the mobility of the intermediate ion by the methods shown in Figure 5 for each month of the year, we obtain a curve shown in Figure 6. In general we see that in the winter months the mobility is low. During the summer months it rises to a maximum value of 0.01 for the negative in June. The positive ions attained the maximum mobility-value during September. As a whole this shows that the intermediate ion does not have a stable mobility but fluctuates during the course of the year.

If the value of k_1 for the diurnal counts is calculated by methods

previously outlined, the relationships between k, N_I, and N_H are shown in Table D. One index of expressing the degree of interrelation between several factors is the correlation-coefficient. Since these coefficients are calculated from a population of 24 they should be considered only as an index of the tendency of variation between the factors involved. In general one notices that the number of intermediate ions varies inversely with the mobility of that ion, that is, the higher the mobility the lower the

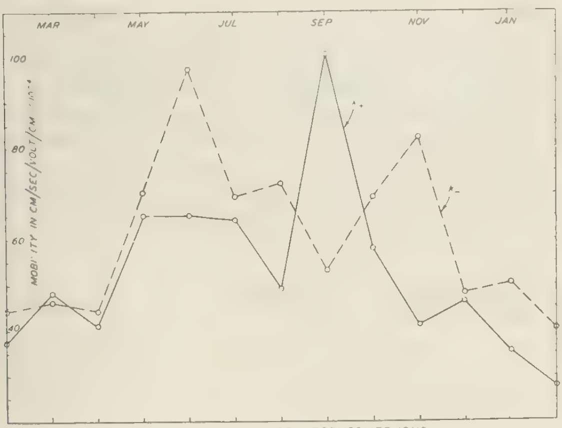


FIG. 6—MOBILITY OF INTERMEDIATE IONS

number of ions. The mobility of the intermediate ion also varies inversely with the number of heavy ions during the heating season and directly with the number of heavy ions during the non-heating season. Finally, there is a highly significant negative correlation between intermediate and heavy ions of the same sign than of different signs. These facts show that there is a different relationship existing in the intermediate-ion group during the winter and summer months, and indicate that either the heavy ions

TABLE D—*Relationships between k, N_I, and N_H: Diurnal correlation-coefficients*

Group	Season[a]	k (+)	k (−)	N_I+	N_I-	N_H+	N_H-
$k+$	h	......	+.493	−.710	−.060	−.158	−.279
	n	......	+.322	−.225	−.107	+.596	+.084
$k-$	h	+.493	......	−.302	−.363	−.355	−.249
	n	+.322	......	−.076	−.325	+.340	+.426
N_I+	h	−.710	−.302	......	+.216	−.030	+.226
	n	−.225	−.076	......	+.073	−.500	+.129
N_I-	h	−.060	−.363	+.216	......	−.287	−.800
	n	−.107	−.325	+.073	......	+.025	−.530
N_H+	h	−.158	−.355	−.030	−.287	......	+.787
	n	+.595	+.340	−.500	+.025	......	+.471
N_H-	h	−.279	−.249	+.226	−.800	+.787	......
	n	+.084	+.426	+.129	−.530	+.471	......

[a] h = heating season; n = non-heating season.

for the two seasons are different as pointed out by Wait and Torreson[1], or that the nature of the intermediate ions is different.

The variation of the ion-content with various weather-factors shows, in general, that an increase in sunshine causes an increase in the number of light ions and a decrease in the numbers of heavy ions and intermediate ions. Wind-movement as a whole plays a small part. South winds favor the heavy-ion numbers while north or land winds favor the light-ion numbers. Finally, a high relative humidity tends to reduce the number of small ions and to increase the number of large ions while the intermediate-ion numbers do not seem to be affected. In addition, it might be mentioned that over the year as a whole there is a fair inverse relationship between the number of light ions and intermediate ions, while the intermediate-ion number shows a direct change with the heavy-ion number. This latter phenomenon is the reverse of that observed for the diurnal counts where an inverse relationship was found between the heavy and intermediate ions.

The authors wish to express their appreciation to L. P. Herrington, E. J. Rodee, K. L. Smith, and F. N. Hagan, all members of the Laboratory Staff, for their assistance in taking the routine observations and diurnal counts.

John B. Pierce Laboratory of Hygiene,
New Haven, Connecticut

A TABLE OF INVERSE TRIGONOMETRIC FUNCTIONS IN RADIANS

By Irwin Roman

In the integration of numerous expressions, the results are expressible in the inverse trigonometric functions measured in radians. For three- or four-place evaluations, the customary conversion-formulas may be applied to tables using the sexagesimal system. For more than four places, the tables become bulky and inconvenient and the conversion tedious, especially when values are wanted to seven or more places. In 1927, the writer needed arctangents for torsion-balance calculations. At the time, Gifford's eight-place tables were not available and it soon became apparent that time would be saved by constructing abridged tables of the inverse functions, together with the proper coefficients for interpolating.

A preliminary investigation disclosed several facts:

(1) The arctangent formulas are the simplest to handle, as they involve no irrational elements.

(2) Any one inverse function can be obtained from any other by well-established relations of a comparatively simple form.

(3) Series for the arctangent converge with convenient rapidity for small values of the argument.

(4) Simple reduction-formulas are available for arguments near unity.

(5) The argument may be restricted to the range from zero to one, because of simple reduction-formulas.

The tables of arctangents as constructed in 1927 were sufficient for immediate needs but the writer was not sufficiently confident of their accuracy to offer them for publication. Recently, the tables were needed again and more careful calculations were made for them. The numerical calculations involved were made independently by the writer and by a student assistant, John Unsworth Allen, to whom the writer extends his thanks and appreciation for careful and accurate assistance. The tables are considered accurate and the results obtained from them should be reliable to the customary degree—the forcing errors being less than a few units in the last digit. It is advisable to interpolate from the nearest tabular entry rather than consistently forward—as for most tables where linear interpolation is not sufficiently accurate. The writer will be grateful for notification as to any errors occurring in the tables.

For most purposes, the tables may be used without reference to their construction, but for more accurate values recourse may be had to the formulas and methods used herein. The calculations of the tables may be divided into two parts; that of arctangent a where a is a multiple of a hundredth and that of the A_n's used in interpolation.

The calculation of arctan a was based on the following relations:

$$\arctan x - \arctan y = \arctan [(x - y)/(1 + xy)] \qquad (1)$$

If we set $x = a$ and $y = (a - h)$, this leads to

$$\arctan a = \arctan (a - h) + \arctan \{ h/[1 + a (a - h)] \}$$

If we set $x = h/[1 + a\,(a - h)]$ and $y = h$, equation (1) leads to

$$\arctan \left\{ h/[1 + a\,(a - h)] \right\} = \arctan h - \arctan z$$

where

$$\begin{aligned} z &= [a\,(a - h)\,h]/[(1 + h^2) + a\,(a - h)] \\ &= h - [h\,(1 + h^2)]/[(1 + h^2) + a\,(a - h)] \end{aligned}$$

The second form for z has its first term independent of a while its second term decreases monotonically in a so that z increases monotonically in a. If we keep a between zero and unity, the maximum value of z is

$$z_{max} = [(1 - h)\,h]/[(1 + h^2) + (1 - h)]$$

For $h = 0.01$, z_{max} is less than 0.005 so that $(1/7)\ z^7{}_{max}$ is less than 1.12×10^{-17} and we may write

$$\arctan z = z - z^3/3 + z^5/5$$

with an error not exceeding one unit in the sixteenth decimal place. For $h = 0.01$, we have

$$\arctan h = 0.009\ 999\ 666\ 686\ 665$$

to fifteen places and we may write

$$\arctan a = \arctan (a - 0.01) + 0.009\ 999\ 666\ 686\ 665 - z + z^3/3 - z^5/5 \tag{2}$$

Starting with $a = 0.01$, we thus may calculate arctan a by steps of $h = 0.01$ from 0 to 1. These calculations were made to fifteen decimal places, the cumulative error being 27 units. This error was distributed uniformly over the entire table.

To check these values, differences of all orders to the ninth were calculated and showed no gross errors. In addition, twelve decimal calculations were made for multiples of a tenth, in two groups, namely, (*A*) and (*B*).

(*A*) For $a = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6$, and also $a = 0.01$, the values of arctan a were calculated to twelve places by each of the formulas

$$\arctan x = \sum_{n=0}^{\infty} \frac{x^{4n+1}}{4n+1} - \sum_{n=0}^{\infty} \frac{x^{4n+3}}{4n+3} \quad (x^2 \leqq 1) \tag{3}$$

$$\arctan x = \sum_{n=0}^{\infty} u_n$$

where

$$\begin{aligned} u_0 &= x/(1 + x^2) \quad X = x^2/(1 + x^2) \\ u_n &= [2n/(2n + 1)]\,[X u_{n-1}] \end{aligned} \tag{4}$$

These series were taken from the Smithsonian volume on "Mathematical formulæ and elliptic functions" by Adams and Hippisley.

(*B*) For $a = 0.7, 0.8$, and 0.9, two reduction-formulas were used:

(1) Assuming the value for arctan 0.6 as correct and taking $h = 0.1$, we have

$$\arctan 0.7 = \arctan 0.6 + \arctan x_1$$

where $x_1 = 0.1/[1 + (0.7)\,(0.6)]$ and arctan x_1 may be calculated by series (3) and (4). Similarly, arctan 0.8 may be calculated from arctan 0.7 and arctan 0.9 from arctan 0.8.

(2) For x greater than $\sqrt{3}/3$, we have the relation

$$\arctan x = (1/2) \arctan [2x/(1-x^2)]$$
$$= \pi/4 - (1/2) \arctan [(1-x^2)/2x]$$

where $(1-x^2)/2x$ is less than x. Hence $\arctan [(1-x^2)/2x]$ can be calculated by series (3) and (4).

After applying the twelve-place checks to the fifteen-place calculated values, the results were rounded to eleven figures, corresponding to a nine-bank calculating machine.

The columns headed A_1, A_2, A_3, A_4, are the coefficients in Taylor's series, tabulated so that the corrections will correspond to one unit in the eleventh decimal place when h is referred to one unit in the second decimal place. If $x = (a + h/100)$, Taylor's series is

$$\arctan x = \arctan a + A_1 h + A_2 h^2 + A_3 h^3 + A_4 h^4 + \ldots \quad (5)$$

where

$$A_1 = [1/(1 + a^2)]\, 10^9 \qquad A_2 = -[a/(1 + a^2)^2]\, 10^7$$
$$A_3 = [(3a^2 - 1)/3(1 + a^2)^3]\, 10^5 \qquad A_4 = [a(1 - a^2)/(1 + a^2)^4]\, 10^3$$

The values of A_n were checked against the values of arctan a as computed above. If A_n' refers to $x = a$ and A_n'' refers to $x = (a + 0.01)$, while h is taken as $+1/2$, and $-1/2$, respectively, we have

$$\arctan (a+0.005) = \arctan (a) + A'_1/2 + A'_2/4 + A'_3/8 + A'_4/16 \ldots$$
$$\arctan (a+0.005) = \arctan (a+0.01) - A''_1/2 + A''_2/4 - A''_3/8 + A''_4/16 \ldots$$

Hence

$$\arctan (a+0.01) = \arctan (a) + (A'_1 + A''_1)/2 + (A'_2 - A''_2)/4 + (A'_3 + A''_3)/8 + (A'_4 - A''_4)/16 \ldots$$

The values of A_n were checked by this formula, and no other checks were considered necessary.

For values of x outside the range from zero to unity, we have the relations

$$\arctan(-x) = 3.141\ 592\ 653\ 59 - \arctan x$$
$$\arctan(1/x) = 1.570\ 796\ 326\ 79 - \arctan x$$
$$\arctan(-1/x) = 1.570\ 796\ 326\ 79 + \arctan x$$

These formulas cover all real values of x and select the arctangent as zero or between zero and π radians, π radians being excluded.

For the other inverse trigonometric functions, we have the relations

$$\arcsin x = \arctan [x/\sqrt{(1 - x)(1 + x)}]$$
$$\arccos x = \arctan (\sqrt{(1 - x)(1 + x)}/x)$$
$$\operatorname{arccot} x = \arctan (1/x)$$
$$\operatorname{arcsec} x = \arctan \sqrt{(x - 1)(x + 1)}$$
$$\operatorname{arccsc} x = \arctan [1/\sqrt{(x - 1)(x + 1)}]$$

The use of the tables for multiples of 0.01 in a is direct, the last figure being considered reliable. For other values, a is selected as the nearest argument in the tables and h is the phase, referred to one unit in the second decimal place. Hence $(-1/2) \leqq h \leqq 1/2$. Formula (5) then furnishes the desired arctangent, with the forcing error in the eleventh decimal place.

For illustration, consider y = arctan (0.272 632 417 92); here we have the calculations:

	$a = 0.27$	arctan a = 0.263 711 834 46
$h =$ 0.263 241 792	$A_1 = +$ 9 320 533 13	$A_1h = +$ 2 453 553 84
$h^2 =$ 0.069 296 2	$A_2 = -$ 23 455 53	$A_2h^2 = -$ 1 625 38
$h^3 =$ 0.018 24	$A_3 = -$ 210 87	$A_3h^3 = -$ 3 85
$h^4 =$ 0.005	$A_4 = +$ 1 89	$A_4h^4 = +$ 1
		Sum = arctan (0.272 632 417 92) = 0.266 163 759 08

It should be noted that each successive power of h is needed to two fewer decimal places, and that the result is expressed in radians.

Similarly, for y = arctan (0.866 927 043 36) we have the calculations:

	$a = 0.87$	arctan a = 0.715 991 114 42
$h = -$ 0.307 295 664	$A_1 = +$ 5 691 843 59	$A_1h = -$ 1 749 078 86
$h^2 = +$ 0.094 430 6	$A_2 = -$ 28 185 46	$A_2h^2 = -$ 2 661 57
$h^3 = -$ 0.029 02	$A_3 = +$ 78 11	$A_3h^3 = -$ 2 27
$h^4 = +$ 0.009	$A_4 = +$ 22	$A_4h^4 =$ 0
		Sum = arctan (0.866 927 043 36) = 0.714 239 371 72

For fewer decimal places, the values of h^n and A_n should be rounded off before calculation. Thus, for arctan (0.272 63) to five decimals, we have the calculations:

	$a = 0.27$	arctan a = 0.263 71
$h = +$ 0.263	$A_1 = +$ 932	$A_1h = +$ 2 45
$h^2 = +$ 0.1	$A_2 = -$ 2	$A_2h^2 =$ 0
		Sum = arctan (0.272 63) = 0.266 16

If we wish the value of y = arcsin (0.866 927 043) we have

$$y = \arctan (0.866\ 927\ 043)/\sqrt{(1.866\ 927\ 043)\ (0.133\ 072\ 957)}$$
$$= \arctan (1.739\ 297\ 91) = 1.570\ 796\ 327 - \arctan (0.257\ 494\ 463)$$

It should be noted that for the arctangent of a quantity between zero and unity, a nine-bank computing machine will furnish the function to eleven decimals but where preliminary calculations are needed, the accuracy may be reduced to nine significant figures—in each case, the last figure being subject to the forcing errors.

Of numerous instances in which the radian-value of an inverse trigonometric function is needed, a few may be cited.

Table of inverse trigonometric functions in radian

a	arctan a	$+A_1$	$-A_2$	$-A_3$	$+A_4$
0.00	0.000 000 000 00	10 000 000 00	00 000 00	333 33	00
1	0.009 999 666 69	9 999 000 10	999 80	333 13	10
2	0.019 997 333 97	9 996 001 60	1 998 40	332 53	20
3	0.029 991 004 86	9 991 008 09	2 994 61	331 54	30
4	0.039 978 687 12	9 984 025 56	3 987 23	330 14	40
5	0.049 958 395 72	9 975 062 34	4 975 09	328 36	49
6	0.059 928 155 12	9 964 129 14	5 957 03	326 20	59
7	0.069 886 001 63	9 951 238 93	6 931 90	323 65	68
8	0.079 829 985 71	9 936 407 00	7 898 57	320 74	77
9	0.089 758 174 19	9 919 650 83	8 855 95	317 45	86
0 10	0.099 668 652 49	9 900 990 10	9 802 96	313 82	95
1	0.109 559 526 77	9 880 446 60	10 738 55	309 85	105
2	0.119 428 926 02	9 858 044 16	11 661 72	305 54	112
3	0.129 275 004 05	9 833 808 63	12 571 49	300 92	119
4	0.139 095 941 48	9 807 767 75	13 466 92	295 99	127
5	0.148 889 947 61	9 779 951 10	14 347 12	290 76	134
6	0.158 655 262 19	9 750 390 02	15 211 22	285 26	141
7	0.168 390 157 15	9 719 117 50	16 058 41	279 49	147
8	0.178 092 938 23	9 686 168 15	16 887 93	273 48	153
9	0.187 761 946 51	9 651 578 03	17 699 06	267 23	159
0 20	0.197 395 559 85	9 615 384 62	18 491 12	260 77	164
1	0.206 992 194 22	9 577 626 66	19 263 50	254 11	169
2	0.216 550 304 98	9 538 344 14	20 015 60	247 26	173
3	0.226 068 387 99	9 497 578 12	20 746 92	240 25	177
4	0.235 544 980 72	9 455 370 65	21 456 97	233 09	181
5	0.244 978 663 13	9 411 764 71	22 145 33	225 79	184
6	0.254 368 058 55	9 366 804 05	22 811 62	218 38	186
7	0.263 711 834 46	9 320 533 13	23 455 53	210 87	189
8	0.273 008 703 09	9 272 997 03	24 076 77	203 28	191
9	0.282 257 421 98	9 224 241 31	24 675 12	195 61	192
0.30	0.291 456 794 48	9 174 311 93	25 250 40	187 90	193
1	0.300 605 670 04	9 123 255 18	25 802 47	180 15	194
2	0.309 702 944 54	9 071 117 56	26 331 25	172 38	195
3	0.318 747 560 42	9 017 945 71	26 836 70	164 59	195
4	0.327 738 506 78	8 963 786 30	27 318 82	156 82	194
5	0.336 674 819 39	8 908 685 97	27 777 64	149 07	193
6	0.345 555 580 58	8 852 691 22	28 213 25	141 35	192
7	0.354 379 919 12	8 795 848 36	28 625 77	133 67	191
8	0.363 147 009 95	8 738 203 43	29 015 35	126 06	189
9	0.371 856 073 85	8 679 802 10	29 382 20	118 51	188
0 40	0.380 506 377 11	8 620 689 66	29 726 52	111 05	186
1	0.389 097 231 06	8 560 910 88	30 048 57	103 67	183
2	0.397 627 991 52	8 500 510 03	30 348 64	096 39	181
3	0.406 098 058 32	8 439 530 76	30 627 04	089 22	178
4	0.414 506 874 58	8 378 016 09	30 884 11	082 17	175
5	0.422 853 926 13	8 316 008 32	31 120 20	075 24	172
6	0.431 138 740 72	8 253 549 03	31 335 69	068 44	168
7	0.439 360 887 28	8 190 679 01	31 530 99	061 78	166
8	0.447 519 975 16	8 127 438 23	31 706 52	055 26	161
9	0.455 615 653 21	8 063 865 83	31 862 71	048 89	157
0.50	0.463 647 609 00	8 000 000 00	32 000 00	042 67	154

Table of inverse trigonometric functions in radian—Concluded

a	arctan a	$+A_1$	$-A_2$	$+A_3$	$+A_4$
0.50	0.463 647 609 00	8 000 000 00	32 000 00	−42 67	154
1	0.471 615 567 86	7 935 878 10	32 118 86	36 60	150
2	0.479 519 291 99	7 871 536 52	32 219 76	30 69	145
3	0.487 358 579 51	7 807 010 70	32 303 19	24 95	142
4	0.495 133 263 47	7 742 335 09	32 369 62	19 37	138
5	0.502 843 210 93	7 677 543 19	32 419 57	13 95	133
6	0.510 488 321 92	7 612 667 48	32 453 52	8 71	129
7	0.518 068 528 46	7 547 739 45	32 471 97	− 3 63	125
8	0.525 583 793 55	7 482 789 58	32 475 44	+ 1 28	120
9	0.533 034 110 18	7 417 847 34	32 464 43	6 03	117
0.60	0.540 419 500 27	7 352 941 18	32 439 44	10 60	112
1	0.547 740 013 72	7 288 098 54	32 400 99	15 01	108
2	0.554 995 727 34	7 223 345 85	32 349 57	19 25	104
3	0.562 186 743 90	7 158 708 57	32 285 68	23 32	100
4	0.569 313 191 10	7 094 211 12	32 209 81	27 23	96
5	0.576 375 220 59	7 029 876 98	32 122 46	30 98	92
6	0.583 373 006 99	6 965 728 62	32 024 10	34 56	88
7	0.590 306 746 94	6 901 787 56	31 915 23	37 99	84
8	0.597 176 658 09	6 838 074 40	31 796 30	41 27	80
9	0.603 982 978 25	6 774 608 77	31 667 78	44 39	76
0.70	0.610 725 964 39	6 711 409 40	31 530 11	47 36	72
1	0.617 405 891 75	6 648 494 12	31 383 76	50 18	69
2	0.624 023 052 98	6 585 879 87	31 229 14	52 87	65
3	0.630 577 757 21	6 523 582 75	31 066 70	55 40	62
4	0.637 070 329 28	6 461 617 99	30 896 86	57 81	58
5	0.643 501 108 79	6 400 000 00	30 720 00	60 07	55
6	0.649 870 449 41	6 338 742 39	30 536 54	62 21	52
7	0.656 178 717 99	6 277 857 99	30 346 86	64 22	49
8	0.662 426 293 83	6 217 358 87	30 151 33	66 11	46
9	0.668 613 567 93	6 157 256 33	29 950 32	67 87	43
0.80	0.674 740 942 22	6 097 560 98	29 744 20	69 52	40
1	0.680 808 828 92	6 038 282 71	29 533 30	71 06	37
2	0.686 817 649 76	5 979 430 76	29 317 94	72 49	34
3	0.692 767 835 40	5 921 013 68	29 098 48	73 81	32
4	0.698 659 824 72	5 863 039 40	28 875 19	75 03	29
5	0.704 494 064 24	5 805 515 24	28 648 40	76 15	27
6	0.710 271 007 49	5 748 447 92	28 418 40	77 17	24
7	0.715 991 114 42	5 691 843 59	28 185 46	78 11	22
8	0.721 654 850 86	5 635 707 84	27 949 86	78 95	20
9	0.727 262 688 00	5 580 045 76	27 711 85	79 71	18
0.90	0.732 815 101 79	5 524 861 88	27 471 69	80 39	16
1	0.738 312 572 52	5 470 160 28	27 229 62	80 98	14
2	0.743 755 584 30	5 415 944 54	26 985 86	81 51	12
3	0.749 144 624 61	5 362 217 81	26 740 64	81 96	10
4	0.754 480 183 83	5 308 982 80	26 494 18	82 34	9
5	0.759 762 754 88	5 256 241 79	26 246 68	82 65	7
6	0.764 992 832 71	5 203 996 67	25 998 32	82 91	6
7	0.770 170 914 02	5 152 248 96	25 749 30	83 10	4
8	0.775 297 496 81	5 100 999 80	25 499 80	83 23	3
9	0.780 373 080 07	5 050 249 99	25 249 98	83 31	1
1.00	0.785 398 163 40	5 000 000 00	25 000 00	+83 33	0

Example 1—If U is the gravitational potential at the origin due to the prism $\left[\begin{array}{c|c|c} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \end{array}\right]$, then[1]

$$U_\Delta = \frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2} = K \,\Sigma\, (-1)^{i+j+k+l}\, \theta_{ijkl}$$

where $\theta_{ijk1} = \arctan\,(a_i\, c_k \,/\, b_j\, r_{ijk})$, $\theta_{ijk2} = \arctan\,(b_j\, c_k \,/\, a_i\, r_{ijk})$, and $r_{ijk} = \sqrt{a_i^2 + b_j^2 + c_k^2}$. K is a constant depending on the density of the prism and on the units, and each counter may be unity or two, the summation extending over all combinations. This expression has sixteen terms each term being an arctangent. While in many cases it is possible to convert an expression into equivalent forms involving fewer arctangents, there is usually little real gain over the calculation of each separate arctangent.

As a specific case, we may calculate the "curvature-excess," U_Δ, at the point (5, 7, 0) due to the prism $\left[\begin{array}{c|c|c} 10 & 6 & 6 \\ -10 & -6 & 3 \end{array}\right]$. In this case we have: $a_1 = -15$, $a_2 = 5$; $b_1 = -13$, $b_2 = -1$; $c_1 = 3$, and $c_2 = 6$; we have thence the values shown in Table 1.

TABLE 1

$\tan\theta$	$(ijkl)$	θ
0.172 432	1111	0.170 752
0.129 515	1112	0.128 798
0.250 766	1122	0.245 700
0.333 860	1121	0.322 225
0.013 047	1212	0.013 046
2.935 476	1211	1.242 466
5.560 219	1221	1.392 850
0.024 712	1222	0.024 707
−0.547 453	2112	2.640 707
−0.080 984	2111	3.060 785
−0.152 165	2121	2.990 586
−1.028 634	2122	2.342 081
−2.535 463	2211	1.946 470
−0.101 419	2212	3.040 520
−0.152 400	2222	2.990 356
−3.810 004	2221	1.827 474

Sum = 0.401 411
Curvature-excess = 0.401 411 K

[1]See E. Lancaster-Jones, Computation of Eötvös gravity-effects, Amer. Inst. Min. Metall. Eng., volume on Geophysical prospecting, 1929, p. 519.

Example 2—If we replace the finite prism of the previous example by a rectangular bar infinite in the y direction, the sixteen arctangents reduce to four and the formula becomes

$$U_\Delta = 2\,K\,\Sigma\,(-1)^{i+j}\,\theta_{ij}$$

where $\theta_{ij} = \arctan\,(c_j/a_i)$.

If we have a curved structure of infinite dimensions along the y-axis, we may replace it by a series of rectangular bars as shown in Figure 1. Each of those corners belonging to two rectangles occurs once with a

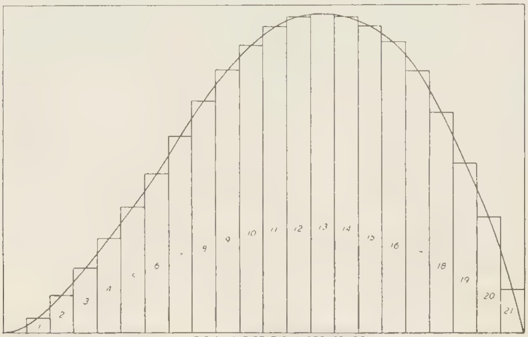


FIG 1—A FICTITIOUS STRUCTURE

positive sign and once with a negative sign so that it may be omitted from the calculations. In the case illustrated, the entire contribution of the bottom-face comes from the left and right edges. The results of calculating the curvature-excess are shown in Table 2. Each bar has a unit horizontal width in the plane of the paper and is infinite in both directions normal to the paper. For the left face of bar k, we have $a = k$ and for the right face we have $a = (k + 1)$. For the bottom, $c = 17$ while for the tops, each bar has the value of c as shown. For the tops of bars 1 to 7, inclusive, and for the left edge of the bottom, the value of c exceeds the value of a and we may tabulate $(\pi/2 - \theta)$ instead of θ.

Example 3—The x-component of the attraction at the origin due to a prism $\left[\begin{array}{c|c|c} x_2 & y_2 & z_2 \\ x_1 & y_1 & z_1 \end{array}\right]$ can be shown to be

$$F = K\,\Sigma\,(-1)^{i+j+k}\,\theta_{ijk}$$

where $\theta_{ijk} = A_{ijk} - B_{ijk} - C_{ijk}$, $A_{ijk} = x_i \arctan\,(y_j\,z_k/x_i\,r_{ijk})$, $B_{ijk} = y_j \log_e\,(z_k + r_{ijk})$, $C_{ijk} = z_k \log_e\,(y_i + r_{ijk})$, and $r_{ijk} = \sqrt{x_i^2 + y_j^2 + z_k^2}$. In this formula both radian-arctangents and natural logarithms are needed. The calculations are long but not difficult.

Example 4.—The vertical magnetic force in dynes per unit-pole due to the horizontal top of a broad dike, with surface density m is[2]

[2]See L. V. King in paper by A. S. Eve, A magnetic method of estimating the height of some buried magnetic bodies, Amer. Inst. Min. Metall. Eng., volume on Geophysical prospecting, 1932, p. 204.

$$F = 2m\,[\arctan(\{x + b\}/y) - \arctan(\{x - b\}/y)]$$

where $2b$ is the width of the dike, y is the height of the observer above it, and x is the horizontal distance from the top center line.

TABLE 2

Top

Bar	Depth	Left edge		Right edge		Excess
k	c	a/c	$[(\pi/2)-\theta_{11}]$	a/c	$[(\pi/2)-\theta_{21}]$	$[\theta_{11}-\theta_{21}]$
1	16.32	0.061 275	0.061 198	0.122 549	0.121 941	0.060 743
2	15.34	0.130 378	0.129 647	0.195 567	0.193 130	0.063 483
3	14.20	0.211 268	0.208 206	0.281 690	0.274 575	0.066 369
4	12.92	0.309 598	0.300 239	0.386 997	0.369 246	0.069 007
5	11.60	0.431 034	0.406 971	0.517 241	0.477 345	0.070 374
6	10.16	0.590 551	0.533 443	0.688 976	0.603 289	0.069 846
7	8.56	0.817 757	0.685 476	0.934 579	0.751 594	0.066 118
		c/a	θ_{11}	c/a	θ_{21}	
8	7.08	0.885 000	0.724 466	0.786 667	0.666 559	0.057 907
9	5.76	0.640 000	0.569 313	0.576 000	0.522 586	0.046 727
10	4.72	0.472 000	0.440 998	0.429 091	0.405 331	0.035 667
11	3.92	0.356 364	0.342 333	0.326 667	0.315 739	0.026 594
12	3.52	0.293 333	0.285 329	0.270 769	0.264 429	0.020 900
13	3.36	0.258 462	0.252 926	0.240 000	0.235 545	0.017 381
14	3.48	0.248 571	0.243 634	0.232 000	0.227 967	0.015 667
15	3.88	0.258 667	0.253 119	0.242 500	0.237 908	0.015 211
16	4.56	0.285 000	0.277 639	0.268 235	0.262 066	0.015 573
17	5.80	0.341 176	0.328 793	0.322 222	0.311 718	0.017 075
18	7.56	0.420 000	0.397 628	0.397 895	0.378 690	0.018 938
19	9.72	0.511 579	0.472 868	0.486 000	0.452 385	0.020 483
20	12.00	0.600 000	0.540 420	0.571 429	0.519 147	0.021 273
21	15.08	0.718 095	0.622 767	0.685 455	0.600 897	0.021 870

Bottom

Left edge $a = 1$ Right edge $a = 22$	$\cot\theta_{12} = 0.058\ 824$ $\tan\theta_{22} = 0.772\ 727$	$[(\pi/2)-\theta_{12}] = 0.058\ 755$ $\theta_{22} = 0.657\ 889$	$-0.854\ 152$
		Sum	$-0.036\ 946$
		Curvature-excess $= U_\triangle$	$-0.073\ 892\ K$

Example 5.—For small angles, we may utilize the approximate equality of x and arctan x, especially when only a few decimal places are desired. For such work, it is convenient to compile an auxiliary table similar to that shown in Table 3, which gives the six decimal values of the arctangent for arguments between 0 and 0.2.

Table 3

a	$-B_0$	$-B_1$	$-B_2$
0.00	0	0	0
0.01	0	1	1
0.02	3	4	2
0.03	9	9	3
0.04	21	16	4
0.05	42	25	5
0.06	72	36	6
0.07	114	49	7
0.08	170	64	8
0.09	242	80	9
0.10	331	99	10
0.11	440	120	11
0.12	571	142	12
0.13	725	166	13
0.14	904	192	13
0.15	1110	220	14
0.16	1345	250	15
0.17	1610	281	16
0.18	1907	314	17
0.19	2238	348	18
0.20	2604	385	18

Table 3 is based on the relation arctan $x = x + B_0 + B_1h + B_2h^2 + \ldots$ where a and h have the same significance as for the principal table. The values of the coefficients, tabulated for one unit in the sixth decimal of the arctangent and one unit in the second decimal for h, have the values $B_0 =$ (arctan a) $- a$, $B_1 = A_1 - 0.01$, and $B_k = A_k$ for k greater than unity. To compute the arctangent of $t = 0.187\ 394$ radian, we have: $a = 0.19$, $h = -0.2606$, $h^2 = 0.07$; $B_1 = -348$, $B_2 = -18$, $B_0 = -2\ 238$, $B_1h = 91$, $B_2h^2 = 1$, $t = 187\ 394$. The sum of the last four is arctan $t = 0.185\ 248$. When calculated to eleven decimals, the last digit is found to be 6 instead of 8 in the sixth decimal place, a permissible variation.

Example 6.—The difference between two nearly equal arctangents may sometimes be calculated more conveniently directly than by calculating the separate values and subtracting. Let $t_1 = [a + (h/100)]$ and $t_2 = [a + (h + m)/100]$.
Then

$$\arctan t_2 = \arctan a + A_1 (h + m) + A_2 (h + m)^2 + A_3 (h + m)^3 + A_4 (h + m)^4 + \ldots$$

$$\arctan t_1 = \arctan a + A_1h + A_2h^2 + A_3h^3 + A_4h^4 + \ldots$$

Hence

$$\theta = \arctan t_2 - \arctan t_1 = A_1m + A_2 (2hm + m^2) + A_3 (3h^2 m + 3hm^2 + m^3) + \ldots$$
$$= m [A_1 + A_2 (2h + m) + A_3 (3h^2 + 3hm + m^2) + \ldots]$$

Thus, for [arctan (0.374 682) − arctan (0.373 564)], we have: $A = 0.37$, $h = 0.3564$, $m = 0.1118$, $A_1 = 0.008\ 796$, $A_2 = -\ 0.000\ 029$, whence $\theta = 0.1118\ [0.008\ 796 - 0.000\ 029\ (0.8246)] = 0.000\ 981$.

Michigan College of Mining and Technology,
Houghton, Michigan

EARTH-CURRENT OBSERVATIONS AT CHESTERFIELD, CANADA

By B. W. Currie

This is a summary of the report on the earth-current observations made near Chesterfield Inlet (63° 20′ north, 90° 42′ west) during the International Polar Year to be published shortly by the Meteorological Service of Canada. The value of the earth-current data in relation to the magnetic and auroral observations at Chesterfield is indicated.

Earth-current observations had not been made in any part of northern Canada prior to 1932-33, and information was lacking concerning the nature and the magnitude of the earth-current that might be expected inside the zone of maximum auroral frequency and close to the magnetic pole. The Department of Terrestrial Magnetism of the Carnegie Institution of Washington, had designed a recording circuit for use at the International Polar-Year Station at College-Fairbanks, Alaska, which provided a large range of sensitivities. That Department kindly loaned an earth-current outfit embodying this circuit to the Meteorological Service of Canada for use at Chesterfield. A description of this circuit and a statement of its advantages can be found elsewhere[1, 2].

Suitable sites for electrodes were difficult to find. The region surrounding Chesterfield consists almost entirely of exposed bedrock. Rings of a fibrous muskeg surround the small sloughs and lakes, and small pockets of sand and gravel, dating apparently from the time when this region was below the waters of Hudson Bay, lie in widely scattered spots. The muskeg seldom extends to a greater depth than two feet, and varies greatly in water-content as the lakes dry up and then fill from the rain and snow.

Fortunately four sand-pockets were found that were situated so that the two pairs of electrodes could be placed on due north and south, and east and west lines, respectively. The distance between the north and south (*N-S*) electrodes was 1.31 km and between the east and west (*E-W*) electrodes was 0.86 km. The cross formed by the electrodes was nearly symmetrical. All the electrode-sites were well-drained, and with the exception of the site for the *S*-electrode consisted of sand, free from gravel and clay.

The electrodes were cross-shaped grids of quarter-inch lead wire. Each arm of the grid was about eight feet long and two and one half feet wide, and contained approximately 30 feet of the wire. The grids were buried horizontally at a depth of three feet except the *S*-grid which could be placed at a depth of only two feet because of the increasingly gravelly nature of the sand. Thermal and chemical electromotive forces at the electrodes were so small that a potential to balance or buck them was unnecessary. The electrodes were connected to the recorders by No. 14 insulated copper wire laid along the surface. This avoided the erection and anchorage of poles to carry the wires, but required considerably

[1] O. H. Gish, Procès-Verbaux, Comm. Année Polaire 1932-33, Innsbruck, 1931, Organisation Mét. Internat. No. 10, App. H, 177-182 (1932).

[2] W. J. Rooney and K. L. Sherman, Terr. Mag., 39, 187-199 (1934).

more wire for slack so that the weight of the snow during the winter would not break the lines over the sharp edges of rocky ledges. Breakdowns in the insulation were never detected, although the low mean ranges of the earth-current potential during June when the lines were buried in melting snow suggested a slow leak of current at that time.

Initially the contact-resistance at each electrode was about 2000 ohms. This value increased rapidly when the sand froze, and continued to increase until resistances of the order of 10^6 ohms were reached in March. By the following summer they had returned to their original values. The unexpectedly large increase in the contact-resistances introduced a considerable error into the final measurements. A substitution-method was used to measure the resistances in which known resistances were added to a millivoltmeter-circuit until the deflection was approximately the same as had been obtained previously with a pair of the electrodes in the circuit. The magnitude of the resistances required a large electromotive force to secure accurately readable deflections. Three hundred volts from *B*-batteries was the largest that was available. It is estimated that the error in the scaled values from this source was not more than six per cent.

Frictional charges from snow-drift caused a large loss of record. Whenever the wind-velocities exceeded 20 miles per hour the frictional charges developed by the flying snow-particles either on the lines, or on the surface in the neighborhood of the electrodes, caused rapid galvanometer-deflections which completely obscured the normal earth-current record. The appearance of the record suggested that the galvanometer had been subject to steady mechanical vibrations, but this was not the case since the zero-marks (when the galvanometer was disconnected from the line) were unaffected. The effect was not noticed until the contact-resistances became commensurate with the resistances in the galvanometer-circuits, and then more so on the *N-S* line than on the *E-W* one. This may have been due to its larger length and lesser covering of packed snow. Attempts to keep the exposed portions of the lines covered with snow were unsuccessful, a few hours of continuous snow-drift removing the cover.

As soon as the recorders were put in operation it was discovered that the earth-current potentials were subject to irregular oscillations of large amplitude and short period. Changes in the potential exceeding one volt per kilometer in less than five minutes were not unusual. The oscillations showed maximum ranges at about local midnight and local noon. The oscillations during the night were more regular than those during the day, and sometimes were so rapid that the galvanometer-deflections could not be registered photographically. Generally when they were registered they were not resolved into their component parts because of the slow motion of the photographic sheet. Because of the disturbed conditions two methods of operation could have been followed —either to have operated the recorders at their lowest sensitivity, so as to get a complete record of all oscillatory disturbances, or to have operated them at a sufficiently high sensitivity, so as to get a measureable record of the diurnal variations of the earth-current potentials on quiet days. The latter method seemed the more feasible. It meant a record of the diurnal variations on at least a portion of the days, as well as the

time of beginning and ending of periods of rapid oscillation. The range of the oscillations would be lost sometimes. Neither method could resolve the rapid disturbances into their component oscillations.

The recorders were in operation from October 1, 1932, to September 9, 1933. During October the recorders were operated at a high sensitivity, and the ranges of many disturbances were lost. During the winter practically the only loss of record was from snow-drift. During June the recorders were stopped for several days while the clock driving the drum carrying the photographic sheet was being repaired.

The records of 81 days were sufficiently undisturbed to make the scaling reasonably accurate. These were divided into two groups:

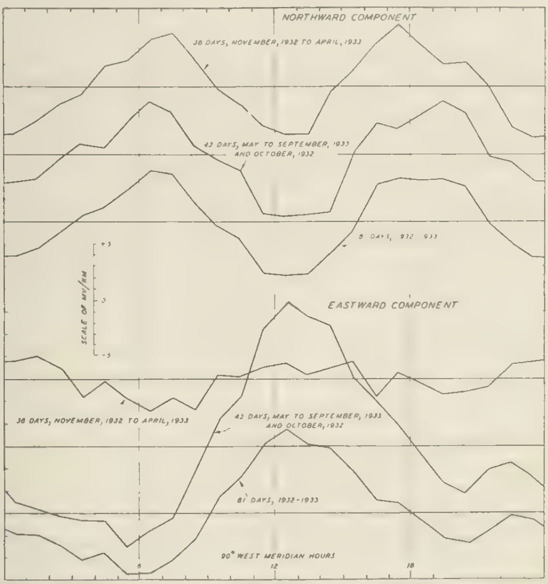


FIG 1—MEAN DIURNAL-VARIATION OF NORTHWARD AND EASTWARD COMPONENTS OF EARTH-CURRENT POTENTIAL-GRADIENTS ON QUIET DAYS AT CHESTERFIELD, OCTOBER, 1932 TO SEPTEMBER, 1933

38 days, November 1932 to April 1933, inclusive; and 43 days, October 1932 and from May to September 1933, inclusive. This division was partly to distinguish between winter and summer conditions, and partly to secure a group that was free from the electrode- and insulation-errors that might have occurred during the late spring, summer, and early fall months.

The mean diurnal *variations* of the northward and eastward components of the earth-current potentials were computed in millivolts per kilometer for each of these groups and for the two groups combined. The values are shown graphically in two ways by Figures 1 and 2. In Figure 1 the diurnal variations of the northward and eastward components are shown separately, while in Figure 2 the corresponding components are

combined to form vector-diagrams showing the average hourly changes in the direction of the earth-current.

The following characteristics of the components can be seen from Figure 1: (a) A double wave in the northward component with minima at approximately local noon and local midnight, (b) a slight increase in the range of the northward component during the day, (c) a slight increase in the range of the northward component during the summer, (d) a double wave in the eastward component during the winter months with a difference in phase of about 180° from the northward component;

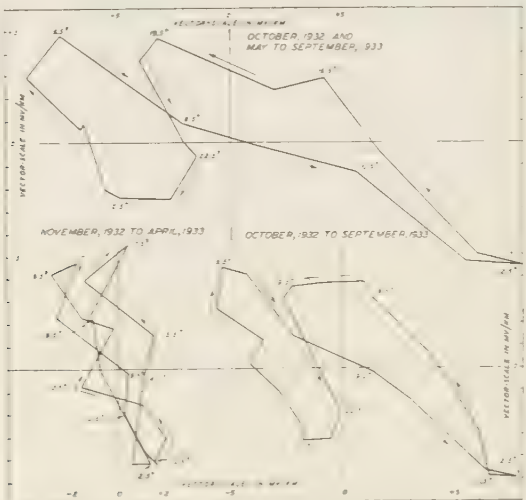


FIG. 2—VECTOR-DIAGRAMS SHOWING CHANGES EARTH-CURRENT DIURNAL-VARIATION ON QUIET DAYS AT CHESTERFIELD, OCTOBER, 1932 TO SEPTEMBER, 1933 (TIMES INDICATED ARE 90° WEST MERIDIAN HOURS)

(e) the small range of the eastward component in the winter compared with the northward component; and (f) the remarkable development of a single wave of large amplitude in the eastward component during the summer. To obtain numerical measures of these characteristics, the quantities used in plotting the curves of Figure 1 were used to compute the Fourier constants for the series

$$\Sigma\, C_n\, (n\theta + \phi_n)$$

θ being counted from 0°, midnight, 90° west meridian mean time, at the rate of 15° per hour. The results for the first four waves are given in Table 1.

The agreement with the results found by Rooney and Sherman[4] for College-Fairbanks, Alaska, is close. They discuss in detail reasons for suspecting that the single wave of large amplitude in the eastward component during the summer is spurious. The fact that it occurred in the Chesterfield records as well as in those from College-Fairbanks is a strong indication that this variation in the potential is real, and not the result of faulty recording circuits.

TABLE 1—*Fourier analyses of diurnal variations of earth-current potential-gradient on quiet days, October 1932 to September 1933, Chesterfield, Canada*

Component	Period	Amplitudes				Phase-angles			
		C_1	C_2	C_3	C_4	ϕ_1	ϕ_2	ϕ_3	ϕ_4
		mv km	*mv km*	*mv km*	*mv km*				
Northward	38 days, Nov.-Apr.	0.6	4.3	0.7	0.2	242	263	66	274
	43 days, May-Oct.	1.3	4.1	0.5	0.1	86	252	34	303
	81 days, 1932-33	0.5	4.2	0.6	0.2	102	256	51	256
Eastward	38 days, Nov.-Apr.	0.6	1.6	0.3	0.3	165	66	28	126
	43 days, May-Oct.	8.2	4.3	0.7	0.5	239	72	211	168
	81 days, 1932-33	4.2	3.0	0.4	0.4	236	72	255	156

A study of Figure 2 shows the following characteristics of the earth-currents: (*a*) The average direction of the variations in the earth-current during the winter is approximately the same as that of the magnetic meridian at Chesterfield (11° west of north); (*b*) this direction changes until it is northwest by west during the summer because of the large increase in the eastward component of the earth-current; and (*c*) the varying portion of earth-current changes at approximately local midnight and local noon from a southeasterly to a northwesterly direction, and then again reverses direction at about 6^h and 17^h during the winter and 6^h and 19^h during the summer.

Apart from the diurnal variations of earth-current potentials little use has been made of earth-current records. Numerous observers have commented on the occurrence of earth-current disturbances during the same period as magnetic disturbances. W. J. Rooney[3] has found a high correlation between oscillatory disturbances and observed aurorae at College-Fairbanks. At Chesterfield the disturbed nature of the earth-current records is involved closely with the exceptionally disturbed nature of the magnetic records, the corresponding records for a given date being easily recognisable by their general appearance. Periods of great auroral activity are also periods of great earth-current activity.

The hourly ranges of the earth-current potentials were found to be a convenient measure of the earth-current activity, and an attempt was made to relate these in a general way to magnetic and auroral disturbances. The average of the hourly ranges of the northward and eastward components of the potential for each day was taken as a measure of its earth-current activity. The sum of the intensities of observed aurorae during a night was taken as a measure of auroral activity for that day. Some adjustments were made to the auroral activity values for cloudy nights from the spectrographic records, the spectrograph being able to record auroral lines through cloud and snow-drift. The days from December 1, 1932, to March 31, 1933, inclusive, were divided into seven groups on the basis of their earth-current activity-values and again on the basis of their auroral activity-values.

The daily activities are indicated graphically in Figure 3, where a system is used much the same as that devised by J. Bartels[4] for comparing international magnetic character-numbers and solar activity character

[3]Terr. Mag., 39, 103-109 (1934).

[4]Terr. Mag., 37, 1-52 (1932); Terr. Mag., 39, 201-202 (1934).

numbers. The international magnetic character-numbers, as given by Bartels for this period, are included so that a comparison can be made between earth-current activity and auroral activity at Chesterfield, and between either of these and magnetic activity over the Earth. In general, periods of great earth-current activity are also periods of great auroral activity, and both are manifestations of increased magnetic activity over the Earth. Occasionally a day of considerable earth-current activity or auroral activity occurs without a corresponding activity in the other. In February the auroral activity was greater than the earth-current activity, while in March this condition was reversed. Auroral forms

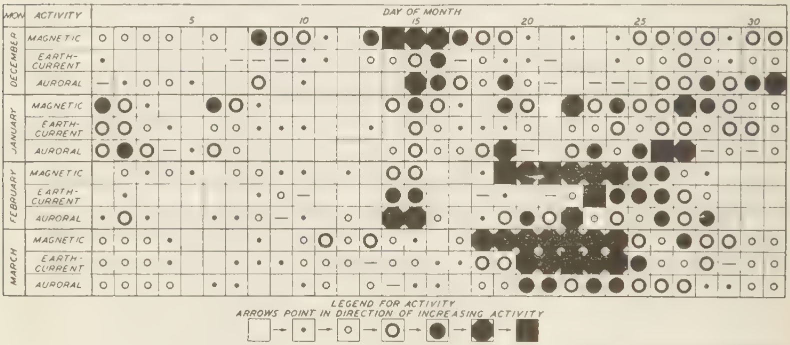


FIG. 3—COMPARISON OF INTERNATIONAL MAGNETIC ACTIVITY AND EARTH-CURRENT AND AURORAL ACTIVITIES AT CHESTERFIELD, DECEMBER, 1932 TO MARCH, 1933

with rapid movement were more numerous during March than other months of the year. If the auroral currents affect the earth-currents inductively the increase in the earth-current activity relative to the given auroral activity during March should be expected, as the auroral activity-values do not distinguish between quiet and active auroral forms.

To see if a better agreement might be found between earth-current activity and auroral activity, only clear days on which aurorae were not obscured by cloud were considered. In addition the earth-current activity-value was computed using the 12 hours centered on midnight instead of the 24 hours starting at 18^h (90° west meridian mean time). Earth-current and auroral data existed for 43 clear nights. The linear correlation-factor between the averages of the hourly ranges of both the N-S and E-W components of the potential for the 12-hour periods and the corresponding sums of the auroral intensities was 0.63. This indicates that visible aurorae affect the range of the earth-current potentials; but practically the same correlation-factor was found when the 24-hour periods were used instead of the 12-hour periods. Obviously an hour-by-hour analysis will be required if the earth-current disturbances at Chesterfield are to be identified with visible aurorae in more than a general way.

The mean diurnal ranges of both components of the potential for the three-month periods, November-December-January, February-March-April, and May-June-July are shown in Figure 4. The various graphs

have been transposed vertically relative to one another to show more clearly their individual features. To avoid confusion on this account the mean range for each period is given. The mean diurnal declination-ranges during the winter and the summer are shown also.[5] These two curves are in their correct relative positions. The graphs of Figure 4 indicate the following: (*a*) Both components of the potential throughout the year have a sharp maximum centered close to midnight and a broad maximum centered close to noon, (*b*) the two maxima occur at about the same times as those in the declination-range, (*c*) the width and the relative magnitudes of the two maxima do not alter appreciably during the year, (*d*) the mean ranges for a given period are very approximately equal for both components, and (*e*) the night maximum in the declination-range weakens during the summer without a corresponding weakening in the night maximum of the potential-ranges.

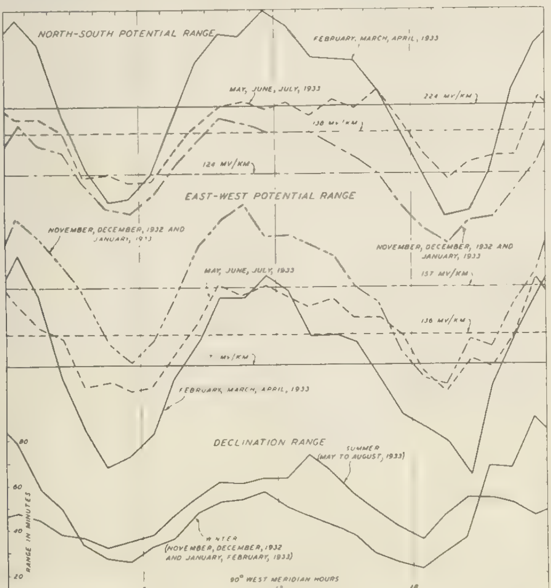


FIG. 4—MEAN DIURNAL RANGES NORTHWARD AND EASTWARD COMPONENTS EARTH-CURRENT POTENTIAL AT CHESTERFIELD FOR 3-MONTH PERIODS, NOVEMBER TO JANUARY, FEBRUARY TO APRIL, AND MAY TO JULY (MEAN DIURNAL DECLINATION-RANGES FOR WINTER AND SUMMER ARE SHOWN FOR PURPOSES OF COMPARISON)

The parallelism between the mean diurnal-variations of the potential-ranges and the declination-range is a fair indication that the same physical agency is responsible for both. Since the mean ranges of the *N-S* and *E-W* components of the potential are very approximately equal the average direction of the rapid variations of the earth-current is along a line either from northwest to southeast or from northeast to southwest. An examination of individual disturbances on the photograms indicates

[5]F. T. Davies, through courtesy of the Director of the Meteorological Service of Canada, kindly supplied the magnetic data from his manuscript now in course of publication.

that the former is the usual direction. Actually there are many disturbances which show a large effect on either the *N-S* component or the *E-W* component, and not on both. If the average direction of the earth-current disturbances is found for each season by using the mean ranges of the components of the potential, and if a reasonable value is assumed for the specific conductivity of the bedrock in the Chesterfield Region, the approximate effect of the seasonal variations of the earth-current disturbances on the declination can be calculated and compared with the observed changes. This is left until the complete magnetic data are available.

A large number of individuals helped in securing and reducing the earth-current data. Particular mention is due Dr. J. Patterson, Director of the Meteorological Service of Canada, for his untiring efforts in the establishment of the Chesterfield Station, and F. T. Davies, in charge of the Station, for its successful operation during the Polar-Year.

University of Saskatchewan,
Saskatoon, Canada

ON THE PRINCIPLES OF MAGNETIC CARTOGRAPHY

By Boris Weinberg

The purpose of a magnetic chart is to represent graphically the distribution of one or several magnetic elements over a portion of the Earth's surface or over the entire surface of the globe. The variation of the element with altitude is rarely charted.

If the net of magnetic stations were sufficiently dense that an interpolation between the observed values of the element in question for points where no observations have been made, would involve no appreciable error, the magnetic chart could be considered as representing the *true* or *real* distribution of the element. Such an interpolation is justified only if we are sure that the survey has disclosed all places of extreme values—maxima and minima—of the element. But this applies only to very detailed microsurveys for which the mean distance between the extremes of magnetic elements is of the order one km[1]. Accordingly the great majority of magnetic charts represent the *smoothed*[2] or *normal* distribution of the magnetic elements[3].

Without insisting on the distinction between the ideas regarding such distributions, I wish to emphasize that neither can give an exact idea of the real distribution inasmuch as the usual distance between the points of observation is far greater than one km. To show how the sinuosities of the smoothed or normal isolines and their approximation to the real isolines augment with increased number of stations we reproduce the maps showing the distribution of the magnetic elements over the Nikopol manganese-ore district[4] where 246 stations were distributed more or less uniformly over an area of 23 square kilometers and practically all the extremes of D, H, and Z probably were detected by the observations. Figures 1 to 9, 14 to 22, and 27 to 35 show the distribution of D, H, and Z (the isolines being traced in the usual manner by determining the isopoints through interpolation between the values at adjacent points) obtained by taking into account only the points of observation whose values satisfy respectively the conditions

$$N = 0 \ (\text{mod. } 9),\ N = 1 \ (\text{mod. } 9),\ \ldots\ N = 8 \ (\text{mod. } 9) \qquad (1)$$

The numbers against the D-isolines are the differences $(D - 3)°$, those against the H-isolines are the differences $(H - 21000)\ \gamma$, and those against the Z-isolines are the differences $(Z - 40000)\ \gamma$.

Figures 10, 11, 12, 23, 24, 36, 37, and 38 are based on observations at points whose values satisfy the conditions

$$N = 0 \ (\text{mod. } 3),\ N = 1 \ (\text{mod. } 3),\ N = 2 \ (\text{mod. } 3),\ \ldots . \qquad (2)$$

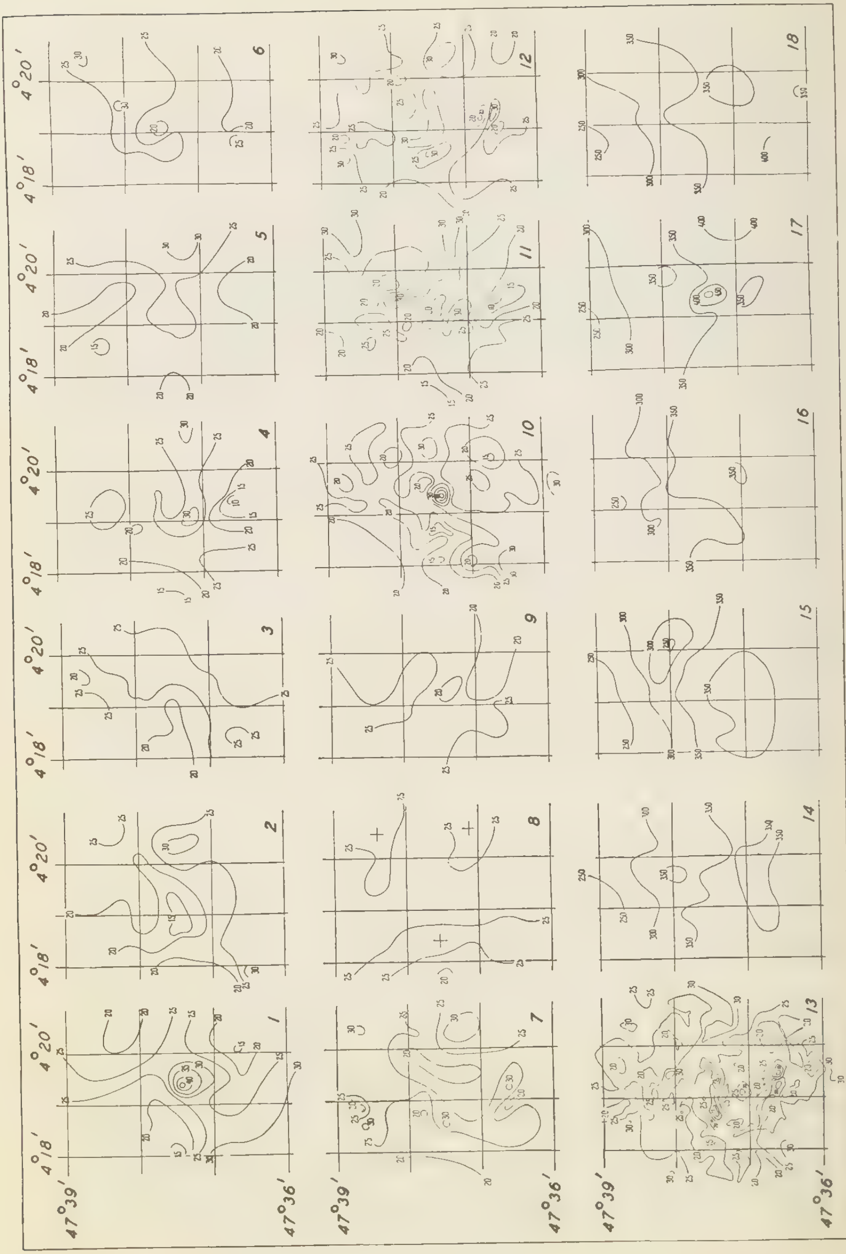
Figures 13, 26, and 39 show the isolines obtained when all the observations are used.

[1]B. P. Weinberg, The magnitude of geomagnetic gradients, Zurn. Russ. Fiz. Obsc., Leningrad, **56**, 677-686 (1924), in Russian language with English abstract; also The magnitude and frequency of local magnetic disturbances (in press).

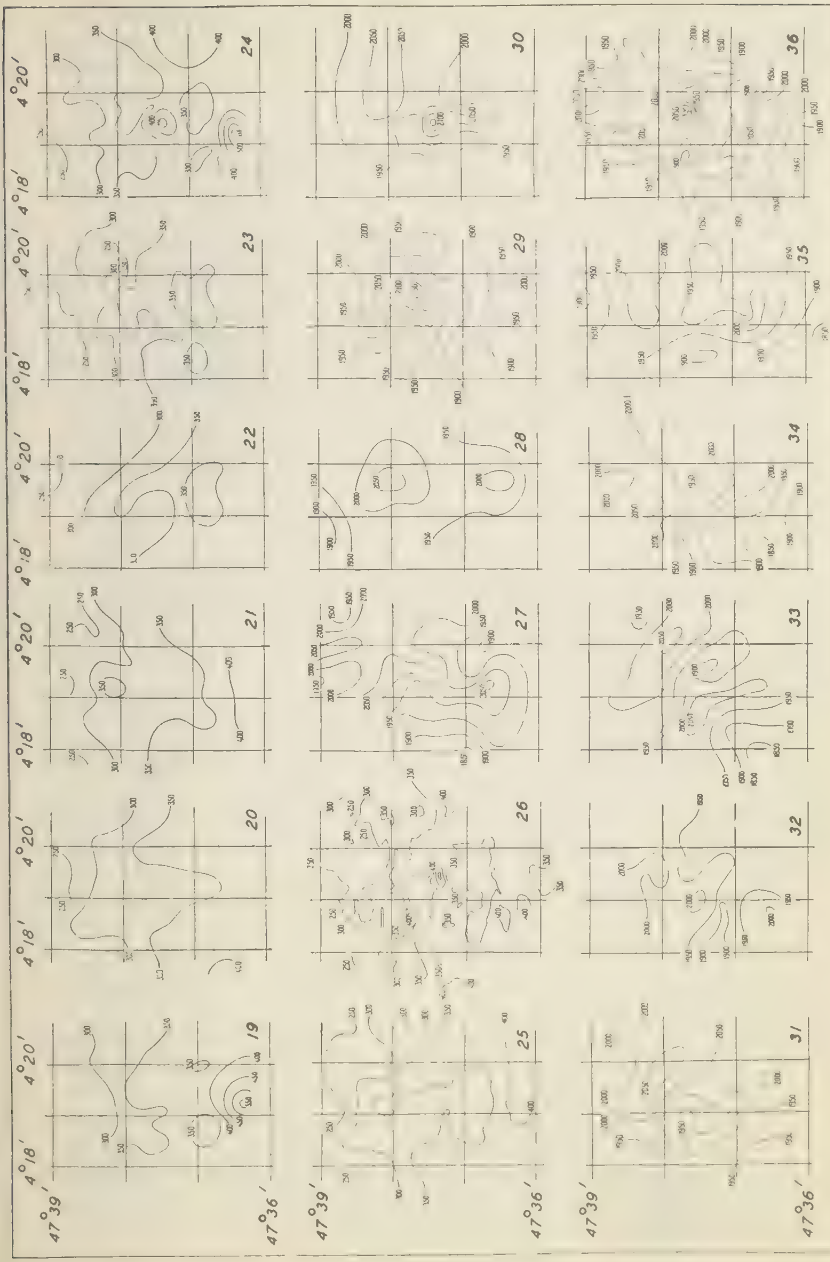
[2]Or terrestrial; cf. for example, G. Ljungdahl, A magnetic survey of Sweden made by the Hydrographic Service in the years 1928-1930, Jordmag. Pub., No. 9 (1934).

[3]Cf. B. P. Weinberg, On the methodology of finding and representing the distribution of a natural element over a certain region of the Earth's surface with special reference to terrestrial magnetism, Terr. Mag., **27**, 137-155 (1922).

[4]N. Trubiatchinski, Magnetometric investigation of manganese-ore deposits (Russian with an English abstract), Trans. United Geol. and Prospect. Serv., U. S. S. R., No. 166 (1932).



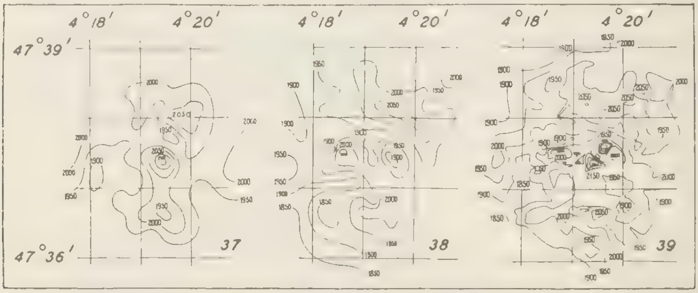
FIGS. 1–18


4°18′
4°20′
47°39′
47°36′
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36

FIGS. 19–36

This example shows clearly how far even from reality are the quasi-real isolines which are intended to show the distribution of a magnetic element as obtained by the determinations usually made on general surveys in which the average distance is of the order of 10 to 30 km.

The most common method of representing the results of a survey of a district is the geographic map on which are traced magnetic isolines corresponding to a definite epoch, but the authors often are dissatisfied with this general method. Some of the objections are listed below.



FIGS. 37-39

The main purpose of a magnetic chart is to make possible the prediction of the value of an element at every point in the region covered by the chart. Many authors, therefore, in order to give an idea of the precision with which prediction may be made, use various methods which are listed without entering into details as follows:

(1) A map giving no isolines but only the points of observation with the observed values of the element entered against them.

(2) A map showing, in addition to the smoothed or normal isolines, the points of observation and the *differences* between the observed value and the value which would be obtained by interpolation.

(3) A map with indication of the isanomals and of the very disturbed regions where the isanomals could be traced on a map of a much larger scale.

(4) A map showing, in addition to the main isolines resulting directly from the observations, the possible deviation of the mean positions of these isolines representing the uncertainty of these positions.

(5) A foot-note to the map indicating the degree of precision which can be attributed to the value of the element obtained by the method of interpolation from the isolines of the map.

(6) In addition to maps representing the normal distribution, the author gives a chart of anomalous components (usually the direction and the magnitude of the anomalous horizontal component at the points of observation and the isolines of the anomalous vertical intensity).

Since the magnetic elements vary with time, most authors indicate the values of the annual variation by one of the following methods:

(7) A foot-note or a remark indicating values of the annual variation.

(8) A numerical table on the map giving the values of the annual variation at equidistant points of the region covered by the map.

(9) The values of the annual variation in different areas of the map are noted on it without precise indication of the points to which these values refer.

(10) A map of the isolines containing also the isopors (lines of equal annual variation) of the element.

(11) An isoporic chart is given in addition to one of the isolines.

(12) An isoporic chart with isolines of the second differences ($d^2 E/dt^2$) where E is the value of the element.

It is rather difficult to state the reasons impelling different authors to choose any one of these methods because in the great majority of cases no explanation of the reason is given in publication. However, it appears that in choosing from among these methods the general tendency is (*a*) to give a distribution of a magnetic element to conform as closely as possible to the observed values and (*b*) to reduce, so far as possible the difficulties of finding the value of the element for any point on the map.

To fulfill these two conditions authors usually substitute for quasi-real isolines, with sinuosities corresponding to all observed values, rather smoothed terrestrial or normal isolines together with the deviations from the distribution which should strictly correspond to such isolines. The indication, by one of the methods (7) to (12), of the secular variations has a similar purpose.

To the two principles of magnetic cartography mentioned above may be added a third, namely, (*c*) to make possible the use of the previously constructed map after a certain interval from the epoch to which the map referred or *after new determinations have been made at points where no observations had been made prior to its construction.*

The latter is easily attained if we accept as a postulate that the characteristic features of a *real* distribution of magnetic elements over a certain region do not essentially vary secularly, that is, that the places and the magnitudes of the extreme deviations do not change appreciably with time and that only the isolines of the smoothed or normal distribution are shifting over the Earth's surface and either converging or diverging. Such a postulate has yet no substantially firm experimental basis, since no detailed microsurvey, which is the only means of finding the real distribution, has been repeated (as far as the author is aware) at precisely identical points after a period of several years. However, consideration of all the observed facts as well as the usual interpretation now accepted of magnetic "anomalies," may be regarded as a sufficient confirmation of such a postulate.

Thus methods (*a*) and (*b*) may be considered as fulfilling the requirement of the third principle when, besides the isolines of the element itself, the map contains the isopors (10). Indeed if new determinations —not too greatly exceeding in number the previous ones—shall have been made in the future, as a whole they can only slightly change (if reduced to the epoch of the previous chart) the system of isolines traced thereon and hence only slightly modify the values of the deviations at the individual stations indicated. These new determinations will therefore merely add a number of new points with new deviations without changing the general features of the earlier map.

The writer, in order to subject such a map to a stricter fulfillment of the second principle, proposes instead of the isolines and isopors that there be drawn *two systems of the isolines* relating (1) to the epoch to which

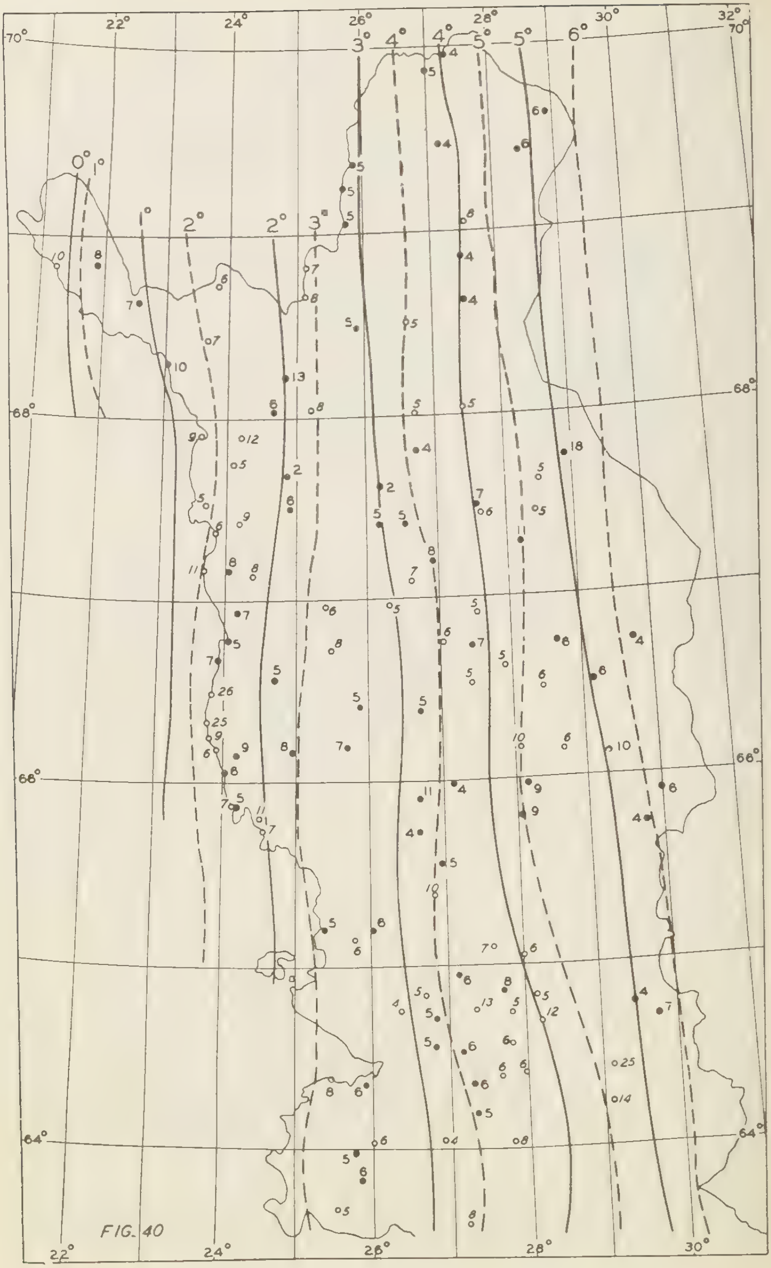


FIG. 40

all the observations have been reduced and (2) to some future epoch. This method permits finding more easily—by means of a "visual" interpolation—the value of the element at any point for an epoch intermediate between the epochs of the two systems of isolines and even for a more remote epoch not much beyond that of the second system of the isolines.

Figure 40 is a sample of this type of magnetic chart, it is the isogonic chart of Finland containing the smoothed isogonics for 1935 and 1940 and indicating all the points where the declination differs more than 0°.5 from the value following from the system of the smoothed isolines. (White circles indicate negative anomalies and black circles indicate positive anomalies; east declination is taken as positive.)

Such a limit has been chosen (*a*) because anomalies of smaller magnitudes have a very restricted practical value (for example for aviation-purposes) and (*b*) because this limit corresponds approximately to the average magnetic anomaly for even much more "quiet" land-regions of the same latitude as Finland which may be generally characterized as a region of continuous anomalies. Figure 40 is based on two maps of J. Keränen.[5]

The Soviet delegation of the Magnetic Commission of the International Meteorological Committee at the next (September 1935) meeting of the Commission will raise the question as to the desirability of organizing an international service of universal magnetic charts. If this proposal should be favorably received by the Commission and practical steps should be taken to get the opinions of magneticians everywhere and of all who use magnetic charts, on the principles of magnetic cartography. These opinions should be carefully weighed in determining the content and the form not only of the world magnetic chart but also of magnetic charts of separate parts of the Earth's surface.

[5]A magnetic survey of North Finland for the epoch 1915.5, Met. Zentralanst. Finn. Staates, Erdmag. Untersuchungen, No. 11 (1924).

CENTRAL GEOPHYSICAL OBSERVATORY,
Leningrad, *U. S. S. R.*

PROVISIONAL SUNSPOT-NUMBERS FOR JUNE TO AUGUST, 1935

(Dependent alone on observation at Zürich Observatory and its station at Arosa)

Day	June	July	August	Day	June	July	August
1	34	41	10	17	38	47[a]	*W*20[c]
2	39[b]	29	11	18	41[d]	46	29
3	34	29	. .	19	38	*E*51[c]	23
4	34	24	23[ad]	20	67	60	*M*52[ca]
5	31[d]	28[d]	18	21	50	36[b]	55
6	41	29	28	22	35[a]	32	39
7	25	22	43	23	29[d]	30	34
8	*E*46[c]	50[d]	49	24	38[a]	29	35[d]
9	*E*60[c]	44	26	25	44	23	. .
10	73[b]	38	33[a]	26	51	19	29
11	*E*74[c]	44[a]	29	27	56	11	13[a]
12	66	*E*59[c]	38	28	52	8	25
13	55	63	. .	29	45	0	28[d]
14	41	51[a]	. .[d]	30	42[b]	*E*8[c]	24
15	49[d]	49	29	31		8	37[d]
16	42	49	26				
				Means. .	45.7	34.1	29.9
				No. days	30	31	27

Mean for the quarter April to June, 1935, 28.6/(89 days)

[a]Passage of an average-sized group through the central meridian.
[b]Passage of a large group through the central meridian.
[c]New formation of a new center of activity: E, on the eastern part of the Sun's disc; *W*, on the western part; *M*, in the central-circle zone.
[d]Entrance of a large or average-sized center of activity on the east limb.

EIDGEN. STERNWARTE,
Zürich, Switzerland

W. BRUNNER

PROVISIONAL SOLAR AND MAGNETIC CHARACTER-FIGURES, MOUNT WILSON OBSERVATORY APRIL, MAY, AND JUNE, 1935

Greenwich mean time						Range hor. int.
Beginning			Ending			
1935	*h*	*m*	*d*	*h*	*m*	γ
May 1	12	46	2	04	. .	115
June 7	6	. .	11	03	. .	159

The storm of May 1 began with a sudden commencement in which the horizontal intensity increased 23 gammas. No observations with the spectroheliograph were made at Mount Wilson from April 27 to May 3. A group of spots, which in four days developed into a large group, was first seen on May 2 in latitude 32° south, 22° east of the central meridian.

An active group which reached its maximum area on June 10 was first seen on June 8 in latitude 32° south, 19° east of the central meridian. On June 9 at 18^h, G. M. T., very bright hydrogen-clouds were observed over this group.

Day	K_2 A	K_2 B	$H\alpha B$ A	$H\alpha B$ B	$H\alpha D$ A	$H\alpha D$ B	No. groups	Mag'c char.
1	2	2	2	2	1	1	0	0
2	2	0.5	2	0.5	1.5	0	0	0
3	...	...	...	...	...	...	...	0
4	1	0.5	...	...	...	...	0	0
5	1	0.5	1	0.5	1	0	0	0
6	1	0.5	1	0.5	1	0	0	0
7	...	...	...	...	...	...	...	0
8	...	...	...	...	...	...	...	0.5
9	1	1	1	0.5	0.5	0	2	1
10	1	1	1	0.5	0.5	0.5	2	0.5
11	1	1	...	...	...	...	2	1
12	1	0.5	...	...	...	...	3	0.5
13	...	...	...	...	...	...	3	1
14	...	...	...	...	...	...	2	0.5
15	...	...	...	...	...	...	...[d]	0
16	1	0.5	1	1	1.5	1.5	1	0
17	1	0.5	1.5	0.5	1	1	1	0
18	1	0.5	1.5	1	1.5	0	1	0.5
19	1	0.5	1.5	0.5	1	0	2	0
20	1	0	1	0	2	0	2	0
21	0.5	0	0.5	0	2	0.5	2	0
22	0.5	0	0.5	0	3	0.5	2	0
23	0.5	0	1	0	2.5	0.5	1	0
24	0.5	0.5	1	0.5	3	0.5	1	0
25	1	0.5	1	1	3	1	0	0
26	...	...	...	...	...	...	0	0
27	1	1	1	0.5	2.5	2	0	0
28	...	...	...	...	...	...	...	0
29	...	...	...	...	...	...	...	0
30	...	...	...	...	...	...	...	0
31								
Mean	1.0	0.6	1.1	0.6	1.7	0.5	1.2	0.2

Day	K_2 A	K_2 B	$H\alpha B$ A	$H\alpha B$ B	$H\alpha D$ A	$H\alpha D$ B	No. groups	Mag'c char.
1	2	0.5	...	...	...	...	3	1
2	...	...	...	...	...	...	4	0.5
3	1.5	1	2	1.5	1	1	4	0
4	1.5	2	2	2	1.5	1	3[d]	0
5	2	2	2	2	2	1	5[e]	0
6	2	2	2.5	2	2	2	5	0
7	2	1	2.5	1	2	1	5	0
8	2	1	2.5	1	2.5	2	5	0
9	2	1	2.5	1	1	0.5	4	0
10	2	1.5	2.5	2	1	0.5	3	0.5
11	...	...	...	...	...	...	...[b, c]	0.5
12	1	1.5	2?	1?	1?	0?	4	0.5
13	...	...	...	...	...	...	...	0.5
14	1.5	1	2	1	1	0	5	0
15	...	...	...	...	...	...	5	0.5
16	...	...	...	...	...	...	...	0.5
17	1	0.5	1	0	1	0	1	0
18	0.5	0	0.5	0	1.5	0	0	0
19	0.5	0	0.5	0	1	0	0	0.5
20	0	0	0	0	1	0	0	0.5
21	0	0	0	0	2	0.5	1	0
22	0	0.5	0	0	2	1	0	0.5
23	0	0.5	0	0	2	0.5	0	0
24	0.5	0.5	0.5	0.5	2	0	2	0
25	0.5	0.5	0.5	0.5	2	0.5	1	0
26	0.5	0.5	0.5	0.5	2	0	1	0
27	0.5	0	0.5	0	2	0	1	0
28	0.5	0	0.5	0	2	0	2	0
29	1	0	1	0	2	0	3	0
30	...	...	...	...	...	...	3	0
31	1	0.5	2	0.5	2	0	3	0
Mean	1.0	0.7	1.2	0.7	1.6	0.5	2.6	0.2

Day	K_2 A	K_2 B	$H\alpha B$ A	$H\alpha B$ B	$H\alpha D$ A	$H\alpha D$ B	No. groups	Mag'c char.
1	1	1	2	1	2	1	4	0
2	2	1	2	1	2	1	4[d]	0
3	2	1	2	1	1	1	2	...
4	2	2	2	2	1	1	4	0.5
5	2	1	2	1	1	1	3	0.5
6	2	0.5	2	0	1	1	3	0
7	2	0	2	0	1	0.5	2	1
8	2	0.5	2.5	0	1	0	5	1
9	2.5[g]	0	3[g]	0	1	0	5	1
10	2	0.5	3	0.5	1	0	6[e]	0.5
11	2.5	2	3	2	1	0	6	0.5
12	2	2	2.5	2	1	0	7	0
13	2	1	2.5	1	0	0	5	0.5
14	2	0	2.5	0	0	0	5	0
15	2	1	2.5	0.5	0	0	5	0
16	2	0.5	2.5	0.5	0	0	4	0
17	2	0	2.5	0	1	0	5	0.5
18	2	1	2.5	1	1	0	4	1
19	2	1.5	2.5	2	1	0	3[f]	0.5
20	2	2.5	3	3	1	0	5	0.5
21	2.5	2.5	3	3	1	0	5	0
22	2.5	2.5	3	3	1	0	3	0
23	2.5	2	3	2	1	0	3[e]	0
24	2	1.5	3	2	1	0	5	0
25	2.5	1	3	1	1	0	4	0
26	2	0	3	0	1	0	4	0
27	2	0	3	0	1	0	4	0
28	2	1	2	0.5	1	0.5	3	0
29	2	2	2	2	1.5	0.5	3[a]	0
30	2	1	2	1.5	2	1	2	0
31								
Mean	2.1	1.1	2.5	1.1	1.0	0.3	4.1	0.3

NOTE—For an explanation of these tables see this JOURNAL, 35, 47-49 (1930).
?Indicates an uncertain value which should be given low weight.
[a]Passage of a large group through the central meridian within 30° of the center of the disc.
[b, c, d, e]Passage of an average-sized group through the central meridian within 20°, 25°, 30°, 35° of the center of the disc, respectively.
[f]Passage of a small group through the central meridian within 15° of the center of the disc.
[g]Very bright $H\alpha$ and K_2 in south central group.

Carnegie Institution of Washington, Mount Wilson Observatory, Pasadena, California

SETH B. NICHOLSON
ELIZABETH E. STERNBERG

AMERICAN *URSI* BROADCASTS OF COSMIC DATA [1] APRIL TO JUNE, 1935

The data for terrestrial magnetism, sunspots, solar constant, and aurorae are the same as given in previous tables.

The first three columns of the Table give (1) the magnetic character according to the scale 0-2 of the International Commission of Terrestrial Magnetism and Atmospheric Electricity, (2) the type featuring the day other than normal by the letters *b*, *p*, *o*, and *i* for days marked by bay, rapid pulsations, long-period oscillations, and irregular oscillations, respectively, and (3) the hour and minute of Greenwich mean time marking the beginning of a storm, the end of the storm being indicated in the footnote to the Table. The next two columns give the data relating to sunspots: (1) the number of groups of spots and (2) the total number of

[1] For previous announcements see Terr. Mag., **35**, 184-185 and 252-253 (1930); **36**, 54, 141, 258-259, and 358-360 (1931); **37**, 85-89, 189-192, 408-411, and 484-487 (1932); **38**, 60-63, 148-151, 262-265, 335-339 (1933); **39**, 73-77, 159-163, 244-247, 353-356 (1934), **40**, 111-115, 220-222 (1935).

Summary American URSI daily broadcasts of cosmic data, April to June, 1935

Date	April							May							June							Date
	Magnetism			Sun-spot		Solar constant		Magnetism			Sun-spot		Solar constant		Magnetism			Sun-spot		Solar constant		
	Character	Type	G. M. T. begin. distur.	Groups	Number	Value	Character	Character	Type	G. M. T. begin. distur.	Groups	Number	Value	Character	Character	Type	G. M. T. begin. distur.	Groups	Number	Value	Character	
			h m			*cal.*				*h m*			*cal.*				*h m*	*	*	*cal.*		
1	0	...	.. ..	0	0	1.951	*f*	0	...	.. ..	3	7	......	...	0	...	.. ..	4	20	......	...	1
2	0	...	.. ..	0	0	1.955	*s*	1	...	.. ..	4	11	......	...	0	...	.. ..	4	19	1.938	*s*	2
3	0	...	.. ..	0	0	1.949	*f*	0	...	.. ..	4	15	......	...	0	...	.. ..	2*	20*	......	...	3
4	0	...	.. ..	0	0	1.938	*s*	0	...	.. ..	3	6	......	...	0	...	.. ..	4*	18*	......	...	4
5	0	...	.. ..	0	0	1.937	*f*	0	...	.. ..	5	10	......	...	1	...	.. ..	3	18	......	...	5
6	0	...	.. ..	0	0	1.945	*u*	0	...	.. ..	5	20	......	...	1	...	.. ..	3	15	......	...	6
7	0	...	.. ..	...	...	......	...	0	...	.. ..	5*	23*	......	...	0	...	15 30	2	6	......	...	7
8	0	...	.. ..	...	...	1.935	*f*	0	...	.. ..	5*	25*	1.938	*f*	2	*i*	.. ..	5	17	......	...	8
9	1	...	.. ..	2	11	1.942	*f*	0	...	.. ..	4	13	1.937	*s*	1	*i*	5 30	5	0	......	...	9
10	0	...	.. ..	2	9	1.945	*f*	0	...	.. ..	3	6	1.939	*u*	1	...	.. ..	6*	36*	......	...	10
11	2	*i*	.. ..	2	3	......	...	1	...	.. ..	...	...	......	...	1	*i*	.. ..	6	40	1.939	*f*	11
12	1	...	.. ..	3	10	......	...	1	...	.. ..	4*	8	1.941	*f*	1	...	.. ..	7	22	......	...	12
13	1	...	.. ..	3	11	......	...	0	...	.. ..	...	...	......	...	0	...	.. ..	5	15	......	...	13
14	2	...	.. ..	2*	13*	......	...	0	...	.. ..	5	8	1.935	*u*	0	...	.. ..	5*	14	......	...	14
15	0	...	.. ..	...	...	......	...	0	...	.. ..	5	6	1.939	*f*	0	...	.. ..	5	14	......	...	15
16	0	...	.. ..	1	3	1.941	*f*	1	...	.. ..	...	...	1.941	*s*	0	...	.. ..	4	13	......	...	16
17	0	...	.. ..	1	4	1.948	*f*	0	...	.. ..	1	1	1.944	*f*	0	...	.. ..	5*	10*	......	...	17
18	0	...	.. ..	1	5	......	...	0	...	.. ..	0*	0*	......	...	1	...	.. ..	4*	13*	......	...	18
19	0	...	.. ..	2*	3*	......	...	0	...	.. ..	0	0	......	...	1	...	.. ..	3	12	......	...	19
20	0	...	.. ..	2	2	......	...	1	...	.. ..	0	0	......	...	1	...	.. ..	5	19	......	...	20
21	0	...	.. ..	2*	3*	1.953	*u*	0	...	.. ..	1	1	......	...	0	...	.. ..	5	22		..	21
22	0	...	.. ..	2	3	1.949	*f*	0	...	.. ..	0	0	......	...	0	...	.. ..	3	13		..	22
23	0	...	.. ..	1	3	......	...	0	...	.. ..	0	0	......	...	0	...	.. ..	3	15	. .	..	23
24	0	...	.. ..	1	1	1.947	*u*	0	...	.. ..	2	3	......	...	0	...	.. ..	5*	16	. .	..	24
25	0	...	.. ..	0	0	......	...	0	...	.. ..	1	1	......	...	0	...	.. ..	4*	22	. .	.	25
26	0	...	.. ..	0	0	......	...	0	...	.. ..	1	1	......	...	0	...	.. ..	4*	10	......	...	26
27	0	...	.. ..	0	0	1.941	*s*	0	...	.. ..	1	1	......	...	0	...	.. ..	4*	19*	1.946	*s*	27
28	0	...	.. ..	...	...	1.938	*f*	0	...	.. ..	2	2	1.948	*f*	0	...	.. ..	3*	14*	1.943	*s*	28
29	0	...	.. ..	...	...	......	...	0	...	.. ..	3*	5*	1.939	*s*	0	...	.. ..	3*	13	1.938	*s*	29
30	0	...	.. ..	...	...	......	...	0	...	.. ..	3	5	......	...	1	...	.. ..	2*	12*	1.931	*s*	30
31								0	...	.. ..	3	16	......	...								31
Mean	0 2	...	.. ..	1.1	3.5	1.945	...	0.2	...	.. ..	2.6	6.9	1.940	...	0.4	...	.. ..	4.1	16.6	1.939	...	Mean

*A revision of value originally broadcast.
Greenwich mean time for ending of storms: 11h 30m, June 8; 3h 23m, June 11.

Kennelly-Heaviside Layer heights, Washington, D. C., April to June, 1935
(Nearest hour, Greenwich mean time, of all observations is 17)

Date	Frequency	Height	Date	Frequency	Height	Date	Frequency	Height
1935	*kc/sec*	*km*	*1935*	*kc/sec*	*km*	*1935*	*kc/sec*	*km*
Apr. 3	2,800	120	May 1	7,000	400	Jun. 5	4,000	230
" "	3,490	190	" "	7,100	370, 410	" "	4,400	380
" "	3,550	180	" "	7,500	390, 570	" "	4,700	990
" "	3,900	270	" "	7,700	430	" "	4,900	470
" "	4,000	260	" "	8,000	470	" "	5,100	410
" "	4,700	330	" "	8,200	530	" "	5,500	420
" "	5,300	290	" "	8,300	640	" "	5,700	470
" "	6,100	350	May 8	3,300	120	" "	5,800	470, 530
" "	6,200	340, 380	" "	3,400	*	" "	5,900	450
" "	6,500	340, 390	" "	3,550	200	" "	6,300	460
" "	7,100	370	" "	3,800	190	" "	6,400	490
Apr. 10	2,650	140	" "	3,900	280	" "	6,500	500
" "	2,950	160	" "	4,000	240	" "	6,800	600
" "	3,200	*	" "	4,600	630	Jun. 12	2,800	110
" "	3,300	340	" "	5,000	240, 460	" "	3,800	110
" "	3,500	270	" "	5,100	260, 530	" "	4,000	130
" "	4,120	760	" "	5,400	450	" "	4,100	260
" "	4,500	250, 420	" "	5,700	520	" "	4,400	320
" "	4,800	260, 350	May 15	3,500	120	" "	4,700	500
" "	5,000	390	" "	3,520	*	" "	5,100	350
" "	5,100	360	" "	3,550	250	" "	5,700	360
" "	5,700	370	" "	3,700	200	" "	5,900	390
" "	5,900	420	" "	4,400	360	" "	6,100	390, 430
" "	6,000	400	" "	4,700	420	" "	6,300	370
" "	6,300	410	" "	5,000	230, 380	" "	6,500	360
" "	6,700	480	" "	5,200	360, 380	" "	6,900	400
Apr. 17	2,900	120	" "	5,400	390, 520	" "	7,100	470
" "	3,400	*	" "	5,700	440	Jun. 19	2,500	110
" "	3,480	220	" "	6,000	450	" "	3,300	120
" "	3,600	200	" "	6,400	490	" "	3,500	*
" "	4,400	350	May 22	3,350	110	" "	3,630	210
" "	4,700	340	" "	3,370	220	" "	3,750	190
" "	5,100	310	" "	3,520	190	" "	3,920	260
" "	5,300	320	" "	4,320	750	" "	4,020	230
" "	5,700	350	" "	4,500	490	" "	4,400	540
" "	5,900	340	" "	4,700	620	" "	4,500	510
" "	6,300	380	" "	4,900	*	" "	4,600	410
" "	6,600	500	" "	5,500	730	" "	4,700	700
Apr. 24	3,000	120	" "	5,700	*	" "	4,800	800
" "	3,300	*	May 29	3,200	110	" "	4,900	*
" "	3,320	170	" "	3,370	170	" "	5,200	330
" "	3,500	130	" "	3,600	*	" "	5,300	*
" "	3,600	240	" "	3,630	210	" "	5,400	340
" "	4,500	450	" "	3,680	180	" "	5,500	*
" "	4,900	270, 360	" "	3,800	300	Jun. 26	3,000	110
" "	5,100	300, 330	" "	4,000	230	" "	3,630	110
" "	5,300	320, 350	" "	4,400	590	" "	3,700	*
" "	6,100	360, 470	" "	4,500	440	" "	3,820	220
" "	6,500	370	" "	4,600	*	" "	4,400	280
" "	6,800	470	" "	4,700	640	" "	4,500	320
May 1	3,200	120	" "	4,900	470	" "	4,600	310
" "	3,400	*	" "	5,110	500	" "	4,700	400
" "	3,500	220	" "	5,300	690	" "	4,900	*
" "	3,570	190	" "	5,400	*	" "	5,100	330
" "	3,670	250	" "	5,500	560	" "	5,300	320
" "	3,800	210	" "	5,700	*	" "	5,700	340
" "	4,400	380	" "	6,110	570	" "	6,100	340
" "	4,700	410	Jun. 5	3,500	110	" "	6,300	360, 500
" "	4,900	240, 380	" "	3,700	120	" "	6,500	510
" "	6,100	360	" "	3,750	*	" "	6,700	370
" "	6,800	380	" "	3,800	210	" "	6,900	380
						" "	7,100	450

* = No value obtained.

spots. It is to be noted that sunspot-numbers such as those from Zürich can be obtained from the number of groups and spots given in the Table by the formula $N = k\ (10g+s)$, where k for Mount Wilson is about 0.7.

Mount Wilson Observatory is now supplying corrections and additions to the sunspot-data which are broadcast in the *URSI*gram. So far as possible, these additional and corrected values will be used in this tabular summary and will be designated as such in footnotes to the Table.

Beginning January 1, 1934, the magnetic information of the *URSI*-gram is for Cheltenham, Maryland, instead of Tucson, Arizona. In addition to this change in observatory, the data cover the 24 hours ending 8 A. M., 75° west meridian mean time, instead of the 24 hours ending at 7 A. M., 105° west meridian mean time.

The columns headed solar constant show (1) the value in calories of the solar constant, and (2) by letters s, f, and u whether the determination was satisfactory, fair, or unsatisfactory, respectively.

In accordance with information received from Dr. C. G. Abbot, Secretary of the Smithsonian Institution, transfer from Table Mountain to Montezuma solar-constant values was made as of October 23, 1934. Table Mountain for a considerable time has been 0.012 calorie above Montezuma, and above the scale of 1913 to 1930. Hence the value of October 23 and succeeding values are on a scale 0.012 calorie lower than previous ones.

The table of Kennelly-Heaviside Layer heights is self-explanatory.

Department of Terrestrial Magnetism,
Carnegie Institution of Washington,
Washington, D. C.

C. C. Ennis

ELECTRICITY OF RAIN AND THUNDERSTORMS

In his article on "Electricity of rain and thunderstorms," which appeared in the March number of the Journal, Dr. Ross Gunn, referring to the production of electricity when a drop of water breaks up in the air, says: "The effect of a superposed charge in modifying the charge generated by break-up should be investigated in the laboratory."

It is unfortunate that Dr. Gunn did not read the paper, to which he gives a reference, in which I described the breaking-drop theory of thunderstorms dismissed by him so summarily; for if he had he would have read: "Experiments were next undertaken to investigate the extent to which the process is affected by any charge already on the drops, for it is quite conceivable that charged drops might behave quite differently from uncharged drops." The experiments are then described in which over four thousand charged drops were broken up, with the result that I was able to state quite definitely that the positive charge produced when a drop of water is broken in the air is the same within the accuracy of the experiments whether the drop is charged with positive electricity, with negative electricity, or is completely uncharged.

I do not wish at this stage to criticize Dr. Gunn's interesting theory; but I would like to take this opportunity of pointing out that electrification as the result of evaporation and condensation has been put forward over and over again to account for different phenomena of atmospheric electricity. It has recently, however, gone out of favor because the most intense search in the laboratory by a number of experienced physicists

could discover no trace of any such effect; nor is there any theoretical reason why there should be any electrification. If therefore Dr. Gunn has discovered such an effect he will have made a discovery so fundamentally important that the explanation of a thunderstorm will fade into insignificance. I am sure all physicists will look forward with the greatest interest to seeing a full account of the experiments on which Dr. Gunn bases his theory, for the description given in his paper is wholly inadequate.

G. C. SIMPSON

METEOROLOGICAL OFFICE,
London, May 23, 1935

COMMENTS ON DR. SIMPSON'S REMARKS

The Editor of the JOURNAL has kindly offered the opportunity of replying to a criticism of my paper "Electricity of rain and thunderstorms"[1] made by Dr. G. C. Simpson.

I am glad to acknowledge the oversight of the material relating to charged breaking raindrops mentioned by Dr. Simpson but must draw attention to the fact that his quotation did not appear in the paper I cited[2] but in quite a different one appearing nearly 20 years before.[3] His statements relating to this paper are therefore misleading.

In his criticism, Dr. Simpson leaves the impression that the results of my paper depend on a single experiment. This is by no means the case as an attentive reading of the paper will show. Therein it was pointed out that several conceivable mechanisms will account for equilibrium-potentials between a water-droplet and the surrounding ionized air of something like $(3/2)(kT/e)$ or about 60 millivolts.[4] The concentration-cell postulate gave such a potential and was introduced because: (*a*) It correlated in a quantitative and remarkably satisfactory manner a large mass of apparently unrelated data, (*b*) it was specific in nature and satisfactorily supported by experiments both by myself and by others, although these have not been published, (*c*) the existence of such a mechanism *is* consistent with the well-known properties of ions and the experimental foundations of related phenomena.[4, 1]

It was indicated in my original paper that there were difficulties of interpretation and control of experimental details but the broad outlines of the problem are thought to be quite distinct and can be used to forecast observations with considerable success. For example, Dr. Simpson refers to certain experiments performed by him on over 4,000 raindrops. His complete results may very easily be calculated quantitatively from my theory and satisfactory agreement has been obtained. Reference 3 gives two sets of independent data relating to the break-up of water-drops of 0.24-gram mass or 0.385-cm radius. In one set Dr. Simpson atomizes the drop and finds that the mean charge generated by each parent drop is 5.2×10^{-3} e.s.u./drop, or 2.16×10^{-2} e.s.u./gram. Unfortunately he does not specify the degree of atomization attained and since the size of the *droplets* is important in my theory it is necessary to calculate the number of droplets formed from each drop. Reference to

[1]R. Gunn Terr. Mag., **40**, 79-106 (1935).

[2]Proc. R. Soc., **114**, 376-401 (1927).

[3]Phil. Trans., A **209**, 379 (1909).

[4]See, for example, the diverse observed effects enumerated by Gilbert and Shaw, Phys. Soc., London, **37**, 195 (1925), and Verwey, Chem. Rev., **16**, 363 (1935)

line 6 in column 8 of my Table 2 shows that the charge generated per gram by "medium rain" is nearly the same as that measured above and the mass of these droplets (column 2) is 5.2×10^{-4} gram. Therefore, to a fair approximation, each parent drop produced on the average $0.24/5.2 \times 10^{-4} = 462$ droplets.

Alternately, the number of droplets may be calculated with better accuracy from my equation (22). The initial charge on the drop Q_0 is given by my equation (17). In this equation, taking $a = 0.385$ and $T = 300°$ it is found that $Q_0 = 8.0 \times 10^{-5}$ e.s.u./drop. When the drop breaks up and the surrounding ions are swept away by the air-jet, a charge (according to Simpson's measurements) of 5.2×10^{-3} e.s.u. drop is developed, whence by equation (22)

$$N = [(5.2 \times 10^{-3})/(8.0 \times 10^{-5})]^{3/2} = 525 \text{ droplets}$$

Thus in his described experiments the initial drop was broken, on the average, into about 500 equal-sized droplets.

Having deduced the size of the droplets by our theory we turn to Dr. Simpson's second set of data in which he studies the effect of a superposed charge. He finds that the average charge put on each drop by his method of inducing a charge is 1.9×10^{-2} e.s.u./drop or 7.9×10^{-2} e.s.u./gram and that by breaking up this drop by an air-jet a charge of 2.39×10^{-2} e.s.u./drop or 9.96×10^{-2} e.s.u./gram is measured. Thus the charge generated by break-up was 0.49×10^{-2} e.s.u. drop or 2.04×10^{-2} e.s.u./gram.

Now, as we have seen from the first set of experiments, each large drop in Dr. Simpson's work breaks up on the average into about 525 "medium-rain" drops and if the superposed induced charge on the original drop is evenly divided between each, the charge on each will be $1.9 \times 10^{-?}/525$ or 3.62×10^{-5} e.s.u./droplet. Now, from column 7 of my Table 2, it is seen that the equilibrium-charge on the droplets due to the concentration-cell phenomena is 1.1×10^{-5} e.s.u./droplet and this is superposed on the induced charge. Therefore, the ratio of the charges measured on the evaporating drops in Simpson's experiment, when broken up, and when not broken up, should (if our theory is correct in its essentials) be

$$[(3.62 + 1.1)\ 10^{-5}/3.62 \times 10^{-5}] = 1.305$$

This is in good agreement with Dr. Simpson's measurements which gave

$$[2.39 \times 10^{-2}/1.9 \times 10^{-2}] = 1.26$$

It is thus clear that my theory predicts with satisfactory accuracy Dr. Simpson's experimental results and instead of his data discrediting my analysis of the electricity of rain they support it in every measurable detail. It is unfortunate that Dr. Simpson did not carry out more experiments using a variety of induced charges on the drops.

Additional experiments are under way which will clarify the phenomena discussed in my original paper and in this note.

Ross Gunn

Naval Research Laboratory,
Washington, D. C., August 19, 1935

PRINCIPAL MAGNETIC STORMS

Sitka Magnetic Observatory

April to June, 1935[1]

(Latitude 57° 03′.0 N., longitude 135° 20′.1 or 9^h $01^m.3$ W. of Gr.)

April 10—This storm began at about 10^h 20^m, G. M. T., with moderate, long-period fluctuations in all the elements, which continued for about ten hours, followed by about five hours of relative calmness. Then came a period of more rapid fluctuations, increasing irregularly until they reached their maximum intensity about 11^h, April 11. By 13^h these had ceased, and they were followed by several large oscillations with a period of about two hours. The storm proper ended at 17^h, but the record continued moderately disturbed for two days longer. Ranges: Declination, 107′, horizontal intensity, 670 gammas, vertical intensity, 741 gammas.

John Hershberger, *Observer-in-Charge*

Cheltenham Magnetic Observatory

April to June, 1935[1]

(Latitude 38° 44′.0 N., longitude 75° 50′.5 or 5^h $07^m.4$ W. of Gr.)

April 9—Beginning at 2^h 09^m, G. M. T., H decreased about 75 gammas within one hour, after which there was a decrease of about 30 gammas in Z. D slowly decreased as H decreased.

April 10-12—This period was marked by continuous long-period fluctuations of fairly large amplitudes in D, H, and Z, beginning about 10^h 20^m on April 10

May 1—A mild storm began suddenly at 12^h 47^m with a sudden increase of about 18 gammas in H. It continued for about 15 hours.

May 20—At 3^h 20^m there was quite a sharp increase of about 60 gammas in H followed by a few long-period fluctuations in H. At the same time D decreased about 20′ and returned to normal in about one hour, and Z fluctuated about 40 gammas in two long-period waves.

June 7-11—Beginning at 12^h 12^m on June 7, there were many fluctuations of H and D having periods of about 20 minutes and varying in amplitudes up to 18′ for D and about 90 gammas for H. The fluctuations in Z were moderate but of longer periods.

June 17-20—There was a moderate disturbance of all elements.

June 29—An earthquake near Calima, Mexico, was recorded at 7^h 05^m on the H-trace.

H. E. McComb, *Observer-in-Charge*

Watheroo Magnetic Observatory

April to June, 1935

(Latitude 30° 19′.1 S., longitude 115° 52′.6 or 7^h $43^m.5$ E. of Gr.)

May 1-2—This moderate disturbance began May 1 at 12^h 47^m, G. M. T., with a sudden commencement. Horizontal intensity increased 15 gammas in five minutes, vertical intensity decreased five gammas, and declination 0′.5 in the same interval. The disturbance ended May 2 at approximately 2^h.

W. C. Parkinson, *Observer-in-Charge*

[1]Communicated by R. S. Patton, Director, United States Coast and Geodetic Survey.

Huancayo Magnetic Observatory
April to June, 1935

(Latitude 12° 02′.7 S., longitude 75° 20′.4 or $5^h\ 01^m.4$ W. of Gr.)

Greenwich mean time						Range		
Beginning			Ending			Decl'n	Hor. int.	Ver. int.
1935	*h*	*m*	*d*	*h*	*m*	′	γ	γ
Apr. 10	12	37	10	21	..	5	265	36
May 1	12	46	2	12	..	5	296	32
May 10	11	..	11	10	..	4	220	21
May 19	3	20	19	05	..	1	61	6
June 7	12	12	11	22	..	9	293	39
June 18	1	..	21	01	..	7	200	42

April 10—The disturbance which began at $12^h\ 37^m$ April 10 was of a minor nature with a short life. It affected the *H*-trace principally, interposing large bays with an approximate period of 30 minutes. The most rapid change was in the 10-minute period beginning at $16^h\ 32^m$. During this interval the horizontal intensity increased 112 gammas.

May 1-2—This minor disturbance began with a sudden commencement in all three elements, most marked in *H*, which increased 11 gammas in two minutes, decreased five gammas in the next minute and increased 41 gammas in the following three minutes. *H* oscillated moderately during the period from 13^h to 17^h, the maximum occurring at $16^h\ 01^m$. From 17^h to 22^h the trace was quiescent after which the horizontal intensity decreased 145 gammas in the last two hours of the day. The minimum value of *H* occurred at $23^h\ 50^m$ May 1, G. M. T., and thereafter, until 2^h May 2, *H* maintained a very low value, showing small, short-period fluctuations, after which there was a slow, smooth recovery to normal value by 12^h. *D* and *Z* were lower than normal simultaneously with the low values of *H* between 0^h and 3^h May 2.

May 10-11—This disturbance (character 1) is chiefly notable for the large, continuous decrease in *H* between the maximum at $16^h\ 19^m$ on May 10, and the minimum at $1^h\ 03^m$ on May 11, the range being 220 gammas. The commencement was inconspicuous at about 11^h May 10, minor fluctuations occurring in all three elements for five hours thereafter, superposed on the normal diurnal trend. *H*, after the minimum, slowly recovered to normal value by about 10^h on May 11. *D* and *Z* were considerably disturbed throughout the whole period. The two following days, May 12 and 13, were disturbed by numerous moderate peaks and bays of relatively long period in *H*, and small but numerous fluctuations in *D* and *Z*, though the diurnal trend was fairly well maintained.

May 19—A brief disturbance (character 1) unimportant except for a sudden commencement at $3^h\ 20^m$, with an instantaneous increase in *H* of six gammas, followed by a slow increase of 46 gammas in 17 minutes. *D* and *Z* showed a barely perceptible decrease and increase, respectively, at the commencement. The end of the disturbance, about two hours after the commencement, was followed by a generally disturbed day in all three elements, though diurnal trends were generally maintained.

June 7-11—A disturbance of four and one-half days duration, with a conspicuous sudden commencement in all three elements at $12^h\ 12^m$

June 7. *H* increased 56 gammas in four minutes, *D* first decreased 0′.2 in one minute, then increased 2′.9 in four minutes, and *Z* increased 5.4 gammas in six minutes. Following the commencement, prominent rapid fluctuations were superposed on the diurnal trend until 16^h, after which *D* and *Z* were less affected, while *H*, commencing at 16^h 19^m, began a very large steady decrease which continued until 1^h 53^m June 8, with a range of 256 gammas. The minimum in *H* at 1^h 53^m on June 8, is the low value for the whole disturbance; the maximum occurred June 11 with a sudden peak at 16^h 56^m. Recovery in *H* after the minimum, was approximately attained by 10^h June 8, but for three days thereafter there was considerable disturbance. The usual diurnal maxima June 8 and 9 were largely obliterated by large peaks and bays of 20- to 60-minute period and with ranges between 50 and 100 gammas, while the maximum June 10 was entirely absent, with the trace very ragged. A final series of large peaks and bays, superposed on the diurnal trend, between 13^h and 20^h June 11, marked the termination of the storm.

June 18-21—A mild disturbance of three days' duration, with inconspicuous commencement at about 1^h June 18, which is chiefly noteworthy for the deep bay in *H* between 16^h June 18 and 6^h June 19. The range from crest to trough was 165 gammas. On the bay were superposed minor peaks and bays of 25 to 50 gammas range, with periods of 10 to 30 minutes. Before and after the large bay, *H* showed minor bays and peaks of 50 to 100 gammas range, with periods of 30 minutes to two hours. The maximum on each of the three days was lower than normal. *D* and *Z* were but little disturbed throughout.

O. W. Torreson, *Observer-in-Charge*

Apia Observatory
April to June, 1935

(*Latitude 13° 48′.4 S., longitude 171° 46′.5 or* 11^h $27^m.1$ *W. of Gr.*)

April 9—There was a fairly low minimum value of horizontal intensity at 8^h 15^m G. M. T.

May 1—A slight disturbance beginning with sudden increases of 11 gammas in horizontal force and three gammas in vertical force occurred at 12^h 15^m G. M. T., and culminated in a minimum value of horizontal force at 3^h 50^m G. M. T. the next day. The range of the disturbance was 114 gammas in horizontal field and 14 gammas in vertical field.

May 20—There was a low minimum value of horizontal force at 7^h 50^m G. M. T., with a corresponding smaller and weaker minimum in vertical force.

June 7—A slight disturbance began at 12^h 10^m with increases of 11 gammas in horizontal force and of two gammas in vertical force. The total range of the disturbance, which developed slowly at first, was 154 gammas in horizontal force and 10 gammas in vertical force.

June 9—A slight disturbance commenced at 5^h 30^m G. M. T. The initial increases were 14 gammas in horizontal force and three gammas in vertical force; the total ranges were 69 gammas and 8 gammas in the horizontal and vertical fields, respectively.

J. Wadsworth, *Director*

NOTES

(See also page 280)

26. *River-ranges for controlling compass-deviations*—The need for more intensive data for magnetic declination in rivers and harbors has been demonstrated recently. A vessel navigating the Delaware River found excessive compass-deviations by making observations on one of the river-ranges near Wilmington, Delaware. This is an area of considerable local attraction, declination on land varying from 4° to 13° west. The distribution over the water-area is now being determined by observations along both shores of the River and the United States Coast and Geodetic Survey hopes later to supplement these by observations on water along the channel-courses.

27. *Magnetic work of the Academia Sinica.*—The National Research Institute of Physics of the Academia Sinica, Shanghai, China, has completed its plans for erecting a magnetic observatory on Purple Mountain in Nanking and actual construction work on the absolute and variation buildings has been begun. As soon as these structures are completed and the instruments mounted and tested, it is planned to send out a field-party for magnetic work during the coming winter.

28. *Magnetic activity in New Zealand*—The magnetic instruments previously used at the Christchurch Magnetic Observatory have now all been transferred to the new observatory at Amberley. Some magnetic work has been done in connection with geophysical prospecting. Most interesting results occur in andesitic country where the first stages of propylitization show up very well in the magnetometer and indicate the areas where electrical observations are worth while for the determination of lodes. A bulletin devoted to the subject is to be issued soon.

29. *Precision cosmic-ray meter at Huancayo*—A precision meter for recording photographically the variations in the cosmic radiation, in accordance with the plan of the Cosmic-Ray Committee of the Carnegie Institution of Washington, has been sent to the Huancayo Magnetic Observatory in Peru where it will be installed in a suitable structure designed for that purpose.

30. *Personalia*—Among the names of men of science and those associated with science which appear in the list of honors conferred by the King on the occasion of His Majesty's Birthday and in commemoration of the completion of the twenty-fifth year of His Majesty's reign, are the following who have contributed to the advancement of geophysics: K. C. B.—Dr. *G. C. Simpson*, Director of the Meteorological Office, London; C. B. E.—Prof. *A. Fowler*, Emeritus Professor of Astrophysics, Imperial College, South Kensington, and Dr. *E. Marsden*, Secretary, Department of Scientific and Industrial Research, Dominion of New Zealand; O. B. E.—*A Walter*, Director of the Meteorological Service, British East Africa.

Dr. *W. S. Adams*, Director of the Mount Wilson Observatory of the Carnegie Insitution of Washington, has been elected a corresponding member of French Academy of Sciences, in the Section of Astronomy.

W. J. Peters, research associate of the Carnegie Institution of Washington, associated since 1905 with the Department of Terrestrial Magnetism and chiefly responsible for the successful inauguration and development of its magnetic work at sea and for many years in command of the research vessels *Galilee* and *Carnegie*, has been invited by the British Admiralty to act as consultant in the design of the new non-magnetic vessel *Research*. Mr. Peters will also assist in the design of special instruments for magnetic observations on the *Research*; his wide experience in the design and use of the instruments developed in the magnetic surveys of the *Galilee* and the *Carnegie* especially qualify him in this field. Mr. Peters left for England September 19.

Dr. *J. Bartels*, Professor of Physics at the Forstliche Hochschule at Eberswalde and Lecturer in Geophysics at the University of Berlin, and Dr. *S. Chapman*, Chief Professor of Mathematics at the Imperial College of Science and Technology of London, both research associates of the Carnegie Institution of Washington, arrived in Washington July 16 and 27, respectively. They will spend the summer in America, chiefly at the Department of Terrestrial Magnetism, engaged on research problems in geo-

physics. During August they were in the western United States and took part in two Institution conferences, namely, (*a*) at the Division of Plant Biology at Stanford University regarding correlative studies involving cycles and the best methods of considering such matters, and (*b*) at the Mount Wilson Observatory regarding correlations in the fields of solar physics and geophysics with particular reference to terrestrial magnetism and electricity.

Professor *Emilio Oddone* retired August 1, 1935, from the directorship of the R. Officio Centrale di Meteorologia e Geofisica at Rome, a post which he has held since November 1, 1931, when he succeeded Prof. Luigi Palazzo. By ministerial decree the functions of director were entrused to Prof. *Pericle Gamba*, Chief of the Section of Climatology in the same office effective August 1, 1935.

J. Wadsworth, Director of the Apia Observatory, Western Samoa, has been granted a six-month leave of absence during which he will visit Europe and attend the meetings of the International Meteorological Organization in Warsaw in September. He visited the Department of Terrestrial Magnetism in Washington during August.

On June 1, 1935, *J. Wallace Joyce* relieved *A. K. Ludy* of charge of the Tucson Magnetic and Seismological Observatory. The latter took charge July 1 of the Cheltenham Magnetic Observatory, relieving *H. E. McComb* who had been in temporary charge in addition to his administrative duties at the Washington Office of the Coast and Geodetic Survey.

Dr. *R. W. Brock*, since 1914 Deputy Minister of Mines of Canada, dean of the Faculty of Applied Science, University of British Columbia, and Mrs. Brock were killed July 30, when their plane nose-dived into trees. Dr. Brock was for seventeen years connected with the Canadian Geological Survey, for eleven years as geologist, and from 1908 to 1914 as director. He was sixty-one years old.

We learn with regret of the death, at Rome, on May 21, 1935, of Prof. *G. Magrini*, editor of the Bibliographia Oceanographica.

LIST OF RECENT PUBLICATIONS

By H. D. Harradon

A—Terrestrial and Cosmical Magnetism

Afrique Occidentale Française. Carte magnétique. Sahara Mauritanien et Soudanais. Déclinaison en degrés au 1er janvier 1935. Echelle 1:2500000. Dakar, Service Géographique de l'A. O. F., 1935 (70 + 50 cm).

Agamennone, G. Le perturbazioni magnetiche in relazione con i terremoti. Estratto, Roma, Boll. Soc. sismol. ital., v. 32, fasc. 3-4, 1934, 26 pp.

Antipolo Observatory. Results of the observations made at the Magnetic Observatory of Antipolo near Manila, P. I., during the calendar year 1931. (Part IV of the annual report of the Weather Bureau for the year 1931.) Manila, Bureau of Printing, 1934, 47 pp. 29 cm.

Besairie, H., P. Gruyelle, A. Lenoble et A. Savornin. Etude géologique et magnétique de la falaise de la Mandraka (Madagascar). Paris, C.-R. Acad. sci., T. 200, No. 19, 1935 (1614-1616).

Burgaud, M. Sur quelques observations magnétiques récentes faites dans le sud et le sud-ouest de la Chine. Paris, C.-R. Acad. sci., T. 200, No. 18, 1935 (1543-1545). Observations magnétiques dans le sud et le sud-ouest de la Chine, et carte des isogones et isodynames. Paris, C.-R. Acad. sci., T. 200, No. 21, 1935 (1776-1778).

Chevrier, J. Le magnétisme terrestre dans les États du Levant sous mandat français. Ann. Phys. Globe, France d'Outre-Mer, Paris, 2e année, No. 8, 1935 (39-40).

De Bilt, Meteorological and Magnetic Observatory. Annuaire. Quatre-vingt-cinquième année 1933. B. Magnétisme terrestre. (K. Nederlandsch Met. Inst. No. 98.) 's-Gravenhage, 1934 (vii + 24). 34 cm.

Debrach, J. Le magnétisme terrestre au Maroc. Ann. Phys. Globe, France d'Outre-Mer, Paris, 2e année, No. 8, 1935 (36-38 avec cartes).

Gruyelle, H., et C. Poisson. Sur les anomalies magnétiques du massif cristallin de Madagascar. Paris, C.-R. Acad. sci., T. 201, No. 1, 1935 (11-14).

L'anomalie magnétique du Tsiafajavona. Paris, C.-R. Acad. sci., T. 201, No. 2, 1935 (111-114).

Hofmann, J. E. Das magnetische Erdfeld. Natur u. Kultur, München, Jahrg. 31, Heft 9, 1934 (368-373).

Hubert, H. Une carte du XVIIIe siècle donnant la déclinaison magnétique en Afrique. Ann. Phys. Globe, France d'Outre-Mer, Paris, 2e année, No. 8, 1935 (53-57).

Jenny, W. P. Some practical examples of magnetic prospecting. Oil and Gas J., Tulsa, v. 33, No. 49, 1935 (33-34, 48). Abstract, Geophys. Abstr., Washington, D. C., No. 75, 1935 (1489-1490).

Kato, Y. Seismic and volcanic activities and changes in the Earth's magnetic field. (The third paper.) Jap. J. Astr. Geophys., Tokyo, v. 12, No. 2, 1935 (237-243).

Kuwahara, S. On the change in magnetic declination at Ogasawara Gunto. Hydrogr. Bull., Tokyo, 14th year, No. 5, 1935 (167-170). [Text in Japanese language.]

Lasserre, A. Déclinaison magnétique en Algérie et dans les territoires du sud de l'Algérie. Ann. Phys. Globe, France d'Outre-Mer, Paris, 2e année, No. 9, 1935 (65-72).

London, Meteorological Office. The observatories' year book 1933, comprising the meteorological and geophysical results obtained from autographic records and eye observations at the observatories at Lerwick, Aberdeen, Eskdalemuir, Cahirciveen (Valentia Observatory), and Richmond (Kew Observatory), and the results of soundings of the upper atmosphere by means of registering balloons. London, H. M. Stationery Office, 1935 (449). 31 cm.

Lovö. Ergebnisse der Beobachtungen des magnetischen Observatoriums zu Lovö (Stockholm) im Jahre 1931. Stockholm, Kung. Sjökarteverket, 1935 (95). 31 cm.

Lubiger, F. Ueber die vom Samoa-Observatorium registrierten erdmagnetischen Pulsationen. Zs. Geophysik, Braunschweig, Jahrg. 11, Heft 3, 1935 (116-126).

Maurer, J. Die Schwankung der Kompassnadel in der Umgebung unserer Alpen nach langjährigen Aufzeichnungen. Zürich, Vierteljahrsch. Natf. Ges., Jahrg. 79, Nr. 1/3, 1934 (24-28).

Mauritius, Royal Alfred Observatory. Results of magnetical and meteorological observations for the months of October 1933 to May 1934 (new series, v. 19, pts. 10-12; v. 20, pts. 1-5). Port Louis, R. W. Brooks, Govt. Printer, 1934 (163-208); 1935 (iv + 60). 34 cm.

Annual report of the Royal Alfred Observatory for the year 1934 (N. R. McCurdy, Director). Mauritius, 1935, 8 pp. 25 cm. [Contains brief report of the magnetic work for 1934.]

Nakamura, S. T., and Y. Kato. Researches on the terrestrial magnetism and the Earth's potential specially referred to the occurrence of earthquake and volcanic activity. Ann. Rep. Saito Grat. Found., 10, 1934 (50-71). Abstract, Jap. J. Astr. Geophys., Tokyo, v. 12, No. 3, 1935 (21).

Anomalous secular-variation of magnetic dip in the epicentral area of the destructive earthquake of November 26, 1930. Tokyo, Proc. Imp. Acad., v. 10, 1934 (256-258). Abstract, Jap. J. Astr. Geophys., Tokyo, v. 12, No. 3, 1935 (21).

Poisson, Ch. Le magnétisme terrestre à Madagascar. Ann. Phys. Globe, France d'Outre-Mer, Paris, 2e année, No. 8, 1935 (40-42).

Prague, Institut Géophysique National Tchécoslovaque. Bulletin magnétique. No. 4. Année 1932-1933. Praha, 1935 (iii + 51). 24 cm. [Contains the values of magnetic declination at the Observatory of Stará Dala (O'Gyalla) during the years 1932-1933.]

Bulletin magnétique. No. 5. Année 1934. Praha, 1935 (iii + 27). 24 cm. [Contains the values of the magnetic declination at the Observatory of Stará Dala (O'Gyalla) during the year 1934.]

Rio de Janeiro, Observatorio Nacional. Anuário publicado pelo Observatorio Nacional do Rio de Janeiro para o ano de 1935. Ano LI. Rio de Janeiro, Imprensa Nacional, 1934 (xiii + 460 com mappa). 18 cm. [Contains tables of magnetic declination at various points in Brazil reduced to epoch 1935.0 and an isogonic map of Brazil for January 1935.]

Romer. Le magnétisme terrestre à la Martinique. Ann. Phys. Globe, France d'Outre-Mer, Paris, 2e année, No. 8, 1935 (42-44).

Salles-Homery, Mme G. Le magnétisme terrestre dans la France d'Outre-Mer. Ann. Phys. Globe, France d'Outre-Mer, Paris, 2e année, No. 8, 1935 (34-36).

Wiechert, E. Untersuchungen an Baystörungen unter besonderer Berücksichtigung der Störmerschen Theorie. Königsberg/Pr., Mitt. Geophys. Warte Gr. Raum der Albertus-Universität, Nr. 22, 1934 (26). 30 cm.

B—Terrestrial and Cosmical Electricity

Ackemann, M. Untersuchung von Sekundäreffekten der kosmischen Ultrastrahlung bei grossen Bleidecken. Zs. Geophysik, Braunschweig, Jahrg. 11, Heft 3, 1935 (154-156).

Appleton, E. V., and S. Chapman. Report on ionization-changes during a solar eclipse. New York, N. Y., Proc. Inst. Radio Eng., v. 23, No. 6, 1935 (658-669).

Appleton, E. V., and R. Naismith. Some further measurements of upper atmospheric ionization. London, Proc. R. Soc., A, v. 150, No. 871, 1935 (685-708).

Auger, P. Nature of cosmic rays. Nature, London, v. 135, May 18, 1935 (820-821).
Une hypothèse sur la nature et les propriétés des rayons corpusculaires cosmiques. J. Physique et Le Radium, Paris, T. 6, No. 5, 1935 (226-228).

Auger, P., et F. Bertein. Analyse du rayonnement corpusculaire cosmique sous un écran de 8 m de sol. J. Physique et Le Radium, Paris, T. 6, No. 6, 1935 (253-254).

Auger, P., et P. Ehrenfest, Jr. Clichés de rayons cosmiques obtenus avec une chambre de Wilson-Blackett dans des conditions spéciales. J. Physique et Le Radium, Paris, T. 6, No. 6, 1935 (255-256 avec 6 figs.). [Une chambre de Wilson-Blackett transportable a été construite. Des clichés de rayons cosmiques ont été obtenus en haute altitude, au niveau du sol sous des écrans épais de plomb, et dans un laboratoire souterrain.]

Auger, P., et A. Rosenberg. Absorption et effets secondaires des rayons cosmiques. J. Physique et Le Radium, Paris, T. 6, No. 5, 1935 (229-232).

Banerji, S. K. Registration of earth-current with neutral electrodes. Delhi, Mem. India Met. Dept., v. 26, pt. 1, 1933 (11 with 4 pls.). 30 cm.

Barnóthy, J., und M. Forró. Bemerkung zur Arbeit von W. Kolhörster "Höhenstrahlung und Nova Herculis." Zs. Physik, Berlin, Bd. 94, Heft 11/12, 1935 (773-774).

Böggild, J. Schnellregistrierung von kleineren Hoffmannschen Stössen. Naturw., Berlin, Jahrg. 23, Heft 23, 1935 (372).

Clay, J. Comparison of the decrease of intensity of the primary cosmic radiation in different materials. Physica, The Hague, v. 2, No. 7, 1935 (645-649).

Absolute intensity of cosmic rays. Physica, The Hague, v. 2, No. 7, 1935 (650-651).

Clay, J., and H. R. Woltjer. Diurnal variation of cosmic rays. Physica, The Hague, v. 2, No. 6, 1935 (582-584).

Compton, A. H. An attempt to analyse cosmic rays. London, Proc. Phys. Soc., v. 47, No. 261, 1935 (747-773).

Compton, A. H., and I. A. Getting. An apparent effect of galactic rotation on the intensity of cosmic rays. Phys. Rev., Lancaster, Pa., v. 47, No. 11, 1935 (817-821).

Das, A. K. Nova Herculis and cosmic rays. Nature, London, v. 136, July 6, 1935 (29-30).

Dauvillier, A. Étude du champ électrique terrestre, de l'ionisation atmosphérique et du courant vertical au Scoresby Sund pendant l'Année Polaire. Paris, C.-R. Acad. sci., T. 200, No. 26, 1935 (2209-2211).

Doan, R. L. Statistical fluctuations of cosmic-ray ionization in new recording meter. Phys. Rev., Lancaster, Pa., v. 48, No. 2, 1935 (167).

Harang, L. Filteraufnahmen von rot gefärbten Nordlichtern. Beitr. Geophysik, Leipzig, Bd. 44, Heft 3, 1935 (229-235).

Johnson, T. H. The nature of the cosmic radiation. Philadelphia, Pa., J. Frank. Inst., v. 220, No. 1, 1935 (41-67).

Lenz, E. Die elektrische Ablenkung von Ultrastrahlungsteilchen. Ann. Physik, Leipzig, Bd. 23, Heft 3, 1935 (207-239).

McCrea, W. H. Cosmic rays and novae. Nature, London, v. 135, May 18, 1935 (821).

Messerschmidt, W. Ionisationsmessungen zum Zusammenhang zwischen Ultrastrahlung und Nova Herculis. Zs. Physik, Berlin, Bd. 95, Heft 1/2, 1935 (42-45).

Millikan, R. A., and H. V. Neher. Equatorial longitude-effects on cosmic rays. Washington, D. C., Proc. Acad. Sci., v. 21, No. 6, 1935 (313-315).

Norinder, H. Lightning-currents and their variations. Philadelphia, Pa., J. Frank. Inst., v. 220, No. 1, 1935 (69-92).

Noto, H., and K. Takeda. On the charge carried by raindrops and snowflakes. Jap. J. Astr. Geophys., Tokyo, v. 12, No. 2, 1935 (219-235).

Scholz, J. Polarlichtuntersuchungen auf Franz-Josephs-Land. Beitr. Geophysik, Leipzig, Bd. 44, Heft 2, 1935 (145-156).

Die Ionenbilanz in der Arktis. Beitr. Geophysik, Leipzig, Bd. 44, Heft 3, 1935 (306-320).

Steinmaurer, R. Messungen des luftelektrischen Potentialgefälles in 2300 m Höhe am Hafelekar bei Innsbruck. Beitr. Geophysik, Leipzig, Bd. 44, Heft 3, 1935 (337-345).

STÖRMER, C. Measuring of aurorae with very long base-lines. Geofys. Pub., Oslo, v. 11, No. 3, 1935 (15 with 2 pls.).

STREET, J. F., R. H. WOODWARD, AND E. C. STEVENSON. Absorption of cosmic-ray electrons. Phys. Rev., Lancaster, Pa., v. 47, No. 12, 1935 (891-895).

VEGARD, L., AND E. TÖNSBERG. Continued investigation on the temperature of the upper atmosphere determined by means of bands appearing in the auroral spectrum. Geofys. Pub., Oslo, v. 11, No. 2, 1935 (14 with 5 figs.).

WHITEHEAD, S., AND W. G. RADLEY. Experiments relating to the distribution of alternating electric currents in the Earth and the measurement of the resistivity of the Earth. London, Proc. Phys. Soc., v. 47, No. 261, 1935 (589-614).

WINTER, H. Die Gewitterhäufigkeit in Beziehung zum Dampfdruck und zur Temperatur. Met. Zs., Braunschweig, Bd. 52, Heft 4, 1935 (140-143).

C—Miscellaneous

ALIVERTI, G., UND G. ROSA. Zur Frage der Adsorption von RaEm an Kernen. Beitr. Geophysik, Leipzig, Bd. 44, Heft 2, 1935 (107-111).

APPLETON, E. V. Temperature-changes in the higher atmosphere. Nature, London, v. 136, July 13, 1935 (52-53).

BAILEY, V. A. AND D. F. MARTYN. The influence of electric waves on the ionosphere. Melbourne, Council Sci. Indust. Res., Bull. No. 88 (Radio Res. Board, Rep. No. 7), 1935 (40-52).

BOOKER, H. G. The application of the magneto-ionic theory to the ionosphere. London, Proc. R. Soc., A, v. 150, No. 870, 1935 (267-286).

CABANNES, J., ET J. DUFAY. Les bandes de Vegard-Kaplan dans le spectre du ciel nocturne. Paris, C.-R. Acad. sci., T. 200, No. 18, 1935 (1504-1506).

CHAPMAN, S. The Götz inversion of intensity-ratio in zenith-scattered sunlight. London, Phil. Trans. R. Soc., A, v. 234, No. 737, 1935 (205-230).

DOBSON, G. M. B. The upper atmosphere. Sci. Prog., London, v. 30, No. 117, 1935 (1-15).

FARMER, F. T., AND J. A. RATCLIFFE. A new test of the magneto-ionic theory. Nature, London, v. 135, May 18, 1935 (831-832).

HARTREE, D. R. The differential analyser. Nature, London, v. 135, June 8, 1935 (940-943).

KIRBY, S. S., AND E. B. JUDSON. Recent studies of the ionosphere. New York, N. Y., Proc. Inst. Radio Eng., v. 23, No. 7, 1935 (733-751).

KREBS, A., UND N. WOLODKEWITSCH. Zur Frage des Zurückbleibens der Ionen in den Lungenalveolen. Bioklim. Beibl., Braunschweig, Jahrg. 2, Heft 2, 1935 (90-95).

LAINÉ, P. Recherches sur l'ozone et ses propriétés magnétiques. Ann. Phys., Paris, Sér. 11, T. 3, mai-juin 1935 (461-554).

MARTYN, D. F. The propagation of medium radio waves in the ionosphere. Melbourne, Council Sci. Indust. Res., Bull. No. 88 (Radio Res. Board, Rep. No. 7), 1935 (9-24 with 4 figs.).

MARTYN, D. F., AND A. L. GREEN. The characteristics of downcoming radio waves. Melbourne, Council Sci. Indust. Res., Bull. No. 88 (Radio Res. Board, Rep. No. 7), 1935 (25-39).

MITRA, S. K., AND P. SYAM. Absorbing layer of the ionosphere at low height. Nature, London, v. 135, June 8, 1935 (953-954).

MOUNT WILSON OBSERVATORY. Summary of Mount Wilson magnetic observations from September 1934 to February 1935. Pub. Astr. Soc. Pacific, San Francisco, Cal., v. 46, 1934 (358-359); v. 47, 1935 (54-57, 102-103).

MÜLLER, E. A. W. Berechnung der oberen Grenze der in der Ionosphäre möglichen Temperatur aus den Messungen der Ionisierungsschichtdicken. Zs. Geophysik, Braunschweig, Jahrg. 11, Heft 3, 1935 (126-127).

Norton, K. A. Propagation of radio waves over a plane earth. Nature, London, v. 135, June 8, 1935 (954-955).

Paul, M. Der Rotationsmechanismus der Sonne und die Rotationsdauer des Sonnenkerns. Beitr. Geophysik, Leipzig, Bd. 44, Heft 4, 1935 (376-398).

Pollak, L. W., und F. Kaiser. Neue Anwendungen des Lochkartenverfahrens in der Geophysik. Hollerith Nachr., Berlin, Heft 44, 1934 (574-583).

Ueber die numerische Methode von J. Fuhrich zur Ermittlung von Periodizitäten, ihre Erprobung und Anwendung auf die Polbewegung. Prag, Sonderdruck aus Statistický Obzor, Jahrg. 16, Heft 1-3, 1935 (13-54).

Schafer, J. P., and W. M. Goodall. Diurnal and seasonal variations in the ionosphere during the years 1933 and 1934. New York, N. Y., Proc. Inst. Radio Eng., v. 23, No. 6, 1935 (670-681).

Schlundt, H. The radioactivity of the spring-water on the Hot Springs Reservation, Hot Springs, Arkansas. Amer. J. Sci., New Haven, Conn., v. 30, No. 175, 1935 (45-50).

Smith, F. W. Zodiacal light notes. Pop. Astr., Northfield, Minn., v. 43, 1935 (38, 131, 187-188, 253-254, 320-321, 393-394).

Storm van Leeuwen, W., J. Booij, H. Isräel, und J. van Niekerk. Studien über die physiologische Wirkung des Föhns. Beitr. Geophysik, Leipzig, Bd. 44, Heft 4, 1935 (4C0-439).

Tönsberg, E., und F. W. P. Götz. Ozon in der Polarnacht. Naturw., Berlin, Jahrg. 23, Heft 22, 1935 (354).

Zeleny, J. The rôle of surface instability in electrical discharges from drops of alcohol and water in air at atmospheric pressure. Philadelphia, Pa., J. Frank. Inst., v. 219, No. 6, 1935 (659-675).

Terrestrial Magnetism
and
Atmospheric Electricity

VOLUME 40 DECEMBER, 1935 No. 4

THE ELECTRIC CURRENT-SYSTEMS OF MAGNETIC STORMS

BY S. CHAPMAN

1. This paper continues my discussion of the average characteristics of the field of geomagnetic disturbances or storms, given in two previous papers[1] which will be referred to as *1* and *2*; the irregular part of magnetic disturbance is not considered. A condensed representation of the main average features of magnetic storms is given in *1* (pp. 61-72) and *2* (pp. 242-264), and evidence is described which supports the view that the type of the field remains fairly constant throughout a considerable range of intensity for the field as a whole.

In *2* it is shown[2] that certain electric current-systems in the Earth's atmosphere[3] could produce a disturbance-field of the observed type (this will be called the D- or disturbance-field; the letter D is also used in terrestrial magnetism for the declination but I think no confusion need arise from its use in this further sense). These current-systems are indeed somewhat too simple, because so far as their high-latitude portions are concerned they apply less to the actual Earth than to an ideal Earth for which the magnetic and geographic axes coincide; but I believe that they constitute a useful first approximation to the more complicated (atmospheric) current-system appropriate to the real Earth.

2. It is, of course, in principle impossible to infer uniquely, purely from observations of a magnetic field (of external origin) at the Earth's surface, the location of the current-system which is the source of the field. So long as no further considerations are taken into account, the problem has not only one but an infinity of solutions. This may be illustrated as follows: Imagine any current-system outside the Earth, whether linear, or distributed over a surface or throughout a volume of any form. Observation of its field at the Earth's surface enables the potential of the field to be determined and expressed as a series of spherical harmonic terms. Along any radius from the Earth's center O the variation of the term of degree n ($n=1, 2, \ldots$) is proportional to r^n. This expression for the potential is valid up to the boundary of any sphere centered at O

[1] *1*—An outline of a theory of magnetic storms. Proc. R. Soc., A, **95**, 61-83 (1918); *2*—On certain average characteristics of world-wide magnetic disturbance, Proc. R. Soc., A, **115**, 242-267 (1927).

[2] The current-system diagram given in *1*, p. 76, took no account of the disturbance-fields in high latitudes, and is superseded by the diagrams of *2*, p. 263.

[3] These currents induce secondary electric-currents within the Earth which themselves contribute to the observed disturbance-field; cf. S. Chapman and T. T. Whitehead, Trans. Camb. Phil. Soc., **22**, p. 463 (1922), and S. Chapman and A. T. Price, Phil. Trans. R. Soc., A, **229**, pp. 427-460 (1930). The present paper deals mainly with the external part of the disturbance-field and the external current-systems, and references to the field and currents are to be read in this sense.

which does not intersect the external current-system. The same surface-field can be produced by an infinite variety of external current-systems differing from the actual one; in particular, it can be produced by current flowing in any sphere concentric with th Earth. The current-distribution in such a spherical sheet can be very simply deduced from the spherical harmonic series for the potential. The radius R may have any value whatsoever (of cours $R>a$, a being the Earth's radius). The greater the radius, the more prominent will be the spherical harmonic terms of higher degree in the current-distribution, since their relative magnitudes there will be proportional to $(R/a)^n$.

The current-distribution over a spherical sheet can easily be represented by a diagram using any projection of the sphere upon a plane. This is one method of representing the potential of the field graphically; it is of value in this way even if the field is produced by a non-spherical current-distribution. When used for this purpose, the current-distribution must be drawn for a definite radius, which may conveniently be a itself, or some radius R differing so slightly from a that the relative proportions of the main harmonic terms are not seriously different from those corresponding to a; in other words, $(R/a)^n$ must be small for the values of n which are important. Within the range of R satisfying this condition, the current-system may be considered independent of R; of course this range is fairly small, but if the field is so simple that only the first three or four harmonics in it are important, the range may be considered to include values of R differing from a by not more than 300 or 400 km, which is the thickness of the atmosphere up to and including the main known ionized layers; this is because the ground value of the ratio of the n^{th} to the first harmonic is increased at 400 km by the factor $(6770/6370)^{n-1}$, and up to $n=4$ this does not exceed 1.2.

Considerations of continuity indicate that if a field is produced by a current-sheet that departs from the spherical concentric form, but only by a moderate amount, the character of the current-distribution in the sheet can be inferred approximately from that of the spherical current-distribution of the same mean radius, which would give the same surface-field.

3. In attempting to decide between the infinite number of possible current-systems that could produce a given field, additional "non-magnetic" considerations of various kinds may be employed.

One of these concerns the space-distribution of electric conductivity, or, what is equivalent in the case of a rare medium such as alone comes into question for the space surrounding the Earth, the distribution of ionized gas. Any information as to this may indicate a certain region as a likely location for the current-system of a particular field, or may show that it is an improbable or impossible situation for that current-system.

For example, the spherical current-distributions corresponding to the fields (S and L) of the solar (S) and lunar (L) daily magnetic variations are much more intense over the sunlit than over the dark hemisphere, and even over the latter their intensity decreases perceptibly from sunset to dawn; (see Fig. 1 for the S current-system, as drawn by Bartels from my spherical harmonic analysis of S). Now there are ionized regions in the atmosphere which share these characteristics, and it is natural to suppose, in the absence of evidence to the contrary, that the

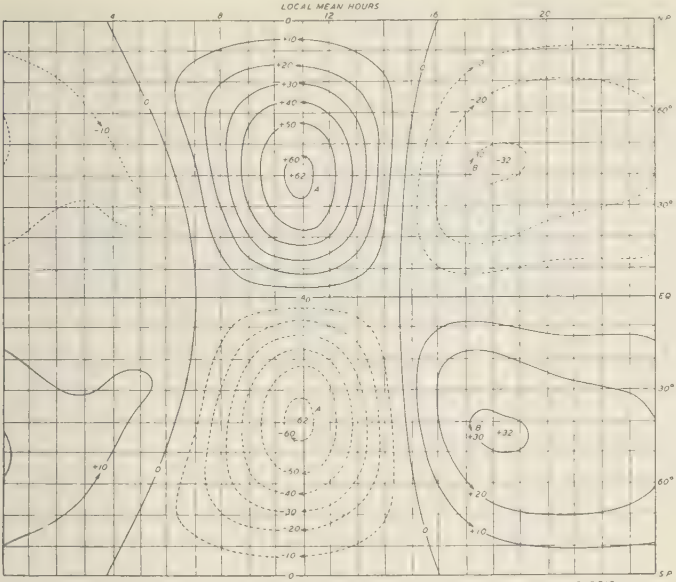


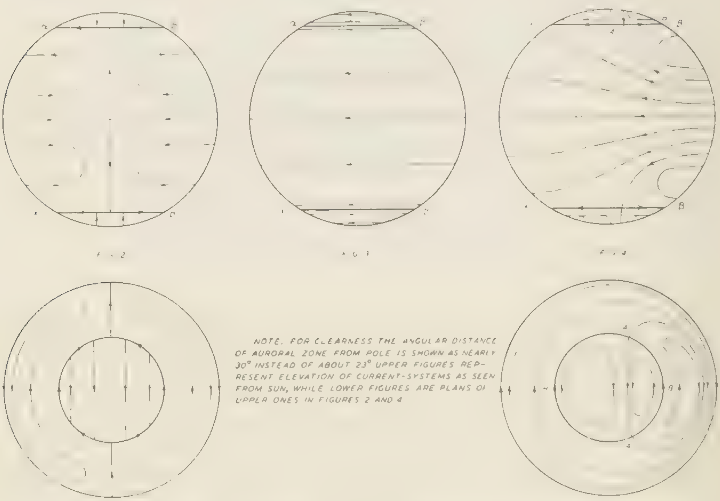
FIG. 1—CURRENT-SYSTEM, S_q, CORRESPONDING TO EXTERNAL PART OF SOLAR-DIURNAL MAGNETIC VARIATIONS ON QUIET DAYS, EQUINOX, SUNSPOT-MINIMUM (10000 AMPERES FLOW BETWEEN SUCCESSIVE CURRENT LINES)

S and L current-systems flow in one or other of these layers. The conclusion is reasonable (though provisional) despite the dependence of the current-system upon the distribution of electromotive forces as well as upon the conductivity. The conclusion will gain strength if it is found that the S and L fields vary in intensity throughout the year and from year to year, and also irregularly, in unison with one or other of the ionized layers; in this way it may become possible to associate S with a particular ionized layer, and likewise L with another. There is, in any case, little or no reason to doubt that the S and L currents flow in our atmosphere in a layer which is very nearly spherical and concentric with the Earth.

4. The (spherical) current-system (D) corresponding to the average field of magnetic disturbance is shown in Figures 2, 3, and 4 reproduced here from *2*, p 263. The complete system, shown in Figure 4, may be analyzed into a part (Fig. 3) symmetrical about the Earth's axis, and a non-symmetrical part (S_D) in which there is no resultant flow along the circles of latitude (Fig. 2); Figures 2 and 4 show the systems as viewed from the Sun, and also (below) as viewed from above the North Pole.

One of the outstanding features that distinguishes the D from the S and L current-systems is that the D-system is not more intense by day than by night; in the ("interzonal") belt between the two auroral zones the plane that divides the more intense from the less intense parts of the system passes through the Sun, the currents being stronger over the p. m. (post-meridiem) than over the a. m. hemisphere. In the auroral zones the order of intensity is reversed, but the dividing plane still passes

nearly through the Sun. Until recently no ionized layer of the atmosphere was known in which the ionization increased from a minimum at dawn to a maximum at sunset, hence it seemed doubtful whether the interzonal part of the D current-system flowed in the atmosphere. Now, however, it appears that there is a layer (F_2) whose properties somewhat resemble this; it is the highest of the known ionized layers, its height at the equator being about 250 km. Moreover the general theory of the daily variation of atmospheric ionization indicates that the higher the layer, the later in the daytime should be its hour of maximum ion-content. It may be, therefore, that the F_2-layer, or possibly a layer somewhat higher, still unknown to us because screened by the F_2-layer, is the seat of this part of the D current-system. At least it no longer seems *necessary* to exclude the atmosphere as a possible situation for these currents, and consequently to locate them in the free space beyond.



FIGS 2,3,4 – CURRENT SYSTEMS, D, CORRESPONDING TO EXTERNAL PART OF AVERAGE FIELD OF MAGNETIC DISTURBANCE

(FIG 2 – NON-SYMMETRICAL PART S_D; FIG. 3 – SYMMETRICAL PART ABOUT EARTH'S AXIS; FIG 4 – COMPLETE SYSTEM)

5. On the other hand, the latter possibility requires consideration, particularly since it has often been suggested that a current-ring encircling the Earth, far outside the atmosphere, is responsible for the decrease in the horizontal magnetic force during and after magnetic storms. The reasons prompting this suggestion seem to have been of two kinds. One is the high degree of regularity, both in geographical distribution, and in azimuth at each station, shown by the average disturbing-force[4], and also the slowness of its decline over a period of many days. It seems to have been thought that the force was too regular and perhaps too enduring to be produced by a current-system in our atmosphere. But there

[4] J. A. Broun, Trans. R. Soc. Edin., **22**, Part 3 (1861); W. van Bemmelen, Met. Zeits., **12** (1895); G. Angenheister, Gött. Nachr., Math. Phys. Kl. (1924); Ad. Schmidt, Zs. Geoph., **1**, pp. 3-13 (1925). The force referred to occurs during what I have called the main phase and the phase of recovery of a magnetic storm; Angenheister has used the terms "impetus" and "after-disturbance" for these two phases; van Bemmelen calls the "after-disturbance" the "post-perturbation."

seems no obvious reason why a ring-current outside should be more regular than a current-system in our atmosphere nor, indeed, why the decay of the latter should be too short, in view of our present knowledge that the decline of ionization in any atmospheric layer will be slower, the higher the layer; certainly the slow decrease of the symmetrical part of the *D*-field would require the currents, if atmospheric, to be located high up.

The second reason for suggesting the existence of a ring-current round the Earth was given by Störmer,[5] who showed that a certain difficulty in his auroral theory might be overcome by postulating such a current; he assigned to it a very large radius (placing it beyond the Moon's orbit), and suggested that the ring might consist of electrons sweeping round the Earth, as their path from the Sun becomes subject to the deflecting influence of the Earth's magnetic field. The hypothesis of a ring-current with so large a radius seems, however, to create more difficulties than it solves. The chief argument against it is that no such ring-current could hold together against the mutual electrostatic repulsion of its parts if composed of charges of one sign only, particularly if these are electrons, as Störmer proposed. If, however, there are charges of both signs present, the suggested mode of formation of the ring breaks down, because the heavier ions would not follow the path calculated for the easily deflected electrons. What kind of compromise path would be arrived at by the two sets of particles moving together is not easy to determine. Ferraro[6] and I have attempted to solve the problem, and conclude that the ring, if it exists, is likely to be far smaller than Störmer supposed. The current in it will depend on the relative motion of the ions and electrons, and would disappear if they had the same velocity. Collisions between them will tend to equalize their speed, but Ferraro showed that a ring in which there was a moderate difference between the two speeds could carry the necessary current and endure for days in equilibrium, so far as concerns the decay of current due to collisions, at a distance of a few Earth-radii from the Earth's center. The calculation did not, however, take into account the daily rotation of the Earth's magnetic axis, which describes a cone of 22°-angle about the geographic axis; it may be that this would disrupt the ring, though only calculation can decide this question. A relatively small ring of this size would require far less energy[6] than a ring of the size suggested by Störmer, and it would be free from the objection against the latter, that it far more than nullifies the Earth's field in the region where it is set up, thus destroying and reversing the forces that are supposed to call it into being. (Störmer's large ring-current would modify the Earth's field appreciably throughout an immense volume of surrounding space, and would seem likely during a magnetic storm to alter considerably the normal paths of cosmic rays.)

While there seems to be no good reason for maintaining the hypothesis of so large a ring as Störmer's, the question as to a possible smaller ring is somewhat less easy to dismiss. It must be confessed, however, that we have no clear indication whether or how such a ring could be formed. Ferraro and I made some tentative suggestions on this point, but in their present form these have little compelling force, being proposed merely as

[5] C. Störmer, C.-R. Acad. sci., 151, 736-739 (1910).

[6] S. Chapman and V. C. A. Ferraro, Terr. Mag., 36, 77-97, 171-186 (1931); 37, 147-156, 421-429 (1932); 38, 79-96 (1933); also Terr. Mag., 37, 269-272 (1932).

the least improbable mechanism to account for such a ring, if on other grounds it is believed to exist. Of themselves, even when combined with the demonstration (under certain limited conditions) that the ring if formed might endure for several days, these suggestions hardly suffice to weigh the balance in favor of this hypothesis to explain the force-reduction during storms, as against the view that the currents responsible are located in the atmosphere, should a layer possessing appropriate properties be found to exist.

6. Added strength is given to the latter view by the consideration that the symmetrical and S_D parts of the D-field result only from a convenient analysis of what appears to be a unitary phenomenon. One mechanism should explain both together. This is somewhat difficult with any closed and long-enduring current-system circulating freely in the space round the Earth. The horizontal-intensity reduction on the equator during the moderate storms considered in *1* and *2* is about 58 gammas at 6 p. m., and about 30 gammas at 6 a. m.; a ring-current would have to be either very eccentric, or very elliptical, to explain such a difference. The atmospheric theory of the currents, on the other hand, can account for this difference by a divergence of the current-lines flowing from the 6 p.-m. to the 6 a.-m. meridian, outwards towards the auroral zones, as shown in Figure 4; such a divergence offers no such difficulty in a spherical current-sheet, as it does in regard to currents in free space. For the present, therefore, it seems proper to consider the interzonal currents of Figures 2 to 4 as real; at least the probability of this justifies the discussion of the magnitude of these currents if real.

7. We next consider the concentrated currents along the auroral zones represented in Figures 2 to 4 by the heavy lines $\alpha\beta$ and $\alpha'\beta'$. The case for locating these in the atmosphere is here very much stronger and clearer. In the first place, we have evidence from radio measures that during magnetic disturbances the ionization along the zone is enhanced and intense at levels from about 90 km upwards; moreover, the aurorae themselves afford visual indication of increased ionization and excitation in the rather narrowly concentrated auroral zone at times when the D-field is most intense; this is shown by the negative bands in the auroral spectrum, which are due to nitrogen ions. This creates a presumption that the currents producing the notable magnetic changes near this zone are situated in this ionized region of the atmosphere.

Here another type of "non-magnetic" consideration (cf. paragraph 3) comes into play; it is an argument from *a priori* probability on the grounds of simplicity. The D-field near the zone is highly differentiated locally, and is explicable as due to a current far more concentrated than that suggested by the interzonal part of the D-field; the latter can best be accounted for by a current-sheet in which the current-density varies only gradually from point to point. In the auroral region, on the other hand, a current so limited laterally that it can, on a large-scale view, be regarded as a linear current, is compatible with the local D-field. Its latitude, height, and total intensity can be calculated (cf. paragraphs 16 and 17) as was done by Birkeland[7] and, more recently, by Goldie[8]. Such calculations do not establish a proof that the cause of the local D-field is a current

[7] K. Birkeland, Kristiania, Skr. Vid. selsk., Math-naturv. Kl., No. 1 (1901) [especially pp. 13-38]; Norwegian Aurora Polaris Expedition 1902-1903, **1**, section 1 (1908) [especially chapter 2], and section 2, Part 2, chapter 2, Part 3, chapter 1 (1913).

[8] A. H. R. Goldie, Trans. Roy. Soc. Edin., **57**, 143-177 (1931).

so placed and of such strength; alternate current-distributions remain possible (cf. paragraph 2), but the urther their supposed location is from that calculated for the linear current, the more complicated their character becomes; if placed at a considerable distance away, they may involve an improbable distribution of current-bands of opposite sign.

This can be illustrated by means of the considerations described in paragraph 2. For simplicity we will consider only the symmetrical part of the current in the auroral zone, which flows continuously all round the zone. Such a concentration of current in the system of Figure 3 requires the presence, in the spherical harmonic expression of the field nd current-system, of a cluster of terms of high degree, whose effects are additive near the auroral zone, while elsewhere they nearly nullify one another. If the current-system is located at a considerable distance above the Earth, outside our atmosphere, these terms gain greatly in importance relative to those of lower order, and their own relative magnitudes become modified so that they no longer combine so exclusively in auroral latitudes, and neutralize one another elsewhere; hence they will then represent a highly banded distribution of current round the parallels of latitude This must certainly be regarded as a less probable type of current-system than the approximately linear current along the auroral zone in the atmosphere.

8. The intensity of the current along the auroral zone is not uniform; it increases from zero at two points A, A' which in Figure 4 are at about 2 p. m. and 10 p. m., to maxima at two points B, B' which in Figure 4 are at 6 a. m. and 6 p. m. (on the actual Earth these times may be displaced by a few hours). Thus, all along the zone, except at the maxima, current must be entering or leaving. It is important to know whence comes this current and whither it goes.

Birkeland believed that current came into the zone from outer space, approximately along the Earth's lines of magnetic force, and that it left the Earth again, along the lines of force, after flowing along the zone for some distance. The inflow, whencesoever it comes, must be continuous over the portion BAB', while there must be outflow over the remaining part of the zone, $B'A'B$. Birkeland's view would require that the closed circuit of any elementary tube of this current should be completed by a union, somewhere outside the Earth, of the further ends of the inflowing and outgoing current, possibly greatly diffused and far away; only the nearer parts of the circuit would contribute much to the D-field.

This system of current intake and outflow for the auroral zones may be contrasted with that corresponding to the spherical current-sheet that could also account for the observed D-field. Figures 2 and 4 represent the auroral zone as forming an intense and concentrated part of current-circuits, some of which are interzonal, while in others the flow is across the polar caps within the zones. Of course there are other possible current-systems intermediate between the spherical one and that envisaged by Birkeland; for example, part of the zonal currents might be supplied as he suggested, while the remaining part flows in the Earth's atmosphere, as in Figures 2 to 4. The details of the current-distributions that correspond to the various theoretically possible situations for the current are not clear without examination; for example, it is perhaps doubtful whether a current-system of the kind suggested by Birkeland,

including concentrated currents in the atmosphere along the zone, could explain the observed D-field, particularly in the center of the polar caps, without involving some improbable requirements as to the currents at a distance from the Earth. We may imagine the current-sheets of Figure 4 gradually distorted so that while their concentrated zonal currents remain *in situ*, the two polar sheets, or the interzonal sheet, or all three of these current-sheets, are moved outwards, each polar-cap sheet being stretched upwards over the diameter BB' so as to form two sheets, with downward inflow over the part $BB'A$, and upward outflow over $B'A'B$. It seems to me likely that the current-distribution in these sheets would have to be somewhat singular in order to yield the smooth distribution of the D-field that appears to exist near the center of the polar caps. On the ground of *a priori* probability of simplicity, therefore, the spherical current-sheet distribution over the polar cap seems preferable to the Birkeland theory; but perhaps this argument should not be too strongly pressed.

A more important question is whether free currents of the Birkeland type can occur in nature, or whether they could endure sufficiently long if they were started, at a reasonable distance from the Earth. This question is one that I hope to discuss in a separate paper. If we assume, however, that the D-field within the polar cap and in the interzonal region, not too near the auroral zone in either case, is due to an atmospheric current-sheet only a few hundred km above the ground, it is still possible that the total current which their currents could supply to the auroral zone would not account for the observed currents there; in that case we should gain a definite indication of the necessity for some outside supply to the zone; similarly, if the supply from the current-sheets outside the zone was excessive, that would imply the entry of opposed currents from outside. So far as I know, no attempt has hitherto been made to examine this point, or indeed, to estimate the strength of the D-currents outside the zone.

The known spherical-sheet current-distributions responsible for S and L have been derived from the observations of the S and L surface-fields by a somewhat complicated mathematical process—spherical harmonic analysis. Their general nature and main circuits can however be divided by simple inspection from the daily variation-curves for the three magnetic elements from many stations; the same applies to the D spherical-sheet also, which cannot be represented by any simple series of spherical harmonic terms (cf. *2*, paragraphs 10-15). Fortunately it is possible also to make an approximate *quantitative* determination of the total current in the various circuits of the D-field in a manner which, though quite simple, does not seem to have been described hitherto. The method will be illustrated, and its accuracy tested, by first applying it to the S-field, and comparing its results with those derived from the spherical harmonic analysis.

9. Consider the daytime current-systems at the equinoxes (Fig. 1). They encircle points (A, A') in each hemisphere on the meridian 11 a. m., in latitude 40°. On this meridian the horizontal-force S-variation attains its maximum (between latitudes 40° north and south) or minimum (beyond these latitudes); at latitude 40° the horizontal-force variation is reversed. The east-force daily-variation passes through one of its zeros at the same hour, 11 a. m. The maxima or minima of horizontal force

occur, for each circle of latitude, at the points where the current-flow is wholly eastward or westward, as is the case on the 11^h meridian AA'. From A to A' the current-flow is eastward, and its intensity i rises from zero at A or A' to a maximum i_0 at A_0 on the equator; near this maximum the current-intensity i varies slowly from point to point in the sheet, and for points outside the sheet, at a small distance above or below A_0, the field of the sheet will be nearly the same as that of an infinite plane easterly current-sheet of intensity i_0—that is, it is horizontal, northerly, and of amount $2\pi i_0$ (independent of the distance from the sheet if this is truly an infinite plane). This, however, is not the observed intensity F_0 at the point on the ground immediately below A_0, because F_0 contains a contribution due to the secondary currents induced in the Earth by the "primary" field of the atmospheric sheet. Suppose that a fraction f of F_0 is due to the primary field, then $fF_0 = 2\pi i_0$, or

$$i_0 = fF_0/2\pi$$

The value of f depends on the conductivity of the Earth and the degree of the main spherical harmonics in the series-representation of the field. The conductivity has been determined from the analysis of the S-field, so that in using the value of f derived from this knowledge we are partly depending on the exact analysis; but this knowledge can be applied with fair accuracy to the D-field without, in that case, having analyzed the field mathematically (though with T. T. Whitehead and A. T. Price I have made such an analysis of part of the D-field). In the case of extensive current-circuits fairly uniformly distributed and covering a considerable fraction of the Earth's area (from 1/5 to 1/10, say), the value 0.6 may be considered a fair approximation to f.

Substituting this value of f in the above equation, and also $F_0 = 20$ gammas, the value appropriate for S in the horizontal force at the equinoxes at the equator at 11 a. m., we find $i_0 = 1.9 \times 10^{-5}$ c. g. s. $= 1.9 \times 10^{-4}$ amp/cm or 21,000 amp per 10°-range in latitude. This may be compared with the value of about 23,000 indicated in Bartels' diagram (Fig. 1). To get the total current (I_{day}) flowing round the daytime current-circuit, we require to know the position of the center A of this system, and the mean current-density $\bar{i}$ between that point and the equator. The latter may be estimated by assuming that the current varies "parabolically" from its zero-values at the two circuit-centers to its maximum at the equator; this implies $\bar{i} = (2/3)i_0$. If $\bar{i}$ were supposed to vary between these two points like sin x between $x = 0$ and $x = \pi$, the factor $\bar{i}/i_0$ would be $2/\pi$, which is not materially different. The latitude l of A is that of reversal of type of the horizontal-force daily-variation, namely, about 40° at the equinoxes (35° in summer). Thus, $I_{day} = \bar{i}l$, where $\bar{i}$ is reckoned in amperes per degree of latitude, or $I_{day} = (2/3)i_0 l = (2/3)\ 2100 \times 40 = 56{,}000$ amperes. This is a reasonably good approximation to the value of 62,000 amperes derived from the spherical harmonic analysis.

The same method may be applied to the night current-circuits, whose centers B, B' lie in approximately the same latitude (40°). Taking the night maximum value of F at the equator as 12 gammas, we have $I_{night} = (12/20)\ I_{day} = 33{,}600$ amperes (Fig. 1 shows 32,000).

The longitude of B and B' is 18^h, as is best shown by the vanishing of the variation of the east force from the mean at that hour in latitude.

Between A and B, in latitude l, there flows northward (between 11^h and 18^h) the combined current of the two circuits, namely, 89,600 amperes according to our approximate calculations, or 94,000 according to Figure 1. This combined value can be inferred independently from the east-force variation in latitude l. The distance in cm between A and B,

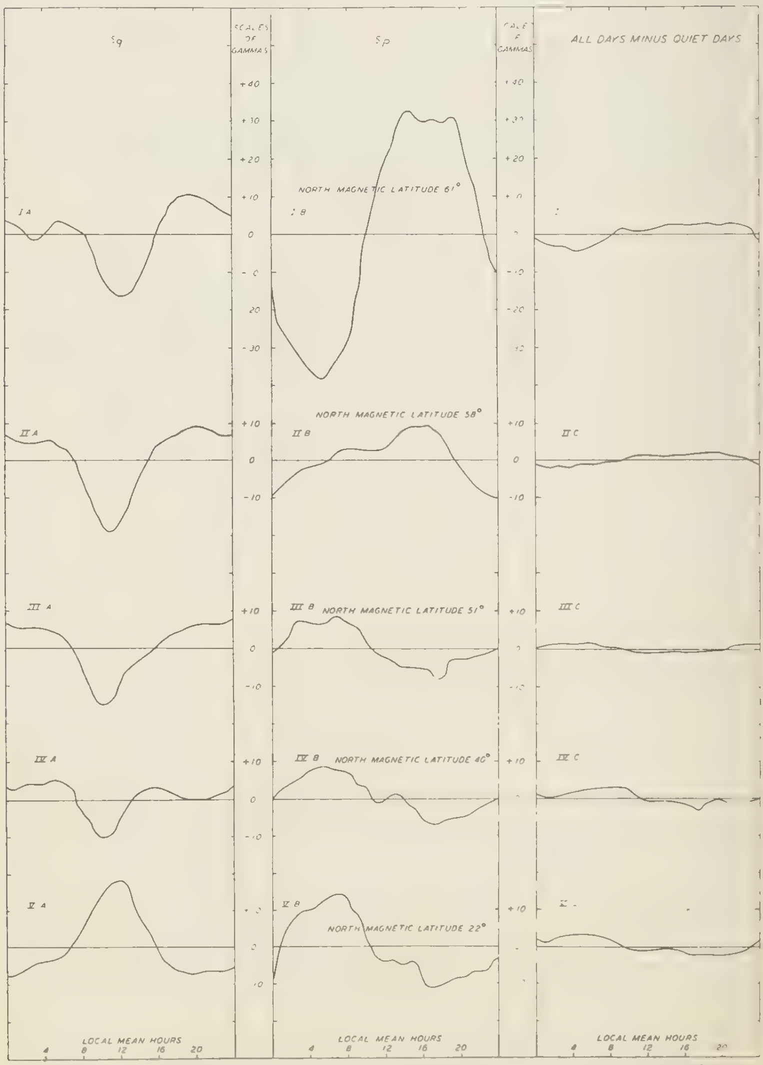


FIG. 5—MAGNETIC DIURNAL VARIATION OF HORIZONTAL FORCE AT FIVE GROUPS, I TO V, OF OBSERVATORIES IN NORTHERN HEMISPHERE; SECTION A FOR QUIET DAYS (Sq), SECTION B FOR ADDITIONAL VARIATION ON STORM-DAYS (Sd), SECTION C FOR (ALL DAYS MINUS QUIET DAYS)

in latitude 40° and 7^h apart in longitude, is 8.9×10^8. The value F_0 (now eastwards), from the east-force daily-variation, is about 16 gammas, giving $i_0 = 1.5 \times 10^{-4}$ amp/cm, and $\bar{i} = 1.0 \times 10^{-4}$ amp/cm. Hence, $I = 1.0 \times 10^{-4} \times 8.9 \times 10^8 = 89{,}000$ amperes, in reasonable accord with the value of 89,600 based on horizontal-force data alone.

The northward current between B and A is smaller in intensity and covers a far greater range in longitude, namely, 17 hours. Its distribution, with the intensity fading away towards the dawn meridian, can be approximately inferred from the east-force variation during the night. The departure of east force from the daily mean is almost zero in the 10 1/2-hour interval between 17 $1/2^h$ and 4^h, and the current-density must be very small during this interval, as shown by Fig. 1. Over the 7-hour interval 4^h to 11^h, the maximum divergence of east force from the mean is 14 gammas, giving a total southward current of $(14/16) \times 89{,}000 = 78{,}000$ amperes (by comparison with the northward current between 11^h and 18^h, covering an equal distance in longitude but with a larger F_0); the remaining 11,000 amperes must be spread over the long interval 17 $1/2^h$ to 4^h. This is in reasonably good agreement with Figure 1.

The present method of approximately determining the intensity of a spherical current-system corresponding to a given surface-field should not supersede the much more accurate method of spherical harmonic analysis, where this is practicable, as is the case for S and L. Even for these, however, it may usefully supplement the analytical method; not much is gained by taking more than a few terms in the harmonic expression for the potential, yet the few main harmonic terms fit the derived data only moderately well. It would be worth while to consider the part of the daily variations, at a number of observatories, which remains after subtraction of the part represented by the harmonic series, and from this residual part to determine the corresponding current-system, in the above manner. This could then be combined with the current-system corresponding to the harmonic series, to give the whole S current-system. It would, of course, vary throughout the day. The same method might usefully be applied to L.

10 The above method will now be applied to determine the current-flow in the S_D current-system (Fig. 2). The observational material on which the following considerations are based is shown in Figures 5 to 7, reproduced here from paper *2*. Five groups of observatories, here referred to as I to V, have been represented. In each of the figures, the left-hand section, marked a, shows the curves for diurnal variation S_q on quiet days (five per month); the center section, marked b, shows the curves for the additional diurnal variation S_D during the first days of 40 selected magnetic storms [that means, the total diurnal variation on these storm-days was $(S_q + S_D)$]; the right-hand section, marked c, shows, in the average for a number of years, the difference of the diurnal variations on all days minus that on quiet days. All curves correspond to the average conditions throughout the year.

The centers of the two (a.-m. and p.-m.) circuits between the auroral zones are at local times 6^h and 18^h approximately. Their latitude l can be inferred from the S_D-curves given in Figure 5, section b, for the first day of the series of moderate magnetic storms considered in *1*. The lati-

tude of reversal of the S_D-variation in horizontal force occurs between Curves II*b* and III*b*; the former is for Pavlovsk (magnetic latitude 58°) and the latter for the mean of Pola, Potsdam, and Greenwich (mean magnetic latitude 51°); thus l may be taken as 55°, the distance from the equator being 6.1×10^8 cm. Curve V*b* of Figure 5 gives the maximum horizontal-force divergence from the mean at magnetic latitude 22° as

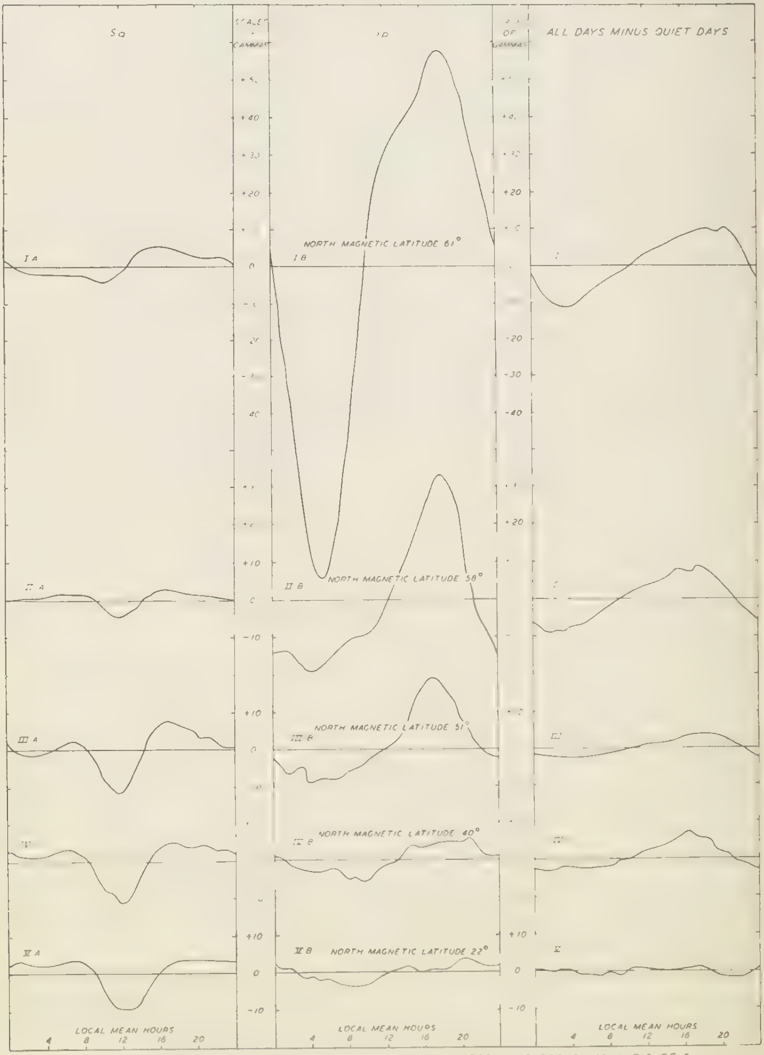


FIG 6—MAGNETIC DIURNAL VARIATION OF VERTICAL FORCE FOR SAME GROUPS AS IN FIGURE 5

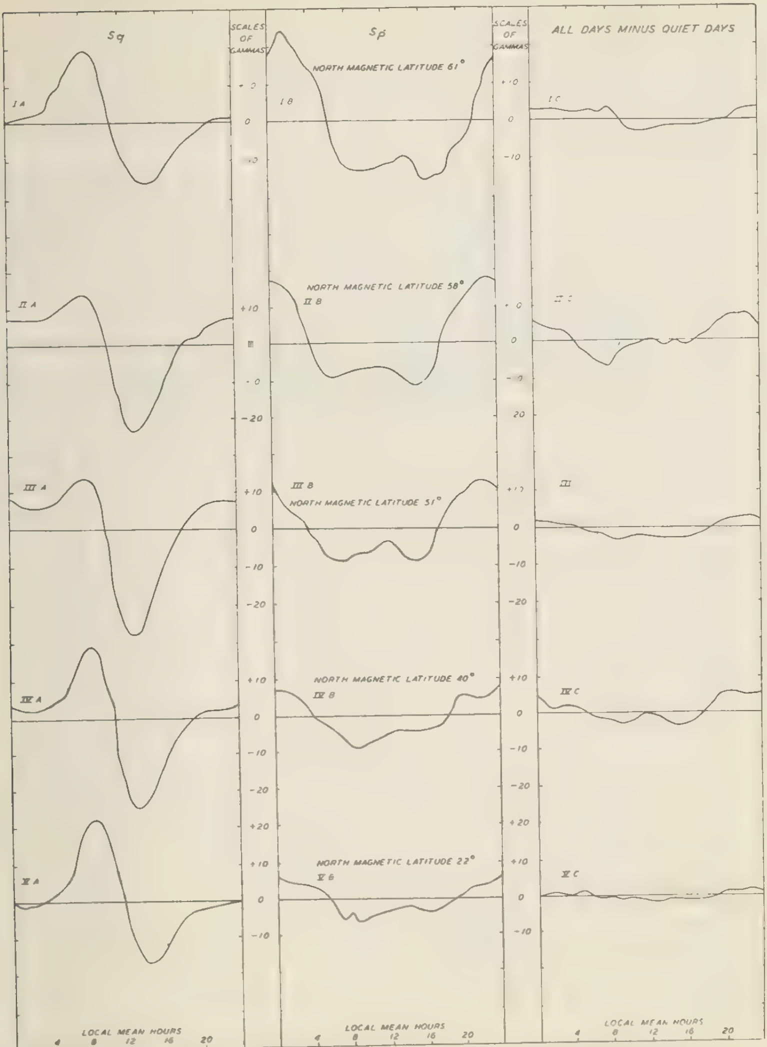


FIG. 7—MAGNETIC DIURNAL VARIATION OF EAST DECLINATION FOR SAME GROUPS AS IN FIGURE 5

12 gammas, and the equatorial value may be estimated as 13 gammas. This gives $i_0 = 1.2\times10^{-4}$ amp cm, $\bar{i} = 8.3\times10^{-5}$ amp cm, and $I = 51{,}000$ amperes, for any one of the four current-circuits between the two auroral zones.

This total current can also be estimated from the S_D-variation in the east force at latitude l (55°); estimating F_0 as 13 gammas, from curves II*b* and III*b* of Figure 7, we have $i_0 = 1.2\times10^{-4}$ amp cm, $\bar{i} = 8.3\times10^{-5}$

amp/cm, and $I=48{,}000$ amperes, the width of the northward or southward current-flow being 5.8×10^8 cm (namely, 90° of longitude on the 55° latitude-circle). This is in reasonably good agreement with the quite independent estimate from the horizontal-force variations. In round figures, the total current in each S_D current-circuit may be taken as 50,000 amperes.

As might be expected from a comparison between the magnitudes of the S_q and S_D daily variation-curves (*a* and *b* respectively in Figs. 5 to 7), this is about equal to the average of the currents in the night and day circuits of the S_q-system [namely, half of (62,000+32,000) or 47,000 amperes; cf. Fig. 1] in a sunspot-minimum year. In a sunspot-maximum year the S_q-currents are stronger, just as the S_D-currents are stronger during more intense storms.

The curves (*c*) in Figures 5 to 7 refer to S_D on ordinary (all minus quiet) days, that is, to the average amount of disturbance present on five out of six days (one-sixth of all days being selected as quiet) over a period of years. The ratio of the amplitudes of corresponding curves (*b*) and (*c*) differs somewhat from one curve to another, presumably because of accidental irregularities in the data, but the average ratio seems to be about five, implying that these four S_D current-circuits on ordinary days each carry a total current of about 10,000 amperes. This is only a small fraction of the S_q current-strength; it is comparable with the strength of the main (daytime) L current-circuit in summer.

11. The symmetrical (or storm-time) part of the disturbance-field between the auroral zones will next be considered. This is due to westerly currents flowing round the parallels of latitude, as illustrated in Figure 3. The reduction of the horizontal force at maximum-phase in the moderate magnetic storms of *1*, below the initial value of the horizontal force, is shown in Figure 8 to be $[+9-(-29)]=38$ gammas at magnetic latitude 22° (left-hand curve). The reduction decreases, with increasing latitude, to about $[+7-(-15)]=22$ gammas at latitude 53° (right-hand curve). Near this latitude the reduction reaches a minimum and further north it appears to increase rapidly towards the auroral zone. We will estimate the total westward current between the equator and 55°, the latitude of the center of the S_D current-circuits. Using a simple graph showing the horizontal-force reduction as a function of latitude, from the three curves of Figure 8, we find its value to be 20 gammas at 55°, 41 gammas at the equator, with a mean value of 34 gammas over this 55°-range. This gives $\bar{i}=3.25\times10^{-4}$ amp/cm, flowing across a section of width 6.1×10^8 cm; hence $I=198{,}000$ amperes, or, in round figures, 200,000 amperes. This great current is four times as strong as the current in each S_D-circuit.

The combination of the symmetrical current-flow with the S_D current-circuits gives the combined current-flow, between ±55° latitude, shown in Figure 4. On the 18^h-meridian its total amount is 250,000 amperes, on the 6^h-meridian 150,000 amperes, between the equator and 55° latitude, or twice these values, 500,000 and 300,000 amperes, between ±55° latitude. The difference, 100,000 amperes in each hemisphere, represents the decided asymmetry of the interzonal current-system, already referred to in paragraph 6; this current flows across the 55° latitude-circle towards the auroral zone.

FIG. 8—GENERAL STORM-TIME VARIATIONS DURING FIRST 48 HOURS OF 40 SELECTED MAGNETIC STORMS FOR THREE GROUPS OF OBSERVATORIES

The average intensity of these currents on ordinary (all minus quiet) days is about one-fifth of the above values. After a prolonged period of magnetic calm the currents may decline to decidedly lower values.

12. We next consider the symmetrical part of the current-flow (Fig. 3)

from latitude 55° to 70°, and from this to the pole. The mean westward current-intensity probably has a sharp maximum between latitude 55° and 70°, from which it decreases towards the north, rapidly at first, to zero at the pole. Data for many periods of moderate magnetic storms not being available to me at the moment, the current-strengths on average (all minus quiet) days will be considered. The reduction of the daily mean horizontal force under the zone [(all minus quiet)-days] may perhaps be estimated as 20 gammas (cf. the Bossekop curve in Fig. 6 of paper *2*); this may be contrasted with about four gammas at latitude 55°, and about 10 gammas at the equator. The maximum current-density of the westward (symmetrical) current in the zone 55° to 70° cannot on this account be estimated as just twice that at the equator, because near the auroral zone the current-density probably varies too rapidly with latitude for the current-sheet to be treated as if uniform and plane. But we are perhaps not likely to fall seriously into error if we estimate the total average westward current-intensity [for (all minus quiet)-days], between latitude 55° and 70°, as corresponding to that of a uniform plane current-sheet for which $F_0 = 10$ gammas. This would give $\bar{i} = 10^{-4}$ amp/cm, and I, across the 15°-belt of latitude between 55° and 70°, as 15,000 amperes.

If on days of moderate storms this current is magnified five times, as appears to be so for S_D, the corresponding value is 75,000 amperes.

Within the auroral zones, from latitude 70° to the pole, the storm-time current is still westward, presumably along the isochasms or parallels of magnetic latitude. The average reduction of horizontal force [(all minus quiet)-days] may be estimated as 10 gammas, giving I, for the 20°-belt, as about 20,000 amperes. On days of moderate storm it is perhaps five-fold, or 100,000 amperes. The estimates in the present section are less reliable than those made in paragraphs 10 and 11, because of the paucity of the data used, and because of the more rapid variation of intensity of the currents with latitude (which calls for a more refined treatment of the data). Unfortunately no check can be derived from the declination-data, as in paragraph 10. The vertical-force data should, however, aid in estimating these current-strengths.

13. We next consider the S_D-currents north of latitude 70°, for average (all minus quiet) days. Near the center of the polar cap the horizontal-force vector-diagrams are nearly circular, and uniformly described. Their radius is somewhat uncertain; note the difference between the curves for Kingua Fjord and Cape Evans in Figure 6 of paper *2*, which, however, refer to quite different years. Here we shall assume that it is 30 gammas, which lies between the values for these two stations. As indicated in *2* (paragraph 14. 3), this type of diagram may be ascribed to a fairly uniform current-sheet in this region (cf. Fig. 7, plan, of paper *2*). Its intensity i_0 at the pole will be 3×10^{-4}; this must decrease to zero near 6^h and 18^h in the auroral zones, where the current-direction is reversed; applying the usual factor to obtain $\bar{i}$, we find 2×10^{-4} amp/cm. Taking the limit of the sheet as latitude 70°, the whole (diametral) breadth of the cap is 4.4×10^8 cm. Hence I across this breadth is 88,000 (or, say, 90,000) amperes.

On days of moderate magnetic storm this will be increased say five-fold, to 450,000 amperes; this estimate needs to be checked by reference to actual polar data for such days.

14. We have thus estimated that on average (all minus quiet) days there is a total current-flow northward into the latitude-zone 55° to 70° of amount 20,000 amperes (paragraph 10) and southward into this zone, of amount 90,000 amperes (paragraph 13); and that on days of moderate storm these estimates may be magnified five-fold.

Thus on average days 110,000 amperes flow into the zone, from the north and from the south, and this extra-zonal current-supply appears to divide and flow half eastward, half westward. These *E*- and *W*-currents will each carry 55,000 amperes, on which will be superposed the continuous westward current of 15,000 amperes. This will increase the westward current at 6^h to 70,000 amperes and decrease that at 18^h to 40,000 amperes. The estimated values on moderate storm-days are 350,000 (*W*) and 200,000 (*E*).

15. We have now arrived at an estimate of the extra-zonal current-supply to the auroral zone (or latitude-belt from 55° to 70°). If this estimate was more reliable, and if reliable independent estimates of the actual *E*- and *W*-currents along the belt could be obtained from stations near the auroral zone, it would be possible to make a test of the hypotheses that the auroral zone does or does not receive current from free space (see the end of paragraph 8). This cannot be done here; the test must await a closer study of the magnetic data, especially those obtained during the Second International Polar Year—a task upon which E. H. Vestine is now engaged in cooperation with me. It is, however, worth while to make a very rough comparison of the extra-zonal current-supply with the estimates at present available for the currents along the zone.

16. Attempts to deduce the height and intensity of the currents along the auroral zone from their magnetic effects seem to have been first made by Birkeland[9], who found intensities of the order of 5×10^5 amperes at heights varying from 150 km to 600 km or more during magnetic storms not of outstanding intensity. Goldie also has made estimates of the zonal currents from the records of Eskdalemuir and Lerwick for ten of the greatest magnetic storms occurring in the year 1926. It is uncertain whether these were of greater or less average intensity than those considered in *1*, but the fact that all the ten occurred in one year suggests that they were not much more than "moderate." The maximum current found from the mean of the ten storms was 595,000 amperes; this was a westward current, at height 290 km, and occurred at 2^h local time. The maximum eastward current found was 480,000 amperes, at 17^h local time; this current was nearly overhead at Lerwick[10] (magnetic latitude 63°), and the estimate of its height was 370 km. These currents are of the same order as those mentioned at the end of paragraph 14, being, in fact, about twice as large; as there, the westward currents are the greater in magnitude, and occur in the morning hours, though at 2^h instead of at 6^h as in Figure 2; the eastward current found by Goldie occurs at 17^h, or nearly at 6 p. m. as in Figure 2. This accordance is satisfactory as far as it goes, and if the storms considered by Goldie were on the average twice as intense as the average of the storms considered in *1*, it would be natural that his currents should be about double those of paragraph 14;

[9]K. Birkeland, The Norwegian Aurora Polaris Expedition, 1902-3, vol. 1, pp. 306-311 (1908).

[10]The daily northward and southward motion of the zonal currents, apparently indicated by Goldie's results, may be due to the inclination of the magnetic to the geographic axis—a complication neglected in this paper.

this could be tested by finding the average storm-time variation for these storms at some low-latitude station. If, however, these storms were found to be less than twice as intense, there would appear to be a discrepancy between Goldie's zonal currents and the extra-zonal current-supply. It would be unsafe, however, to conclude from such a discrepancy that there was a supply of current from free space to the auroral zone. This is not only because of the rough nature of the results of paragraph 14, especially on account of the assumed value (five) of the ratio of the storm to the (all minus quiet)-day D current-system in polar regions; Goldie's estimates of the zonal currents are likewise uncertain, because they were derived without taking account of the influence of the induced internal currents.

17. There is need for further theoretical studies of the nature of the induced earth-currents, and of their magnetic field, in the polar regions. Until this is better understood, the determination of the zonal currents and their height will remain uncertain. It seems likely, however, that $\triangle H_e$ and $\triangle H_i$, the external and internal parts of the (vector) D-change in horizontal force, will usually be similar in direction or sign, while $\triangle V_e$ and $\triangle V_i$, for the vertical force, will have opposite signs. If so, the inclination of the resultant D-vector $\triangle F$ to the horizontal plane will be less than that of its external component. The situation of the zonal current is found by drawing, from each of two stations A, A', say (cf. Fig. 9), a line (l or l') in the vertical plane containing $\triangle F$ at each station, perpendicular to $\triangle F$. Assuming that the disturbance is due to a hori-

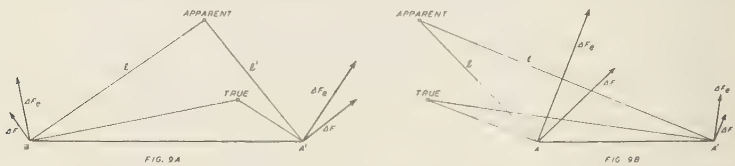


FIG. 9—ESTIMATING HEIGHT OF LINEAR CURRENT IN AURORAL ZONE FROM DRAWING PERPENDICULARS TO MAGNETIC DISTURBING FORCES OBSERVED AT TWO STATIONS ON OPPOSITE SIDES (FIG. 9A) OR ON THE SAME SIDE OF THE AURORAL ZONE (SECTION PERPENDICULAR TO AURORAL ZONE; ΔF, TOTAL DISTURBING FORCE, GIVES APPARENT HEIGHT TOO GREAT; ΔF_e, EXTERIOR PART OF DISTURBING FORCE, GIVES TRUE HEIGHT)

zontal linear atmospheric current between A and A', this must lie along the common perpendicular to l and l', if there is no induced field; the presence of the latter requires that l and l' be drawn normal to $\triangle F_e$ instead of to $\triangle F$. The usual incorrect procedure gives too great a height for the current. The same is probably true in most cases when the situation of the current is deduced from two stations on the same side of the zone (Fig. 9) though it may not be desirable to use two such stations for the purpose.

Neglect of the internal field may also invalidate this estimate of the current-strength I, which is likely to be in excess. To take a simple illustration, suppose one station A is directly below the current: I will be equal to $(1/2)\, h \triangle H_e$, where h denotes the height of the current and $\triangle H_e$ its magnetic effect at A. If $\triangle H$ is used in this expression in place of $\triangle H_e$, which is less than $\triangle H$, then I will be over-estimated on this account; h also is over-estimated, so that I is still more an over-estimate. This may be partly the cause of the excess of Goldie's current-values over those mentioned at the end of paragraph 14.

18. The results obtained in paragraphs 10 to 14 are illustrated in Figures 10 to 15, which are improved quantitative versions of the Figures 2 to 4. The current-flow is indicated in each figure by lines, between each of which there flow 10,000 amperes, as in Figure 1. In the auroral

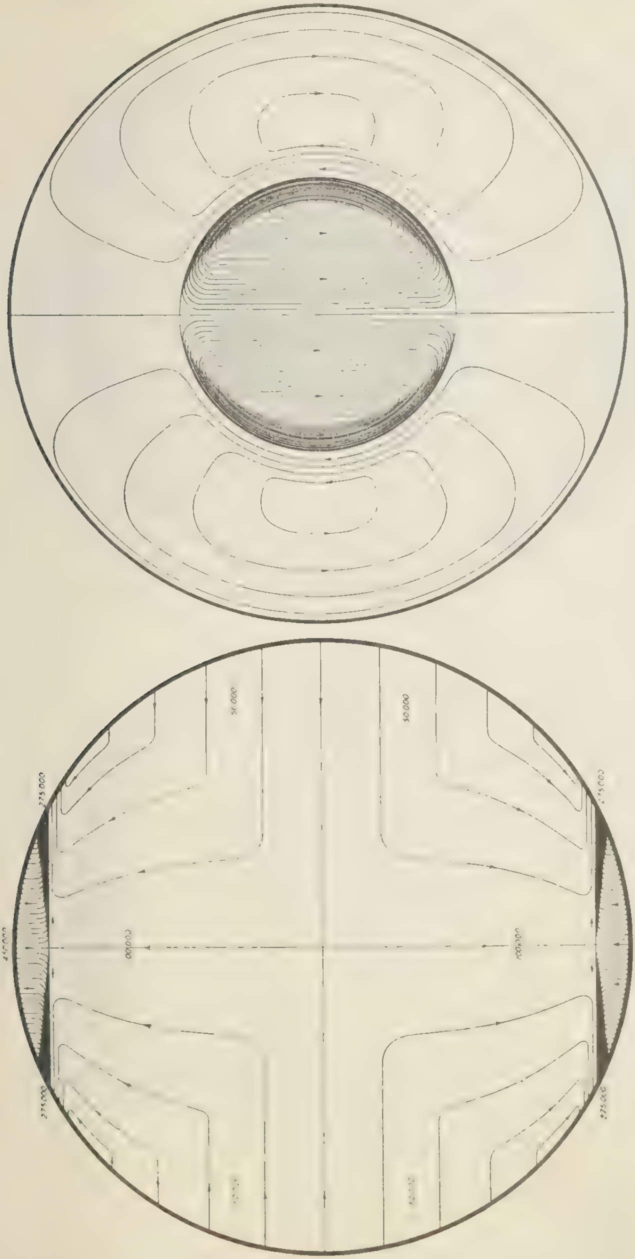
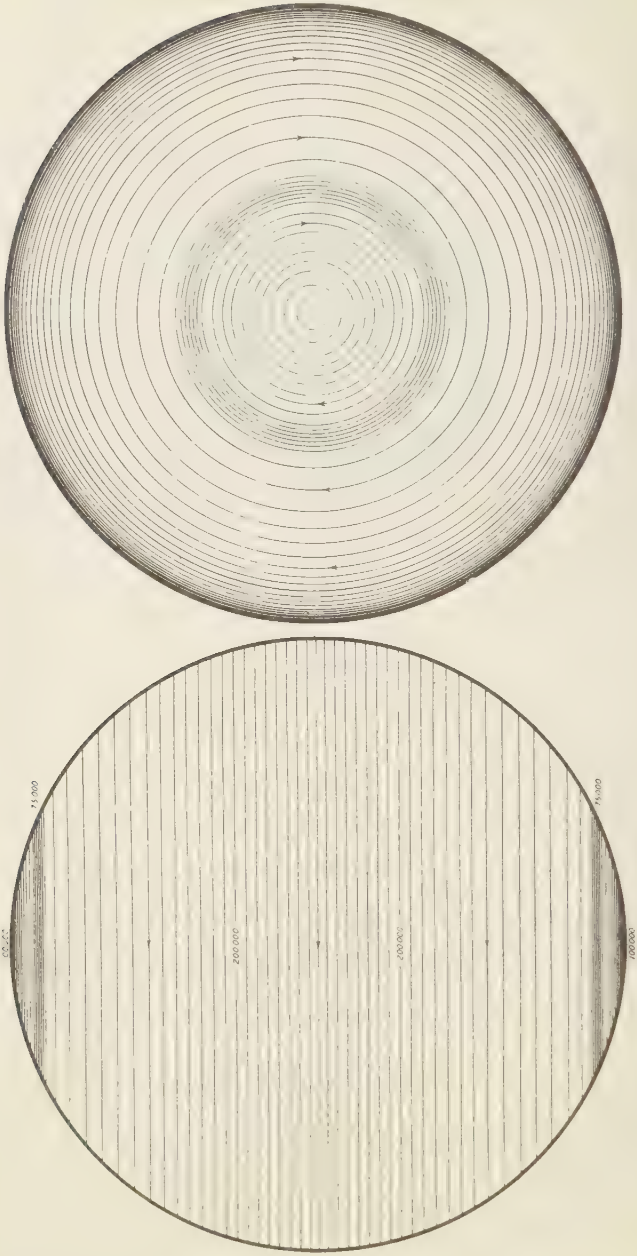


FIG. 11—VIEW FROM NORTH POLE

FIG. 10—VIEW FROM SUN

FIGS 10-11—QUANTITATIVE VERSION OF FIGURE 2—NON-SYMMETRICAL PART S_D
(10000 AMPERES FLOW BETWEEN SUCCESSIVE CURRENT-LINES IN FIGURES 10-15)

zone the lines are so crowded together that they cannot be distinguished individually; the increase of the zonal current from 0^h or 12^h to 6^h or 18^h is indicated by a crowding of lines, giving a wedge-like appearance to



FIGS. 12–13—QUANTITATIVE VERSION OF FIGURE 3—SYMMETRICAL OR STORM-TIME PART

FIG. 13—VIEW FROM NORTH POLE

FIG. 12—VIEW FROM SUN

the zones which must not be interpreted as a real feature of them. The current-flow in each part of the system is indicated in the figures for days of moderate magnetic storm (at maximum storm-time phase); the

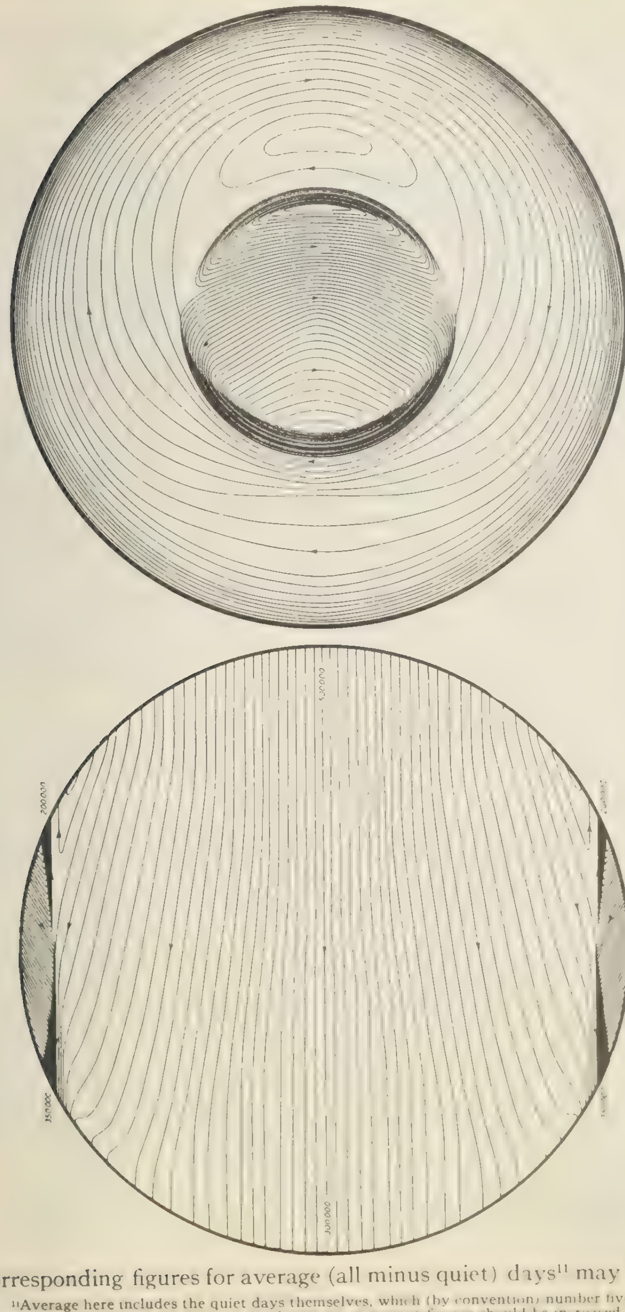


FIG. 14—VIEW FROM SUN

FIG. 15—VIEW FROM NORTH POLE

FIGS. 14-15—QUANTITATIVE VERSION OF FIGURE 4
(COMPLETE D CURRENT-SYSTEM ABOUT 18 HOURS AFTER OUTBREAK OF MAGNETIC STORM)

corresponding figures for average (all minus quiet) days[11] may be taken as

[11]Average here includes the quiet days themselves, which (by convention) number five per month. If these are excluded from the average, the lower set of current-figures should be increased in the ratio 6/5. Magnetic storms are so relatively infrequent that they contribute only a small part to the "average"-day disturbance and currents.

about one-fifth of these. In Figures 10, 11, 14, and 15 the zonal currents are given as calculated on the assumption that there is no current-supply to the zones from outside.

19. The currents shown in Figures 10 to 15, though amounting to 750,000 amperes from pole to pole in Figure 12, are at times very materially exceeded. One of the greatest storms on record occurred May 15, 1921; unfortunately most magnetic observatories lost a large part of their records on this remarkable day, but at Samoa a decrease of horizontal force by 800 gammas was observed during the first six hours of the storm (the rapid attainment of the maximum-phase is in accordance with the result found in paper *1* for great magnetic storms). At Samoa this maximum-phase occurred in the afternoon, so that part of the decrease was doubtless due to S_P. If, as seems reasonable, 600 gammas were due to the symmetrical current-system (Fig. 12), the intensity of the system would then have been 15 times that calculated here (assuming 41 gammas, see section 11) for moderate magnetic storms. It would be interesting to know to what degree on this occasion the relative proportions between the different parts of the current-system shown in Figures 10 to 15 were preserved. One striking fact about the storm of May 1921 was that the auroral zone, which expands equatorwards the more intense the disturbance, approached the zenith of Potsdam; consequently Potsdam experienced a very large vertical-force change, that is, a decrease of 400 gammas to a minimum at about 2^h local time; this was undoubtedly of diurnal-variation type, such as is normally shown in the early morning hours by stations near to, and just outside, the auroral zone (Fig 6, curve I*B*); a large part of the morning variation in horizontal intensity on May 15, 1921 was also of the S_D-type for such a station. In the afternoon at Potsdam the variations were much more like those usually experienced there during magnetic storms; the auroral zone would seem to have by then retreated far to the north of Potsdam.

It is a pleasure gratefully to acknowledge the skilled assistance of Mr. W. C. Hendrix in the preparation of the diagrams of this paper.

Summary

Magnetic disturbance is due to electric current-systems above the Earth's surface, and to secondary induced currents within the Earth. The location of the external currents cannot be uniquely inferred from magnetic observations made at the Earth's surface. Additional considerations are required to resolve this difficulty; knowledge of the distribution of ionized gas above the Earth, such as radio observations afford, is one principal means, while another is the *a priori* probability that the current-system is fairly simple. The question whether or not the currents flow wholly in the atmosphere is discussed, and it is concluded, though not decisively, that the evidence favors this view. The strength of the various parts of the current-system is evaluated, and among other results it is shown that on days of moderate magnetic storm the total additional electric current flowing westward round the Globe, between the North and South poles, rises to 750,000 amperes.

Department of Terrestrial Magnetism,
Carnegie Institution of Washington,
Washington, D. C.

MEASURES OF TERRESTRIAL-MAGNETIC ACTIVITY

By G. van Dijk

In the abstract of his paper[1], "Terrestrial-magnetic activity and its relation to solar phenomena," Professor Bartels states, "A break in the homogeneity of the international character-figures in recent years is discovered." From the contents of the paper it appears that by "recent years" especially the year 1930 is meant.

Bartels bases his conclusion principally on the fact that while the international character-number C shows a considerable increase from 1929 to 1930 (from 0.672 to 0.826) the u- and u_1-measures of magnetic activity indicate a small decrease (u from 1.05 to 1.00, u_1 from 67 to 63).

The measures u and u_1 are not based on direct measurements of the records for the day concerned, but on the difference of the daily means of horizontal intensity for this day and the preceding day; the annual mean of these differences is indicated by U. On quiet or feebly disturbed days this difference, as a rule, is small; on strongly disturbed days the difference may be large or small, depending on circumstances; in cases of disturbances with considerable postperturbation the difference generally is large.

By means of a conversion-factor K, U of a magnetic station is reduced to the magnetic equator; for De Bilt K, according to Bartels, is 1.75. The quantity, reduced to the magnetic equator, indicated as u, is expressed in units of 10 gammas, in order to have a value of about 1, easily comparable with the magnetic character-numbers; u_1 is a quantity, connected with u, and derived with a view to certain statistical investigations.

Methods of Characterisation

Bartels supposes as a cause of the different behaviour of C and u in the years 1929 and 1930, that the characterisation in the latter year was systematically higher than before, and is of the opinion that the supposed break in the homogeneity of the series of international character-numbers may be due to the new practice, adopted by some observatories, of determining the character from actual measurement of ranges.

At the meeting of the Magnetic Commission of September 11, 1905, at Innsbruck, the Commission agreed on the following numerical scale of classification, proposed by Ad. Schmidt: 0 = quiet days; 1 = disturbed days; 2 = very disturbed days. It was left to the discretion of individual directors to fix limits for defining these divisions.[2]

In the annual reviews of the magnetic character of 1906 and following years the methods of classification of a large number of observatories have been communicated. It appears from these reviews (see, for example, 1906, summary, p. 4) that many stations have adopted fixed limits between the classes 0, 1, and 2, expressed in minutes and gammas or in parts of the normal daily range; other stations adhered to the ideas of Ad. Schmidt (method of Potsdam); others chose the scale of Eschen-

[1] Terr. Mag., 37, 1-52 (1932); also Terr. Mag., 39, 1-4 (1934).

[2] See Report of the International Meteorological Conference at Innsbruck, September 1905; London 1908.

hagen as a basis of classification. The system of fixed limits between the classes is as old as 1906, the first year of the publication of the magnetic character-figures.

In March 1911 and March 1924 circular letters were sent out, aiming at an improvement of the efficiency of the magnetic characterisation. In the latter circular De Bilt communicated that it had somewhat modified its system of classification by introducing fixed limits between the classes expressed in minutes and gammas instead of limits expressed in parts of the normal daily ranges. The new limits were chosen so that the annual sum of characters should remain as much as possible unaltered, as compared with the previous system. Some observatories, according to the annual review of the magnetic character of 1923, adopted the new system of De Bilt, to date from 1924.

If this new practice of some observatories would have caused a considerable rising of the character-level, this should have appeared in or soon after 1924. C and u_1 are, however, according to Bartels[3] in good agreement even in 1923 and the following years.

In 1929 and 1930 there were no reasons to modify the system of classification. Out of 43 stations, having contributed both in 1929 and 1930 (one of them after the establishment of the annual mean of 1929), 39 assigned a higher sum of characters to 1930, four a somewhat lower sum. It is not admissible that more than 90 per cent of the stations should have raised, without any outward motive, their standards of classification.

In 1929 the data of 42 and in 1930 of 46 stations were taken into account in establishing the mean annual character-number. The four more observatories having contributed in 1930 are not responsible for the difference; in computing the mean annual character of 1930 from the 42 stations of the year 1929, 0.822 is found, practically the same value as 0.826, derived from 46 observatories.

The above example illustrates the stability of the character-number. Fixed limits between the classes warrant stability. In applying this system, the determination of the character-figure is independent of subjective appreciation and is not a character-estimation, but a character-measurement. The character-figure in this way gets a quantitative meaning; character-figure 1 does not only mean a disturbed day, but also deviation between fixed limits. for example, at De Bilt between 4 and 12 minutes in declination (D), between 15 and 45 gammas in horizontal intensity (H) and between 7 and 21 gammas in vertical intensity (Z).

As will be seen later, the results of many observatories, applying other methods, are in fair agreement with the results of stations, applying fixed limits.

The divergence of C and u in 1930 must have another cause than Bartels' suggestion. It is by no means evident that the u-measure is to be regarded as the best measure of magnetic activity; *a priori* it is just as probable, that 1930, according to the character-number, has to be considered as a very disturbed year, but that the u-measure fails to express this.

Therefore, and also in order to restore confidence of scientists in the international character-number as a measure of magnetic activity, not only for intervals of weeks and months, but also for intervals of years,

[3]Terr. Mag., 37, p. 34, Fig. 13 (1932).

it appears desirable to test the quantities C and u by comparing them with other measures of activity. It will be seen from this investigation that, on the evidence of these measures, 1930 is a very disturbed year, and that a break in the homogeneity of the international character-numbers in recent years does not exist.

Measures of Magnetic Activity

As measures of magnetic activity of a day will be considered: (*a*) the quantity $(HR_H+ZR_Z)/10{,}000$, henceforth indicated by (H, Z), adopted at Stockholm in August 1930; (*b*) the quantity $S.\ R$, the sum of absolute daily ranges of D, H, and Z, expressed in gammas, previously applied in a paper[4]; (*c*) the quantity AS, or absolute storminess, applied by the Observatory at Tromsö, the diurnal sum of the deflections in gammas of the 24 hourly means of a day from a normal line, representing undisturbed conditions (Tromsö considers this quantity, in which the deflections are being taken into account hour by hour, as the best expression for the magnetic activity during a day). Character-number and storminess are functions of the disturbances, that is, of the deviations from the normal undisturbed conditions; (H, Z) and $S.\ R$ comprise both disturbances and normal daily oscillations, the latter giving rise to the systematic seasonal variation of these quantities. The normal daily oscillation may approximately be eliminated from the annual means of (H, Z) and of $S.\ R$ by subtracting the values for quiet days (for individual days this would not hold, because it may give rise to negative values). It seems best to take as quiet days the "five calm days of each month," even although some of these days may be not undisturbed. Excluding one or more of the five days introduces an element of arbitrariness, which is avoided by taking all five days into account.

In this manner it appears possible to separate approximately the disturbances from the normal oscillations in the annual means of (H, Z) and $S.\ R$, thus $(H, Z)-[(H, Z), q]=[(H, Z), d]$; $S.\ R-(S.\ R, q)=(S.\ R, d)$ where q and d indicate normal oscillations and disturbances, respectively.

Since 1921 the magnetic yearbooks of De Bilt give the absolute ranges of D, H, and Z for each day, so that various of the above-mentioned quantities may easily be computed. For this reason the investigation will extend over the years 1921-34; this interval comprises two years of sunspot-minimum, 1923 and 1933, and one year of sunspot-maximum, 1928. It was found that many of the quantities mentioned vary at a large number of magnetic stations in an analogous way as at De Bilt, so that De Bilt may be considered as a representative of these stations.

In Table 1 different annual means have been given, mostly for the period 1921-1934, namely: (*a*) the relative sunspot-number N, according to Wolfer and Brunner; (*b*) the international character-number C; (*c*) u, according to Bartels, for the years 1921-32; (*d*) U, (H, Z) and $S.\ R$ for De Bilt; (*e*) AS, the absolute storminess of the horizontal intensity at Tromsö for the years 1930-34 (the values for 1930-33 are taken from the yearbooks[5] while those for 1934 have been kindly supplied by letter; several days in 1930 are incomplete and the annual mean was computed

[4]Activity of the Earth's magnetism and magnetic characterisation of days, Utrecht 1922.

[5]Results of magnetic observations; the first magnetic yearbook of Tromsö dates from 1930, the magnetic observations having begun in the course of 1929. It is recommended that observatories, having the opportunity to do so, compute and publish values of AS, also before 1930.

from the complete days; from the magnetic character of the missing days it may be derived that, when the records were complete, the annual mean would have been higher); and finally $[(H, Z), q]$, $(S. R, q)$, $[(H, Z), d]$, and $(S. R, d)$ for De Bilt. Figure 1 is a graphic representation to scale of the quantities of Table 1.

Table 1—*Measures of activity, 1921-1934*

Measure	1921	1922	1923	1924	1925	1926	1927	1928	1929	1930	1931	1932	1933	1934
N	26.1	14.2	5.8	16.7	44.3	63.9	69.0	77.8	65.0	35.7	21.2	11.1	5.7	8.
C	0.610	0.663	0.483	0.543	0.564	0.650	0.629	0.632	0.672	0.826	0.663	0.704	0.638	0.56
u	0.98	0.74	0.66	0.74	0.84	1.18	1.01	0.99	1.05	1.00	0.73	0.70	.....	....
U	6.00	4.70	3.66	4.20	4.66	7.34	5.64	6.45	5.85	6.00	4.43	4.30	4.21	3.9
(H, Z)	256.0	253.6	183.2	192.3	237.0	326.5	274.8	286.3	293.9	352.0	252.0	264.5	242.5	222.
$S. R$	187.4	186.1	135.2	141.0	169.4	225.1	196.6	205.4	210.1	245.6	182.0	192.4	175.7	162.
AS	.....	.....	.....	.....	.....	.....	.....	.....	.....	1619	952	1083	926	73
$[(H, Z), q]$	150.7	138.8	119.4	134.3	155.3	167.4	165.9	170.7	166.4	164.1	151.6	138.8	136.7	141.
$(S. R, q)$	108.9	103.6	88.2	97.0	109.1	126.1	123.0	126.4	125.3	121.3	109.7	104.5	101.5	103.
$[(H, Z), d]$	105.3	114.8	63.8	58.0	81.7	159.1	108.9	115.6	127.5	187.9	100.4	125.7	105.8	80.
$(S. R, d)$	78.5	82.5	47.0	44.0	60.3	99.0	73.6	79.0	84.8	124.3	72.3	87.9	74.2	58.

From Table 1 and Figure 1 result: (1) All quantities have a minimum in 1923, with exception of $[(H, Z), d]$ and $(S. R, d)$, where the minimum occurs in 1924. The next minimum of N falls in 1933, likewise that of $[(H, Z), q]$ and $(S. R, q)$; the minima of the other quantities are later. (2) $[(H, Z), q]$ and $(S. R, q)$ correspond most with N, also as to the maximum in 1928, the values are but slightly divergent from 1926 to 1930. (3) C, (H, Z), $S. R$, $[(H, Z), d]$, and $(S. R, d)$ have their maxima in 1930 and show some secondary maxima, as in 1922, 1926, and 1932. AS has a high value in 1930 and a secondary maximum in 1932. (4) u and U have maxima in 1926; they show an analogous course till 1927 and after 1930 but between these years, the courses are in opposite directions (u is the average for some stations while U is the value for De Bilt).

For intercomparison the values of U for Lerwick, De Bilt, and Val-Joyeux from 1927 and 1932 are given below:

Observatory	1927	1928	1929	1930	1931	1932
Lerwick..........	7.84	7.75	8.36	9.63	5.26	5.52
De Bilt...........	5.64	6.45	5.85	6.00	4.43	4.30
Val-Joyeux........	5.81	6.58	6.22	6.09	4.47	4.19

As may be seen, the changes of U from year to year at stations, lying not far apart, may be opposite. The increase of U from 1927 to 1928 at De Bilt is also to be found at Val-Joyeux, but not at Lerwick; the increase of 1929 to 1930 at De Bilt occurs also at Lerwick, but not at Val-Joyeux.

It appears from the data given that the final mean value of u depends on the stations chosen for its calculation and that small differences not always are to be considered as real. The monthly means given by Bartels[6] for 1929 and 1930 are the same as the values given by Duvall[7]

[6]Terr. Mag., 37, p. 9 (1932).

[7]Terr. Mag., 36, p. 311 (1931).

in his article "Magnetic activity, some results of the measure adopted at Stockholm" and have been averaged for Watheroo, Honolulu, and Tucson; the u-values for 1931 and 1932 have been computed for De Bilt, Honolulu, San Juan, and partly Tucson[8].

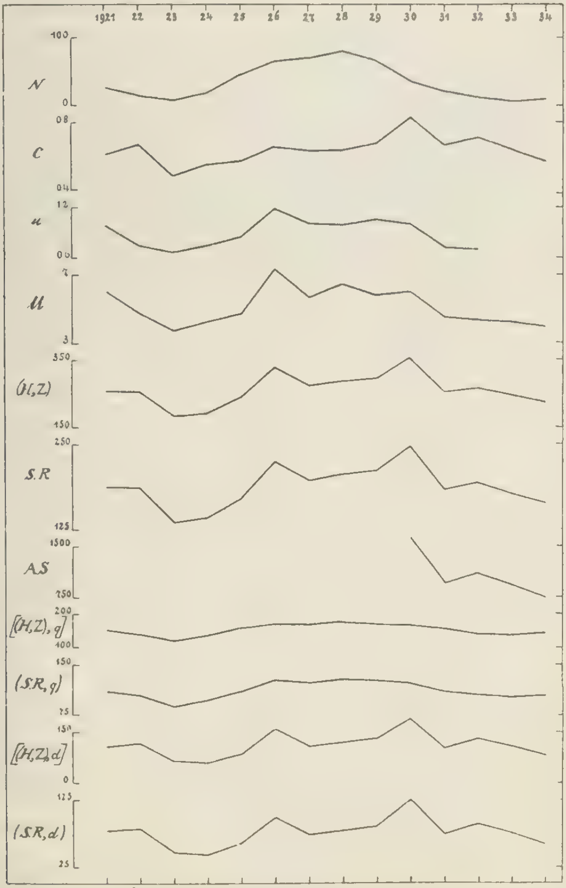


FIG. 1—MEASURES OF ACTIVITY, 1921–1934

$(HR_H+ZR_Z)/10{,}000$.—On the whole C is in good agreement with the activity-measures (H, Z), $S. R$, $[(H, Z), d]$, and $(S. R, d)$ and with AS, as far as this quantity is available, especially with respect to the maxima in 1930 and 1932, which are missing in the u-measure.

Bartels, in communicating his suspicion, that C is too high in 1930, refers to the paper of Duvall, mentioned above. In this paper annual means of (H, Z) at Watheroo and Huancayo are given for 1929 and 1930.

[8] Terr. Mag., 39, p. 1 (1934).

They are respectively: Watheroo, 431 and 496; Huancayo, 583 and 558. When no other data of (H, Z) are available, the conclusion from both stations together may be that there is no outstanding change.

The values for De Bilt are respectively, 293.9 and 352.0, thus showing considerable increase in 1930. It is of interest to consider also the annual means of (H, Z) for other observatories in 1929, 1930, and following years. For 1930 to 1934 they could be calculated from the data in the publication "Caractère magnétique numérique des jours"; as to 1929, they could be computed for some observatories from the values of the absolute ranges of the magnetic elements, to be found in yearbooks and annual reports.

Table 2 gives the annual means of (H, Z) for the observatories having contributed complete data to "Caractère magnétique numérique des jours" during 1930 to 1934, augmented with values for 1929 at Lerwick, De Bilt, Abinger, Tortosa (Ebro), Kakioka, Bombay, Huancayo, and Watheroo, for 1930 and 1931 at Kakioka, and for 1933 at Hongkong.

Moreover there have been inserted in Table 2 values of $(HR_H/10,000)$ for 1930 and 1931 at Meanook, for 1929 to 1934 at Stonyhurst, and for

TABLE 2—*Values of measure* $(HR_H+ZR_Z)/10,000$, *1929-1934*

Station	Latitude	Longitude	1929	1930	1931	1932	1933	1934
	° ′	° ′						
Abisko	68 21 N	18 49 E	...	2483	1610	1839	1653	....
Sodankylä	67 22 N	26 39 E	...	2119	1342	1539	1409	1161
Lerwick	60 08 N	1 11 W	632	1063	589	634	563	465
Lovö (Stockholm)	59 21 N	17 50 E	...	650	373	399	355	299
Sitka	57 03 N	135 20 W	...	1648	826	882	756	584
Copenhagen (Rude Skov)	55 51 N	12 27 E	...	533	342	374	331	294
Eskdalemuir	55 19 N	3 12 W	439−	556−	353−	327	300	261
Meanook	54 37 N	113 21 W	...	553+	286+	829	1240	1049
Stonyhurst	53 51 N	2 28 W	127+	153+	102+	108+	100+	91+
Swider	52 07 N	21 15 E	...	339	254	260	239	221
De Bilt	52 06 N	5 11 E	294	352	252	265	242	222
Abinger	51 11 N	0 23 W	311	364	263	278	260	231
Val-Joyeux	48 49 N	2 01 E	...	276	202	200	189	177
Vienna (Auhof)	48 13 N	16 14 E	...	...	...	213	200	197
Agincourt	43 47 N	79 16 W	...	585	245	267	187	228
Tortosa ("Ebro")	40 49 N	0 30 E	261	298	218	241	217	209
Cheltenham	38 44 N	76 51 W	...	408	262	268	250	216
Kakioka	36 14 N	140 11 E	268	259	227	228	207	203
Tucson	32 15 N	110 50 W	...	267	217	217	213	203
Lukiapang (Zô-Sè)	31 06 N	121 11 E	...	285	236	...	197	204
Helwan	29 52 N	31 21 E	...	...	210	200	207	199
Hongkong (Au Tau)	22 27 N	114 03 E	...	...	...	...	220	222
Honolulu	21 19 N	158 04 W	...	212	183	176	171	171
Bombay	18 54 N	72 49 E	323	317	269	259	256	251
San Juan	18 23 N	66 07 W	...	203	176	168	169	159
Antipolo	14 36 N	121 10 E	...	358	291	269	279	274
Kuyper	6 02 S	106 44 E	...	...	...	393	406	336
Huancayo	12 03 S	75 20 W	583	558	447	387	375	381
La Quiaca	22 06 S	65 36 W	...	259	208	204	205	...
Watheroo	30 19 S	115 53 E	431	496	359	350	357	315
Pilar	31 41 S	63 51 W	...	198	156	155	153	148
Cape Town	33 57 S	18 28 E	...	...	...	...	...	191
Toolangi	37 32 S	145 28 E	...	348	...	...	...	...
Christchurch	43 32 S	172 37 E	150+	192+				...

1929 and 1930 at Christchurch; data of R_Z at these stations were not at our disposal. The values of $(HR_H/10{,}000)$ have been marked with $^+$. Finally values of $[(XR_X+YR_Y+ZR_Z)/10{,}000]$ at Eskdalemuir have been inserted for 1929 to 1931, marked with $^-$. It is to be expected that the relative change of these quantities from year to year will not differ much from the corresponding change of (H, Z).

The year 1930—It appears from Table 2, where the observatories are ranged from north to south, that (H, Z) in 1930 was larger than in 1929 from Lerwick to Tortosa; the difference amounts to 68 per cent for Lerwick and to 14 per cent for Tortosa. At Kakioka, Bombay, and Huancayo (H, Z) is smaller in 1930 than in 1929, being respectively, 3, 2, and 4 per cent; further to the south, at Watheroo, (H, Z) is larger in 1930 by 15 per cent and at Christchurch $(HR_H/10{,}000)$ is larger in 1930 by 28 per cent. Apparently the explanation is, that, although 1930 was more disturbed than 1929 (also according to the character-figures of Kakioka and Bombay, Kakioka having in 1929 and 1930 respectively, 185 and 285 as sums of characters and Bombay 235 and 275; at Huancayo the sums were 145 and 138) the decrease of the ranges of the normal daily oscillation [see Table 1, $[(H, Z), q]$ and $(S. R, q)$] surpassed the increase of ranges, issuing from the disturbances, which are superimposed on the normal oscillation, so that (H, Z) for these stations turned out to be smaller in 1930 than in 1929.

The respective average values of R_H and R_Z of the "five calm days of the month" in 1929 and 1930 were 46.5 and 30.7 and 35.4 and 23.1 gammas at Kakioka and 53.7 and 41.2 and 49.4 and 35.7 gammas at Bombay. The annual means of $[(H, Z), q]$ in 1929 and 1930 were 245 and 185 at Kakioka and 273 and 248 at Bombay; the decrease of $[(H, Z), q]$ is much larger than the decrease of the corresponding quantity relating to all days of the year. For stations at higher latitudes the increase of ranges, issuing from the disturbances surpasses the decrease of the normal daily oscillation, so that here (H, Z) turns out to be larger in 1930 than in 1929; the difference increases with growing distance from the equator.

The year 1932—The year 1932 shows the same character as 1930, but to a feebler degree. According to Table 1 and Figure 1 the values of C, (H, Z), $S. R$, AS, $[(H, Z), d]$, and $(S. R, d)$ are higher in 1932 than in 1931; the other quantities are smaller. It is seem from Table 2 that from 1931 to 1932 (H, Z) increased from Abisko to Kakioka, with amounts of 14 per cent (Abisko, Sodankylä) to below one per cent; only Val-Joyeux shows a decrease of one per cent. Lerwick has an increase of 8 per cent. At Tucson the values of 1931 and 1932 are equal and at the following stations 1932 is lower—for the most, some few per cent—but at Huancayo 13 per cent. The increase noted in 1930, when going further south, does not occur in 1932; Watheroo has still a decrease of 2.5 per cent and Pilar of below one per cent. Data for Christchurch were not available.

The annual mean $[(H, Z), q]$ at Bombay was 221 in 1931 and 207 in 1932, the decrease is somewhat larger than for (H, Z). At Kakioka $[(H, Z), q]$ was 197 in 1931 and 183 in 1932, a decrease of 14, whereas (H, Z) increased by 1.

From 1932 to 1934 all lines of Figure 1 are going down (except N,

[(H, Z), q], and $(S. R, q)$, which show a slight rise after 1933); likewise the values of (H, Z) in Table 2 are, on the whole, decreasing.

The years 1926 and 1930—The measure u has a maximum in 1926 and C in 1930. The graphs of the magnetic character in the annual reviews of 1926 and 1930 clearly show the different character of the two years. The year 1926, especially the last eight months, is characterised by many quiet or feebly disturbed days, separated by many, as it were, isolated disturbances, in the form of towers; the assigned number of 0's (on the average 172 per station) surpasses the number of 1's (148 per station). The number of 2's is rather small, 44 per station; the annual mean character-number is 0.650, not very much outstanding with respect to the neighbouring years.

The graph of 1930, especially in the first ten months, has a more massive basis, above which the disturbances rise; the number of 1's assigned (171 per station) considerably surpasses the number of 0's (129 per station), while the number of 2's is 65 per station. The annual mean character-number, 0.826, is high and is a prominent maximum.

The disturbances in 1930 had but slight postperturbations, the highest value of the day-to-day change of the horizontal intensity at De Bilt was 44 gammas, consequently the annual mean U was rather low, 6.00, only a little higher than in the preceding year. On the contrary in 1926 the postperturbations were more violent, above the maximum of 44 gammas in 1930; there were changes of 101, 90, 68, 66, 65, and 53 gammas, which explains the high value of U in 1926, namely, 7.34, the maximum during the years 1921 to 1934.

The months of 1930—April has the highest mean character of the months of 1930, namely 1.04; this is even the highest monthly mean of the whole series 1906 to 1934. In another way it appears that April 1930 is the least quiet month of the same series; the mean character of the five calm days of April 1930 is 0.41, the highest average during the period 1906 to 1934; it proves that even the quietest days of April 1930 were still rather disturbed (character 0.2 to 0.6).

The graph of the magnetic character of 1930 shows April as a massive chain of mountains with various peaks and some shallow valleys; 46 stations assigned together the character-figure 1 a total of 756 times, the figure 2 a total of 340 times, and the figure 0 a total of 248 times. The monthly mean resulting from these is 1.04. Table 3 gives different means for the months of 1930, namely, N, C, and u, according to Bartels, U and (H, Z) for De Bilt, [(H, Z), m], and AS. [(H, Z), m] is the average (H, Z) of 24 stations, 19 of the Northern and 5 of the Southern Hemisphere, computed from data in "Caractère magnétique numérique des jours." AS is the absolute storminess of the horizontal intensity for Tromsö. At Tromsö some months of 1930 are incomplete, the monthly means have been computed from the complete days. From the magnetic character of the incomplete days it may be concluded, that, if the records had been complete on these days, the monthly means for January, April, May, August, October, and November would have been higher, for June and July lower.

It appears from Table 3 that April has the highest value, not only for C, but also for (H, Z), [(H, Z), m], and AS; then comes May. u and U are rather small in April the highest value of the day-to-day change of the

horizontal intensity in April was only 15 gammas. C, (H, Z), $[(H, Z), m]$, and AS show analogous courses during the months of 1930; u and U are rather similar (even though the maximum of u occurs in September and the maximum of U in May), but show but a slight conformity with the quantities mentioned before; the course of N is independent of the other quantities.

TABLE 3—*Measures of activity, January to December 1930*

Measure	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.	Year
N	65.3	49.2	35.0	38.2	36.8	28.8	21.9	24.9	32.1	34.4	35.6	25.8	35.7
C	0.69	0.89	0.90	1.04	0.93	0.87	0.87	0.88	0.85	0.88	0.61	0.54	0.826
u	0.79	0.82	0.88	0.84	1.13	1.01	0.70	0.77	1.43	1.31	1.01	1.25	1.00
U	4.6	6.8	5.1	4.9	9.3	6.0	5.1	4.4	6.4	7.6	5.7	5.9	6.00
(H, Z)	231	317	360	476	467	419	361	392	366	367	258	213	352
$[(H, Z), m]$	439	578	614	830	771	703	580	667	665	654	419	389	609
AS	858	1499	1897	2499	2277	2177	1900	1594	1853	1536	770	713	1619

Stability of the character-number.—In comparing the changes from year to year of the international character-number C during the years 1929 to 1934—up-down-up-down-down—(see Table 1) with the changes of the annual sums of the character-figures assigned by the individual stations, it is seen that out of 37 observatories, which have contributed during all these years, 17 follow the same course, namely, Lerwick, Sitka, Copenhagen (Rude Skov), Eskdalemuir, Stonyhurst, Swider, De Bilt, Bochum, Abinger, Val-Joyeux, Tortosa (Ebro), Cheltenham, San Miguel, Bombay, Watheroo, Pilar, and Toolangi; 20 stations do not agree, owing mostly to deviation in one of the years. This points to a satisfying stability of the characterisation.

The international character-number C for the years 1929 to 1934 has been derived from the data of 42, 46, 44, 44, 48, and 47 stations, respectively; 34 stations have contributed for all these years. When the annual means are computed from these 34 stations, they are found to be successively beginning with 1929: 0.694, 0.857, 0.658, 0.691, 0.632, and 0.558; the values of C from Table 1 for the corresponding years are: 0.672, 0.826, 0.663, 0.704, 0.638, and 0.566.

The differences are small, sometimes below one per cent; they remain below four per cent and are surely not larger than other measures of activity would give, when making use of different stations. There is a solid nucleus of contributing observatories, guaranteeing stability of the mean character-number; the average is but slightly affected by discontinuation of one or some contributing stations or affiliation of one or some new ones.

The stability of the character-number may be shown in still another way, namely, by comparison with other measures of magnetic activity.

In Table 1, C has the same value or nearly so in 1922, 1929, and 1931, likewise in 1925 and 1934. In the same years equal or almost equal values will be found among the other measures (H, Z), $S. R.$, $[(H, Z), d]$ and $(S. R, d)$. A still closer agreement between these measures is not to be expected on account of the different manner in which they are affected by variations of the magnetic activity.

C and u—Bartels found the following linear relation[9] between the annual means of C and u_1 in the years 1906 to 1930

$$C=0.00392u_1+0.421$$

u_1 is a quantity derived from u for use in certain statistical investigations. u ranges between 0.38 and 2.97 among the monthly means given in Table 2 of Bartels' article just cited. According to the relations chosen between u and u_1 (*l. c.*, p. 16) the ratio (u_1/u) increases within these limits from 21 for $u=0.38$ to 66 for $u=1.1$ to 1.3, then decreases to 45 for $u=3$. From the monthly means of u monthly means of u_1 have been derived and from the latter values annual means have been computed. For the annual means (u_1/u) ranges in the years 1906 to 1930 from 37 (in 1913) to 64 (in 1918, 1919, 1926, 1929) and in the years 1921 to 1934 from 55 (in 1923 and 1934, years of minimum disturbance) to 64 (in 1926 and 1929, years of maximum disturbance).

In computing C (the computed value will be indicated by C') from the above relation for the years 1929 to 1934 (the values of u_1 of 1929 to 1932 according to Bartels,[10] those of 1933 and 1934 according to the observations at De Bilt, $k=1.75$), there is found successively beginning with 1929: $C'=0.684$, 0.668, 0.586, 0.578, 0.586, and 0.570, while $C=$ 0.672, 0.826, 0.663, 0.704, 0.638, and 0.566.

As may be seen, the harmony between u_1 and C is restored for the year 1934. The differences of C' and C between 1929 and 1934 are not to be ascribed to changes in the characterisation of the observatories, but, on account of the close agreement between the international character-number C and the other measures of magnetic activity (H, Z), $S. R$, $[(H, Z), d]$, $(S. R, d)$, and AS, to the u-measure, which failed to bring to light the increase of the magnetic activity in 1930 and 1932.

When considering the special character of the u-measure, that only takes into account the day-to-day change of the daily mean of the horizontal intensity, it is not astonishing that this measure sometimes fails.

The comparison of C with other measures of magnetic activity has shown, that the supposed break in the homogeneity of the international character-number about 1930 does not exist and that the standard of characterisation was stable in the investigated period 1921 to 1934, so that the international character-number can be used as a measure of magnetic activity, not only for intervals of weeks and months, but also for intervals of years.

One may feel grateful to Professor Bartels for his investigations, which have, from different quarters, thrown new light upon the problem of magnetic activity and have given rise to this paper, which, it is hoped, may be of use in discussions of the most adequate measures of terrestrial-magnetic activity.

[9]Terr. Mag., **37**, p. 33 (1932).
[10]Terr. Mag., **37**, p. 15 (1932), and **39**, p. 1 (1934).

De Bilt, August 1935

REMARKS ON DOCTOR VAN DIJK'S PAPER

By J. Bartels

G. van Dijk presents evidence that the discrepancy between C and u (or u_1), especially for the year 1930, may not be due to a break in the homogeneity of C, and stresses the usefulness of the international character-figure C as a measure of magnetic activity not only for intervals of days and weeks (which is undisputed) but also for months and years. Some remarks on the latter point may be permitted, because it now seems quite feasible to ascribe most of the discrepancy found for 1930 not so much to inherent faults or failures of C and u_1, but to the different conceptions of magnetic activity underlying both measures.

(1) April 1930 has the highest average monthly character-figure, $C=1.04$, of the whole series from 1906 to 1934. In the lists of "Principal magnetic storms" published in this Journal, however, all observatories, except Sitka, reported "no storm" for this month. On the other hand, May 1921, famous for two of the most intensive magnetic storms ever recorded, has only $C=0.83$. The u_1-measure gives 52 for April 1930, and 132 for May 1921 (the second highest value for the series 1872 to 1934). This means that the mean monthly C fails to indicate the conspicuous absence of storms in April 1930 as compared with the outstanding storms in May 1921, while u_1 represents these conditions satisfactorily.

(2) April 1930 had only four days with $C \leqq 0.4$, while May 1921 had eleven such days. The measure u_1 fails to indicate the conspicuously low number of quiet days in April 1930, while C expresses this feature.

(3) This indicates the following solution of the discrepancy: According to C, April 1930 appears as a month of very high activity, namely, with very *few quiet days*; according to u_1, the same month appears as a month of low activity, namely, with *no magnetic storm*.

(4) Example: Month A may have five very quiet days with $C=0.0$, and 25 moderately-disturbed days with $C=1.0$; month B may have 15 days with $C=0.0$, five days with $C=1.0$, and ten highly-disturbed days with $C=2.0$. Both months have the same average, $C=0.83$, and may seem equally disturbed. Two extreme and contrasting opinions can be held, however: Either, month A may be judged more disturbed than B, because of the smaller number of quiet days in A; or, vice versa, month B may be judged more disturbed than A, because of the great number of highly-disturbed days in B. Month A is similar to April 1930; month B is similar to May 1921.

Which of these opinions is more reasonable, depends on the circumstances. Imagine two geophysical phenomena P and Q, both depending on terrestrial-magnetic activity, but with different sensitivity so that, in P, the highest possible effect of activity is already reached when $C \geqq 1.0$, while Q is only affected when $C > 1.5$. Then, as far as P is concerned, month A would appear more disturbed than month B, while, for Q, month A is quiet, month B is disturbed. Such cases may arise in radio work on the ionosphere.

(5) The characteristic difference between u (or u_1) and C is one of gradation. The increase of daily magnetic activity from $C=1.7$ (the

highest daily figure in April 1930) to 2.0, judged by u, is much greater than the increase from $C=0.0$ to 0.3, but both increases would affect the monthly mean C alike. W. van Bemmelen[1] concluded that both character-figures C and interdiurnal variability u of horizontal intensity are necessary for a complete description of activity-conditions, but that, on comparison, u is a sharper and more objective measure. In his case, u varied somewhat like the square of his character-figure. If, for instance, $(C-1.0)^2$ is computed for each day on which $C \geqq 1.0$ in the months April 1930 and May 1921 (in order to increase the influence of highly-disturbed individual days on the monthly average), the average value of that quantity is higher for May 1921 than for April 1930, just as indicated by u_1.

(6) Numerous other distributions of weights assigned to the daily values of C [as in section (5)] or to the individual changes of horizontal intensity from one day to the next could be tried for the combination of the daily values of activity to monthly means. However, if activity is conceived as "storminess" and not as "absence of quiet times," u seems to be better than C because it emphasizes the effect of storms. A similar procedure is used in sunspot-numbers, where the sunspot-groups get weight 10, and individual spots only weight 1. And if the correlation-coefficients between magnetic activity (character-figures C or u_1-measure) and sunspot-numbers, N, may be invoked for an impartial decision, the series of annual means 1906 to 1930 gave $+0.82$ between u_1 and N, but only $+0.57$ between C and N; moreover, the correlation between the two terrestrial-activity measures u_1 and C was only $+0.72$, smaller than that between the terrestrial measure u_1 and the solar measure N!

(7) The aversion against "measuring" as compared with "estimating" character-figures, expressed in section (2) of my paper of 1932, is based on Ad. Schmidt's reasons given when he sponsored the introduction of the international character-figure, and repeated in Meteorologische Zeitschrift (volume 33, pages 481-492, 1916). The substitution of range-measurement for estimation appears now even more questionable, owing to the great variability of the amplitudes of the quiet-day diurnal variations, which has been discussed in several papers by N. A. F. Moos, S. Chapman, J. M. Stagg, and myself.

(8) Having made use of more than 10,000 daily international character-figures C arranged in rows of 27 days, I need hardly repeat that none of my remarks is meant to depreciate their great significance for geophysical work.

[1]Met. Zs., **42**, 143-147 (1935).

Department of Terrestrial Magnetism,
Carnegie Institution of Washington,
Washington, D. C.

THE MAGNETIC CHARACTER OF THE YEAR 1934 AND THE NUMERICAL MAGNETIC CHARACTER OF DAYS 1934

BY G. VAN DIJK

The annual review of the "Caractère magnétique de chaque jour" for 1934 has been drawn up in the same manner as for preceding years. Fifty-two observatories contributed to the quarterly tables, forty-seven of them sent complete data.

Table II (reproduced as Table 1 below) of the annual review, contains the mean character of each day for each month. The lists of calm

TABLE 1—*Mean magnetic character-numbers for each day of 1934 from data supplied by 47 magnetic observatories*

Dates	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1934															
January	1.6	1.2	0.3	0.1	0.0	0.1	0.2	0.3	0.2	0.3	0.3	0.1	0.1	1.0	0.8
February	0.1	0.7	0.4	1.1	0.7	0.2	0.3	0.7	1.7	1.3	0.7	0.6	0.5	0.1	1.0
March	0.4	1.2	0.7	1.5	1.7	1.2	1.4	0.5	0.7	1.0	1.0	0.2	0.2	0.1	0.8
April	1.4	0.9	0.6	1.3	1.4	1.1	0.6	0.3	0.2	0.1	0.1	0.2	0.1	0.2	0.5
May	0.2	1.3	1.3	0.7	0.5	0.3	0.2	0.1	0.3	0.6	1.4	1.3	0.6	0.1	0.1
June	0.1	0.0	0.1	0.6	1.4	1.3	0.4	0.8	0.6	0.1	0.9	1.2	0.2	0.7	0.8
July	0.6	0.2	1.5	0.8	0.7	0.4	0.2	0.2	0.5	0.0	0.5	0.2	0.3	0.6	0.7
August	1.0	0.7	1.3	1.1	0.7	0.6	0.5	0.4	0.1	0.3	0.6	1.0	0.9	0.7	0.4
September	0.9	1.3	1.1	0.8	0.1	0.4	0.5	0.7	0.0	0.1	0.6	0.6	0.0	0.1	0.1
October	0.5	0.4	0.2	0.5	0.6	0.6	0.7	0.0	0.0	0.0	0.0	0.7	0.8	0.3	1.3
November	0.0	0.1	0.5	0.2	0.7	0.1	1.7	1.4	0.6	0.3	0.3	0.2	0.6	0.4	0.1
December	1.1	0.8	1.2	1.8	1.0	0.2	0.8	0.5	0.1	0.1	0.8	0.3	0.3	0.6	0.8

Dates	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	Means
1934																	
January	0.6	0.3	1.0	0.4	0.3	0.5	0.9	0.9	0.7	0.4	0.6	0.2	0.7	0.9	0.7	0.7	0.52
February	1.7	1.3	1.1	0.4	0.5	0.3	0.5	0.1	0.8	0.7	0.2	0.3	0.5				0.65
March	0.8	0.5	0.8	0.3	0.1	0.3	1.1	1.0	0.8	1.3	0.6	0.5	0.8	0.9	0.6	1.5	0.76
April	1.0	0.1	0.0	0.5	0.6	0.3	0.6	0.1	0.4	0.3	0.2	0.1	0.2	0.0	0.2		0.45
May	0.0	0.1	1.7	0.9	0.4	0.7	0.8	0.4	0.5	0.7	0.2	0.1	0.0	0.1	0.2	0.1	0.51
June	0.5	0.3	0.9	0.2	0.3	0.0	0.0	0.2	0.1	0.1	0.0	0.8	0.4	0.2	0.1		0.44
July	0.7	0.2	0.0	0.0	0.1	0.1	0.0	0.0	0.4	0.3	0.1	0.1	0.1	0.8	1.8	1.1	0.43
August	0.3	0.8	0.7	0.8	0.1	0.5	0.9	0.5	0.0	0.2	1.1	1.2	1.3	1.3	1.0	0.7	0.68
September	0.9	1.1	0.2	0.9	0.8	0.5	0.9	0.1	1.5	1.8	0.9	1.3	0.8	0.6	1.1		0.68
October	0.3	0.6	0.4	0.0	0.9	0.6	0.3	0.2	1.7	1.6	1.2	0.6	0.6	0.0	0.1	0.1	0.50
November	0.1	0.3	0.5	0.4	0.0	0.0	0.0	0.0	1.2	0.8	0.2	0.1	0.7	0.5	0.0		0.39
December	0.0	0.0	0.6	0.5	0.4	1.1	0.5	0.0	1.1	1.0	0.7	0.1	0.2	1.8	1.5	0.9	0.66

[1]Terr. Mag., **33**, 203 (1928); **34**, 207 (1929); **35**, 178 (1930); **36**, 255 (1931); **37**, 259 (1932); **38**, 301-302 (1933); **39**, 237-238 (1934).

days and disturbed days, and the days recommended for reproduction are also reprinted here as Table 2.

In the introduction a note has been inserted concerning the publication "Caractère magnétique numérique des jours." Volumes X to XIII have been published along with the tables of "Caractère magnétique de chaque jour"; they contain data of 1934 and belated data of 1933. Thirty observatories have sent lists for 1934; all of them were complete.

TABLE 2—*Dates of five magnetically calm and five disturbed days with mean character-numbers during 1934*

Month		Calm days					Disturbed days				
1934											
January	(0.10)	**5**,	**6**,	**9**,	**12**,	**13**	**1** (1.6),	**2** (1.2),	**14** (1.0),	**18** (1.0),	**29** (0.9)
February	(0.15)	**1**,	**6**,	**14**,	**23**,	**26**	**9** (1.7),	**10** (1.3),	**16** (1.7),	**17** (1.3),	**18** (1.1)
March	(0.16)	**12**,	**13**,	**14**,	**19**,	**20**	**4** (1.5),	**5** (1.7),	**7** (1.4),	**25** (1.3),	**31** (1.5)
April	(0.06)	**13**,	**17**,	**18**,	**23**,	**29**	**1** (1.4),	**4** (1.3),	**5** (1.4),	**6** (1.1),	**16** (1.0)
May	(0.04)	**14**,	**15**,	**16**,	**27**,	**28**	**2** (1.3),	**3** (1.3),	**11** (1.4),	**12** (1.3),	**18** (1.7)
June	(0.03)	**2**,	**3**,	**21**,	**22**,	**26**	**5** (1.4),	**6** (1.3),	**11** (0.9),	**12** (1.2),	**18** (0.9)
July	(0.04)	**10**,	**18**,	**19**,	**22**,	**23**	**3** (1.5),	**4** (0.8),	**29** (0.8),	**30** (1.8),	**31** (1.1)
August	(0.12)	**9**,	**10**,	**20**,	**24**,	**25**	**3** (1.3),	**4** (1.1),	**27** (1.2),	**28** (1.3),	**29** (1.3)
September	(0.06)	**9**,	**10**,	**13**,	**14**,	**23**	**2** (1.3),	**24** (1.5),	**25** (1.8),	**27** (1.3),	**30** (1.1)
October	(0.02)	**8**,	**9**,	**10**,	**19**,	**29**	**15** (1.3),	**20** (0.9),	**24** (1.7),	**25** (1.6),	**26** (1.2)
November	(0.02)	**1**,	**20**,	**21**,	**22**,	**30**	**7** (1.7),	**8** (1.4),	**24** (1.2),	**25** (0.8),	**28** (0.7)
December	(0.06)	**9**,	**16**,	**17**,	**23**,	**27**	**3** (1.2),	**4** (1.8),	**24** (1.1),	**29** (1.8),	**30** (1.5)

DAYS RECOMMENDED FOR REPRODUCTION

*February 9 July **3**, July 30, September 25, December **4**, December 29.

KONINKLIJK NEDERLANDSCH METEOROLOGISCH INSTITUUT,
De Bilt, Utrecht, Holland

SUMMARY OF THE YEAR'S WORK, DEPARTMENT OF TERRESTRIAL MAGNETISM, CARNEGIE INSTITUTION OF WASHINGTON

By J. A. Fleming

The activities of the Department of Terrestrial Magnetism during the report-year July 1, 1934, to June 30, 1935, may be grouped under the following general divisions: (1) Reduction and study of accumulated observational data; (2) further development of technique and apparatus to record photographically the electrical conditions and variations in the ionosphere, and their investigation and correlation with other geophysical phenomena; (3) continued theoretical formulation and experimental investigation, in the laboratory, of basic aspects of nuclear physics and magnetism; (4) maintenance of observational program in the field to make for continuity of secular-variation data and of recording at observatories of the Department the seasonal, diurnal, and irregular changes in the Earth's magnetic and electric fields.

As in past years, every opportunity has been taken to encourage and to cooperate with organizations everywhere engaged in geophysical research similar to that of the Department. This is in accord with the growing efforts toward world-wide coordination sponsored by various international scientific bodies and congresses. Such coordination with studied and planned unification of methods and aims in attack is peculiarly necessary for advance in terrestrial magnetism and electricity.

The results obtained and progress made in the investigational and experimental work are briefly summarized below.

Terrestrial magnetism. The experimental evidence available in many branches of geophysics and cosmical physics differs from the readings taken in laboratory-research in so far as, in the laboratory, the observations are taken under fixed conditions or involve only a limited number of variable quantities, while the records obtained at geophysical laboratories show variations in time which are not easily expressed as the effect of a few variables. The analysis of such time-functions, and of their interrelations is, therefore, an indispensable part of geophysical research. This holds in particular for the more or less pronounced fluctuations variously termed cycles, periodicities, or recurrences found in various geophysical and cosmical phenomena.

Every attempt to find cycles has to consider the mathematical fact that any time-function f(t) which is given, in a finite interval of time, by a finite number of values—for instance, hourly or daily values—can be represented as the superposition of a finite series of cyclic functions, such as sine-waves, and for one and the same given function f(t) an unlimited number of such series can be found. If, therefore, a representation of f(t) by means of a series of superposed cyclic functions has been effected, this does not at all entitle one to regard the individual cycles as physically significant constituents of f(t). To prove the physical significance of any cycle, it is in most cases sufficient to demonstrate that it recurs frequently enough, and this consideration of frequency leads, in some way or other, to statistical methods and to the theory of probability.

In a paper on "Random fluctuations, persistence, and quasi-persistence in geophysical and cosmical periodicities" (Terr. Mag., **40**, 1-60, 1935), this fundamental statistical aspect has been considered.

A day-by-day record of terrestrial magnetism and solar activity in the form of a diagram demonstrating the 27-day recurrences in the 11-year cycle 1923 to 1933 was prepared (Terr. Mag., **39**, 201-202, 1934). The complete change in the character of the solar diurnal-variations in vertical intensity observed at the Huancayo Magnetic Observatory during the past sunspot-cycle was investigated. This change is associated with the secular change of the Earth's general magnetic field which consisted partly of a southern shift of the magnetic equator in the region around Huancayo. Detailed features of this change are found to be qualitatively in agreement with the expected changes resulting from such a movement of the magnetic equator. The large magnitude of the changes indicates, however, that the solar diurnal-variation is a complex function of the Earth's general field, such as is called for by the atmospheric-dynamo theory of Balfour Stewart.

Good progress was made on the potential analysis of the magnetic diurnal-variations in the Western Hemisphere using the data from the Huancayo Magnetic Observatory and from the Agincourt, Cheltenham, Vieques, and Pilar observatories for the five international quiet days of the equinoctial months of 1923, employing essentially Schuster's method of analysis and using geomagnetic instead of geographic coordinates.

Work on the lunar magnetic variations at Huancayo, declination 1922-32, and horizontal intensity 1927-32, has been done. In view of the large amplitudes of the solar diurnal-variations (S) in horizontal magnetic intensity at the Huancayo Magnetic Observatory, a study of the lunar diurnal-variations (L) in horizontal intensity also seemed rather promising. The results of the discussion for the six years 1927-32 have fulfilled these expectations to a surprising degree. In southern summer (means of the months November to February), on the average for all days with international character-figure not higher than 1.1, the main term—the 12-hourly lunar wave (L_2)—has a harmonic amplitude of 9γ ($1\gamma = 0.00001$ c.g.s. unit), which is more than one-third of the solar 12-hourly wave (S_2), namely, 25γ. Thus the combined solar and lunar 12-hourly waves vary in amplitude from 34γ when they are in phase to 16γ when they have opposite phases.

This lunar wave is not only in absolute size, but also in relative magnitude compared with the solar wave, the largest so far found for any observatory. It lends itself, therefore, well for studies of the change of L_2 with magnetic activity, sunspot-numbers, etc. For a station so near the equator, the radically different seasonal changes of S and L are significant. While S remains about the same throughout the year, L decreases in the ratio 10:1 from December to June! This fact is a new argument for Chapman's assumption that S and L originate in different layers of the ionosphere and will prove of value when the direct ionospheric research by wireless methods, now in progress at the Huancayo and Watheroo magnetic observatories, is combined with the magnetic results.

A discussion of the data from Little America (magnetic latitude 70° south) for 1929-30, Point Barrow (magnetic latitude 69° north) for 1932-33, and Chesterfield (magnetic latitude 74° north) for 1932-33, showed that for any one station, greatest magnetic and auroral activities

occur at approximately the same time during the night. At all three stations greatest auroral disturbance appeared to occur near magnetic latitude 70°. Greatest magnetic disturbance has been shown to occur also near this latitude. Diurnal variation in magnetic activity was found to be of a double-maximum type with great seasonal variation for Chesterfield; but for Little America, at the same magnetic latitude, it was of a single-night-maximum type. Data from Point Barrow showed a single maximum at night. The data at both Point Barrow and Chesterfield conform in this respect with the results of Stagg's investigation, but Little America data show the "outer"-zone instead of "transition"-zone type. The boundary between these zones is approximately 70° magnetic latitude. The time of the night maximum in diurnal magnetic-activity at these stations appears to be more closely related to geographic than to magnetic latitude.

The auroral and magnetic observations at Little America during June, July, and August 1929, were subjected to statistical analysis and correlation-coefficients were determined. From a comparison of about 1500 pairs of magnetic declination and auroral intensity the following conclusions were reached: (1) The greatest activity in each phenomenon, on the basis of hourly ranges, occurs about 9^h to about 19^h; (2) the times of maximum and of minimum values of the two elements are in good agreement; (3) on the basis of averages, the declination values for times when auroræ were present are greater than those when no auroræ were observed; (4) a more or less loose, in some cases fairly high, correlation is shown.

The data of the Byrd Antarctic Expedition for 1934, a low sunspot-year, may throw light on the apparent non-agreement of magnetic diurnal-variation data for 1929, a high sunspot-year. The results of these studies emphasize the importance of further data from stations near magnetic latitude 70°. The type of diurnal variation in earth-potential activity appears to undergo a marked change similar to the change in diurnal magnetic-activity as the boundary of the transition-zone, magnetic latitude 70°, is crossed. The region of magnetic latitude 70° appears to be that of maximum auroral and possibly earth-potential activity as well as of magnetic disturbance.

An improved design of perminvar vertical-intensity variometer has been constructed which is expected to give more consistent performance over long periods of time, owing to the fact that the moving parts are suspended by quartz and do not rest on knife-edges or pivot-points. Work also has been done on improved methods of using the sine-galvanometer and on the development of electromagnetic instruments.

Terrestrial electricity Further discussion of the atmospheric-electric data from College-Fairbanks Polar-Year station, has yielded interesting information regarding the annual variation in potential gradient and conductivity in polar regions and its possible dependence on other geophysical phenomena. Studies of factors controlling the conductivity of the lower atmosphere have been made, particularly of the small-ion content which depends upon the rates of production and of destruction of small ions. Other investigations during the year have added materially to the available information regarding the intermediate ion of the atmosphere.

The analysis of large-ion data taken simultaneously with Owens

dust-count data by the United States Weather Bureau generally shows a positive correlation between the two elements. During times of high dust-count due to dust-storms in the west, the large-ion content failed to show a corresponding increase. These results were taken to mean that the Owens dust-particles are in general composed of smoke particles, the number of actual dust-particles being more or less negligible. The large-ion content of the air also is affected by the amount of smoke in the air. The dust-particles, while increasing the count with the Owens instrument, for some reason—possibly their large size—do not contribute appreciably to the large-ion content.

Measurements of small-ion production at Washington indicate the existence of a regular diurnal-variation in the production-rate similar in form to the inverse of the daily temperature. The decreased rate of production towards the middle of the day is interpreted as due to a diminution of the radium-emanation content in the lower region of the atmosphere through scattering by turbulence.

The device constructed at the Department for registering air-conductivity in the stratosphere, primarily for use on the flights of the National Geographic Society and Army Air Corps, was further improved. After thorough tests in the laboratory the apparatus was used satisfactorily on two trial balloon-flights to altitudes of about 26,000 and 27,000 feet. The registrations showed a somewhat irregular increase of conductivity, both positive and negative, as the balloon ascended, the values at the top being in both flights about 20 times the values at the surface. Similar trends were shown on the descents. To supplement the measurements of air-conductivity to be made with apparatus fixed in the gondola of the Balloon *Explorer II*, it seemed desirable to obtain similar measurements at the ground-station. A suitable place for such measurements was found near the rim of the "Strato-bowl" where the apparatus was operated for 34 days, 32 complete days of air-potential and 30 of conductivity being obtained.

The series of earth-current records (practically unbroken) now extend for Watheroo over eleven calendar years (to end of 1934) for the north component and eight years for the east component (the first three years of east-component records at the station not being counted because they contain an appreciable spurious diurnal-variation component). For Huancayo the series extends over eight years for both components, the first three years being not as complete as desirable for our purposes. For Tucson the series covers three years.

Final reduction of the Tucson records for the three-year period 1932-34 has been completed and a condensed summary of the results including diurnal-variation data by months for all days recorded and for ten calm days per month prepared for publication (Terr. Mag., **40**, 183-192, 1935). Chief interest in these results lies in the seasonal variation in earth-current flow at this station, which is situated in the "zone of transition" where the diurnal variations of magnetic dip and intensity change from high-latitude to equatorial type. The preliminary findings with respect to seasonal changes have been confirmed by the final records based on the three calendar years reported.

The changes observed in the flow of the earth-currents cannot well be associated with variations in the conductivity of the region or with structural irregularities. Hence it appears that they must be associated

with variations in the inducing phenomenon. This conclusion is strengthened by comparison with the diurnal-variation curves of the magnetic elements. Comparing the northward earth-current curve with the first derivative of the curve for Y, the eastward component of the horizontal magnetic field, and examining the eastward earth-current curve in conjunction with that for X, the northward magnetic force, there is found a decided parallelism in the manner in which the two pairs of curves change from month to month, which extends even to the anomalies noted in the January and March records. This would appear to be an important addition to the evidence linking the two phenomena together, and a day-by-day comparison of the two sets of records suggests itself as a promising study to throw more light on the true relationship between them.

Interest in the method and equipment developed in the Department for measuring the resistivity of undisturbed earth continued to be manifested. Some of the problems in which this method was considered as a possible aid were: Surveys of ground-water; study of the intrusion of salt water into fresh-water wells; location and survey of gravel-beds; surveys for gaining knowledge of geological structure required in engineering projects such as road-building and dam-construction.

Ionosphere-investigations -The first ionospheric data from the South Temperate Zone have become available with the commencement of ionospheric investigations at the Watheroo Magnetic Observatory in January 1935. They represent a notable advance toward the objective of a world-wide determination in the ion-distribution in the upper atmosphere. Results already bear upon the obscure causes of the variation of ion-density in the highest ionospheric region. These are of particular importance because this region is directly exposed to nearly the whole range of solar radiation and, therefore, should give a sensitive measure of solar changes.

Advances in technique of measurement have been accomplished with the commencement of continuous automatic single-frequency recording of ionospheric changes at the Huancayo Magnetic Observatory. Installation of the recording apparatus with the associated power- and control-equipment completes the first step in the program for complete recording of ionospheric details at the observatories. A similar equipment for the Watheroo Magnetic Observatory is well advanced.

Polarization-experiments at the Huancayo Magnetic Observatory show the doubly refracted components from the ionosphere to the plane polarized. This evidence verifies theoretical predictions and establishes Huancayo as a unique location for additional special experiments relating to the terrestrial-magnetic field because of its position on the magnetic equator.

Development of automatic, wide-range, multifrequency equipment has been well advanced. Simplifications on the basis of experiment make possible a rugged and reliable final design.

A study of the combined data from the Huancayo and Watheroo magnetic observatories together with data from the Northern Hemisphere has made possible a complete delineation of the general features of the separation of the upper ionospheric-region at small zenith-angles of the Sun.

Nuclear-physics research -The general objective of the various re-

searches carried out in the Department's experimental laboratory has been the formulation and investigation of basic aspects of magnetism through studies in fundamental physics. During this report-year, two contributions from the Department's laboratory have been of particular interest, one presenting a study of various resonance-transmutations by protons and the other giving new data on the carbon-reactions, relating to the correction of the mass-values of the lighter atomic nuclei.

Probably the most important development of the year in the experimental work was the integration of the Department's nine years of high-voltage experience into the formulation of a comprehensive technical plan for equipment and procedure to encompass the whole range of nuclear investigations in the region below cosmic-ray energies. This plan is the direct result of an effort on the part of the Director to assess the present and future positions o. this phase of the Department's activities in relation to other developments. A critical examination of all known possibilities in respect to the technique of nuclear-physics investigations showed that the Department's work has now provided a sound basis for a full-scale development which eliminates the necessity for technical compromise. Major features of the plan are a high-voltage generator and vacuum-tube installed within a large spherical pressure-tank and insulated by compressed air. The Department's present equipment reaches a maximum of slightly more than one million volts, which is just enough to make a beginning in studies of nuclear interactions. The importance of, and the ultimate necessity for, extending the investigations over a wide voltage-range cannot be questioned. The proposed pressure-sphere generator and associated high-voltage tube are designed to operate at steady and controllable potentials of either sign higher than ten million volts above ground, and to provide ample currents for X-ray studies and for other technical requirements. The characteristics to be desired and the quantitative requirements to be met, together with the highly important matters of flexibility and adaptability to meet the varied demands of a whole array of different applications, are aspects of the design based on the Department's direct experience. The working characteristics and behavior of such an equipment, as well as the magnitude of the attainable voltage, may be considered as already established by reason of the Department's experience with each of the various technical factors.

A project similarly concerned with the future growth of knowledge concerning magnetic and electric interactions on the smallest scale and hence at the highest energies—the subject-matter of nuclear physics—and in a similar way by the development of new technique, is the cooperative effort, now in progress, of the Department's staff with Professor J. W. Beams and Dr. L. B. Snoddy of the University of Virginia. Its aim is to explore the possibilities of the "wave-front" method for producing very high energy-particles, proposed and already partly tested by Beams and Snoddy. Among the twelve distinct methods for high-voltage (high-energy) attack which have been suggested and more or less tested during the past eight years this method alone gives any promise of being ultimately capable of extension into the actual region of cosmic-ray energies above 20-million volts. This cooperative project should determine its potentialities and the rôle the method might play in a comprehensive plan of attack covering the whole range of nuclear energies.

Magnetic survey—The activities of the Section of Land Magnetic Survey have been directed largely, as in past years, toward studies pertaining to secular changes in the Earth's magnetic field. Adverse economic conditions and decreased personnel continue to necessitate curtailment of field-operations and relatively few data have been collected through work of one Department observer in Africa and through cooperative arrangements in China, in Australia, and in Antarctica. The stations occupied were in large part selected from the network of selected magnetic repeat-stations for investigations of secular variation recommended by the International Association of Terrestrial Magnetism and Electricity. The total number of localities occupied during the year was 81, distributed as follows: Africa (southern, eastern, and northern sections) 27, Alaska 1, Australia 9, Central America 4, China 16, Easter Island 1, Galapagos 1, Antarctica 21, and New Zealand 1.

Observatory work—The programs of work in terrestrial magnetism, atmospheric electricity, earth-currents, and meteorology at the Watheroo and Huancayo observatories, and the cooperative work in atmospheric electricity and terrestrial magnetism with the Apia Observatory (Department of Scientific and Industrial Research of New Zealand), and in atmospheric electricity and earth-currents at the Tucson Observatory of the United States Coast and Geodetic Survey were maintained.

At the Watheroo Magnetic Observatory the la Cour rapid running and Eschenhagen magnetographs and the Crichton-Mitchell vertical-intensity inductometer were in operation throughout the year. At the beginning of the report-year the ionospheric equipment was under test. By the end of January 1935 the equipment had been thoroughly tested and calibrated, and since then determinations of layer-heights and critical frequencies have been made biweekly in accordance with the regular schedule. Preliminary reductions of the observational data were made and forwarded to Washington at regular intervals. The use of the higher-powered transmitter of the ionospheric equipment for communication has enabled direct two-way contact to be made with numerous amateur stations in the United States and several reliable channels for the prompt transmission of reports and data were developed. Spectrohelioscopic observations were made daily.

At the Huancayo Magnetic Observatory the la Cour quick-run and Eschenhagen magnetographs were in operation throughout the year. Aluminized cœlostat mirrors for the spectrohelioscope received from Mount Wilson Observatory were installed in November 1934. After a preliminary experimental period, regular observations of solar prominences and spots were begun in March 1935. Of the seismological equipment, the two Wenner horizontal-component seismometers were operated throughout the year. All important disturbances were reported promptly by radio. The automatic ionospheric recording-equipment was received in April 1935. The first record with the photographically recording ionospheric equipment was obtained May 13, 1935. The unique location of the Observatory on the geomagnetic equator, permits separate multifrequency recording of each of the two doubly refracted components. This greatly enhances the utility of these records. Cosmic-ray observations were temporarily discontinued September 1, 1934, pending the delivery late in 1935 of the precision cosmic-ray recording meter as the Observatory is to be one of a world-wide net of stations planned by the Institution's Committee on Coordination of Cosmic-Ray Investigations

The magnetic data from Watheroo for 1933 were completely reduced and the manuscript covering that year has been added to those for 1919 to 1932, which are awaiting publication. Final mean hourly values of the three elements were completed for the Huancayo Magnetic Observatory for 1922 to 1932.

C. I. W. sine-galvanometer No. 1 was installed at the Cheltenham Magnetic Observatory and is to be used as the Observatory's standard horizontal-intensity instrument. In January 1935 the first of the seven precision cosmic-ray recording meters was installed at Cheltenham in accordance with the plan of the Institution's Committee on Coordination of Cosmic-Ray Investigations.

The preparation for compilation and discussion of the magnetograms obtained at Little America by the Second Byrd Antarctic Expedition, which lack only a fortnight of a full year from February 1933 to February 1934, is under way.

Arrangements were made with President Bunnell of the University of Alaska, for the resumption of the important ionospheric studies at College, Alaska.

Oceanographical reductions—The manuscript for the extensive report on, and discussion of, the marine bottom-samples obtained during the last cruise of the *Carnegie* was completed by Roger Revelle at the Scripps Institution of Oceanography; this report completes the manuscripts awaiting publication on physical and chemical oceanography. Good progress is being made on the various reports on the biological materials collected during the cruise.

Miscellaneous—As in the past the policy of the Department of cooperating with other investigators and organizations engaged in similar work was maintained and extended.

An event of outstanding importance to geophysical science was the decision in March 1935 of the British Admiralty to construct a nonmagnetic vessel—to be named *Research*—to continue the magnetic surveys at sea by the *Carnegie*. Upon the request of the British Admiralty, plans and specifications of the *Carnegie* and designs of instruments, as evolved during the many years of the Department's oceanic work, were supplied. Arrangements were completed that William J. Peters, who was for many years in command of the *Carnegie*, proceed to England as a consultant on the design and construction of the new vessel and of her instrumental equipment.

Department of Terrestrial Magnetism,
Carnegie Institution of Washington,
Washington, D. C.

MITTEILUNG ÜBER ERDMAGNETISCHE UND ELEKTRISCHE ARBEITEN IN DER U. d. S. S. R. IN DEN JAHREN 1931-35

Von N. W. Puschkow

Der Umfang der erdmagnetischen und elektrischen Arbeiten in der Sowjetunion wächst von Jahr zu Jahr. Diese Arbeiten werden jetzt von einer grossen Anzahl verschiedener Organisationen geleitet und durchgeführt: Geomagnetischen, hydrographischen, geologischen Instituten, Universitäten, und Hochschulen.

Das Netz der magnetischen Observatorien -Die Sowjetunion verfügt zur Zeit über 18 magnetische Observatorien, von denen sechs sich in der Polarzone befinden. Wir halten diese Zahl aber für ungenügend: es wird beabsichtigt in der nächsten Zeit noch fünf bis sechs Observatorien in der Arktis und im asiatischen Teil der Union zu stiften. Nebst dieser Ausbreitung des Netzes wird auch beabsichtigt seine Ausrüstung bis zu einem Niveau zu heben, das unseren Observatorien gestatten wird eine allseitige Erforschung der magnetischen Erscheinungen mit allen zur Zeit zugänglichen Mitteln zu führen. Die Polarobservatorien werden mit je drei Serien von Magnetographen versehen, inklusive Magnetographe grosser Geschwindigkeit. Auch elektrische Messungsmethoden werden eingeführt.

Die Durchführung dieses Programms wird durch drei folgende Faktoren gesichert: (1) Die Staatsanweisungen für geomagnetische und elektrische Arbeiten wachsen von Jahr zu Jahr; (2) wir organisieren unseren eigenen Betrieb zur Herstellung von magnetischer Apparatur; (3) Die Vorbereitung der wissenschaftlichen Mitarbeiter in den Hochschulen (Studium und Aspirantur) ist bei uns sehr breit angelegt.

Magnetische Generalaufnahme der U. d. S. S. R. -Die dazugehörigen Arbeiten sind im Jahre 1931 angefangen. Die durchgehende Aufnahme des ganzen riesigen Territoriums ist bereits vollendet und es bleiben für die nächsten Jahre bloss Reiseroute-Aufnahmen in schwer zugänglichen Gegenden vorbehalten. Ende dieses Jahres werden die absoluten Bestimmungen aller Elemente—*D*, *H*, und *I* an etwa 13,000 Stationen vollendet sein. Ausserdem wird in den Intervallen zwischen den Stationen eine grosse Anzahl von Bestimmungen der *Z*-Komponente mittels Lokalvariometer durchgeführt. In diesem Jahre nehmen 54 Feldabteilungen an den Aufnahmearbeiten teil. Es werden absolute magnetische Bestimmungen auf 2370 Stationen durchgeführt und die *Z*-Komponente an 10,000 Punkten bestimmt.

Die Erforschung

Sekulärer Gang -Die Gesamtzahl der Beobachtungspunkte des sekulären Ganges ist etwa 300; die Beobachtungen an diesen Punkten werden durchschnittlich alle drei Jahre wieder aufgenommen. Derartige Bestimmungen werden in diesem Jahr von acht speziellen Feldabteilungen durchgeführt. Man ist zur Erforschung der sekulären Variationen im Gebiet der Kursker Magnetischen Anomalie geschritten.

Die Erforschung täglicher Variationen und magnetischer Störungen—Zu den Untersuchungen in diesem Gebiet ist man erst in den letzten zwei Jahren geschritten. Es ist der tägliche, jährliche, und sekuläre Gang der magnetischen Pulsationen geklärt worden. Es wird der tägliche Gang der erdmagnetischen Aktivität untersucht und die Klärung der mittleren Eigentümlichkeiten magnetischer Stürme nach der Methode von Müller-Birkeland gesucht. Im Arbeitsprogramm steht die Erforschung der spezifischen Typen der magnetischen Störungen im Polargebiet und der Einwirkung des Mondes auf die tägliche Variation der magnetischen Elemente. In diesem Jahre soll die Bearbeitung der Materialien des Polarjahres von fünf Stationen vollendet werden.

Das Arktis-Institut hat die magnetischen Beobachtungen der Russisch-Schwedischen Spitzbergen-Expedition herausgegeben.

Methodisch-experimentelle Untersuchungen und Apparatenbau (*Konstruktion*) –In den letzten Jahren wurde darauf Rücksicht genommen, die Methoden der magnetischen Beobachtungen zu vervollkommnen und neue Typen von Apparaten zu bauen. Das Zentral-Observatorium führte eine Reihe von Arbeiten aus, welche den Zweck verfolgten, die Koeffizienten p und q von Magneten zu bestimmen nach der Methode von Schmidt, mittels seines Magnettheodoliten. Es wurde dabei festgestellt, dass diese Methode keine genügende Genauigkeit zur Bestimmung der Koeffizienten leistet, welcher Umstand uns Anlass gab, ellipsoidale Magnete in die Messungspraxis einzuführen.

Herrn Dr. Janowsky ist es gelungen ein neues System von Magnetographen zu konstruieren und einen entsprechenden Apparat zu bauen. Die Eigentümlichkeit des Systems besteht darin, dass hier zum Unterschied von allen zur Zeit verwendbaren Systemen, die Registrierung der Z-Komponente auf dem alten Lamont'schen Prinzip gegründet ist, bloss wird statt Eisen Permaloy verwendet. Das System ist sehr portativ und für Feldarbeiten gut geeignet. Die Details sind aus den Abbildungen ersichtlich.

Ausserdem wurde ein elektrisches Z-Magnetometer für unmittelbare Bestimmungen der Z-Komponente gebaut. Es ist auch eine Methode zur Bestimmung der Z-Komponente mittels eines Magnetrons ausgearbeitet und bei Feldarbeiten geprüft worden.

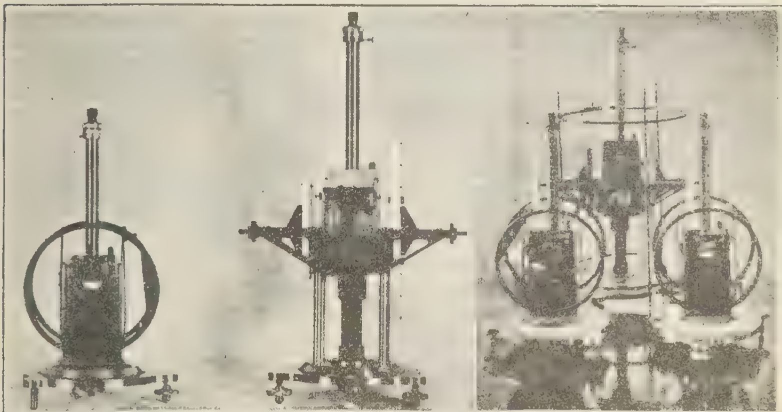
Es wurden 38 neue Magnettheodolite vom Universaltypus für Feldarbeiten eigener Konstruktion des Zentralinstituts für Erdmagnetismus angefertigt. Die Bestimmung der Deklination geht hier auf die gewöhnliche Weise vor sich, die Bestimmung der Horizontalkomponente aber nach der elektrischen Methode und der Methode der Ablenkungen unter Anwendung von Magneten aus Nickelaluminiumstahl. Die Bestimmung von I wird nach der Lamont'schen Methode unter Anwendung von Magneten aus Dynamostahl durchgeführt.

Die Lamont'sche Methode der Bestimmung von I gibt eine Genauigkeit, welche jener des Induktionsinklinators nicht nachsteht, bei etwa10 Minuten Zeitaufwand pro Beobachtung. Der Theodolit kann ausserdem als Z-Lokalvariometer verwendet werden. Genauere Auskunft über das ganze Verfahren geben die Abbildungen.

Mathematische Untersuchungen -Vor ungefähr drei Jahren wurde am Zentralinstitut für Erdmagnetismus eine theoretische Abteilung gegründet, dessen Aufgabe die Ausarbeitung von genauen und Näherungsmethoden für die Berechnung der Magnetfelde von Körpern gegebener

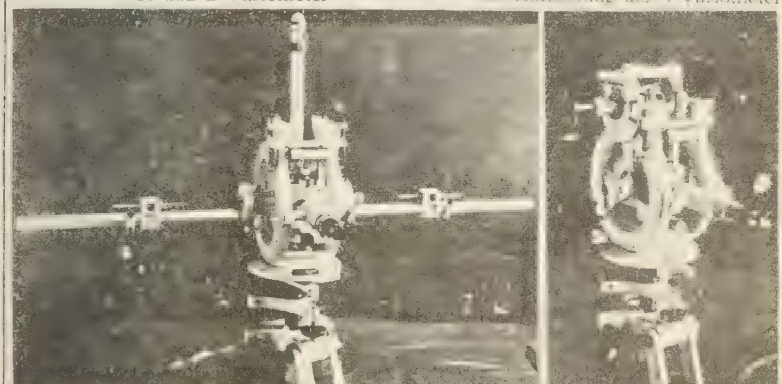
Form ist. Zur Zeit sind etwa 20 Untersuchungen vollendet, von denen ein Teil im ersten Band der Arbeiten des Instituts veröffentlicht wurde.

Luftelektrizität—Systematische Messungen der luftelektrischen Elemente werden in neun Beobachtungspunkten der Union, von denen drei sich im Polargebiet befinden, geführt. In den nächsten Jahren soll das



H und Z Variometer

Die Aufstellung der 3 Variometer



Magnetischer Reisetheodolit für Bestimmungen von D. H und Z

Stationsnetz und das Programm der Arbeit eine wesentliche Erweiterung erfahren.

In letzter Zeit ist man bestrebt eine Abhängigkeit zwischen dem elektrischen Zustand der Atmosphäre und den Luftmassen verschiedenen Ursprungs aufzufinden. Es ist festgestellt, dass es eine scharf ausgeprägte Abhängigkeit zwischen dem Gehalt der Radiumemanation und dem Luftmassentypus gibt. Am grössten ist die Radioaktivität für *c P L*[1], am geringsten für *m A L*[1]. Ausserdem fand man, dass die Luft polaren Ursprungs durch die grösste, dagegen die *m A L* durch die

[1] =continentale Polarluft; maritime Atlantikluft.

geringste Leitfähigkeit charakterisiert wird. Es ist ausserdem die Konstruktion eines neuen Ionenzählers vom leichtübertragbaren Typus und die Herstellung einer speziellen Apparatur für luftelektrische Messungen in der Stratosphäre zu erwähnen.

Die Ausbreitung der Radiowellen und die Ionosphäre—In den letzten Jahren werden in Slutzk und auf dem Franz-Joseph-Land systematische Beobachtungen der Radiowellenausbreitung geführt. Man untersucht die Abhängigkeit zwischen der Intensität des Radioempfanges und dem Zustand des erdmagnetischen Feldes. In Leningrad und Murmansk werden von Zeit zu Zeit Höhenbestimmungen der Kennelly-Heaviside Schicht unternommen. . Im Arbeitsprogramm steht die Bestimmung der Ionenkonzentration.

Erdströme—Die systematischen Beobachtungen der Erdströme wurden bis jetzt bloss in einem Punkt durchgeführt—auf der Kola Halbinsel, wo zu diesem Zweck zwei Telegraphenlinien benutzt wurden. Die Analyse der erhaltenen Materialien weist auf die Notwendigkeit einer vollkommenen Organisation der Erforschung dieser Erscheinungen hin. Es ist demnach ein Projekt ausgearbeitet, nach dem drei spezielle Stationen organisiert werden sollen, von denen die eine übertragbar und für kurze Serien von Beobachtungen verwendet werden soll.

Die Erforschungen der magnetischen und elektrischen Erscheinungen in der Sowjetarktis—In diesem Jahre ist im Arktis-Institut ein Sektor des Erdmagnetismus und der Elektrizität gegründet. Seine Hauptaufgabe ist die mehrseitige Erforschung des ganzen Komplexes der erdmagnetischen und elektrischen Erscheinungen in der Polarzone der Sowjetunion.

Das Arbeitsprogramm des Sektors schliesst ein: Untersuchung der erdmagnetischen und elektrischen Erscheinungen, der Erdströme, der Höhenstrahlung (an der Erdoberfläche und in der Stratosphäre), der Polarlichterscheinungen (photographische Höhenbestimmung und spektroskopische Forschungen), der Ionosphäre mittels der Radiowellenmethode (Untersuchung der Radiowellenausbreitung, Bestimmung der Höhen und Ionenkonzentration). Der Sektor verfügt über sechs permanente magnetische Polarstationen, zwei Beobachtungspunkte für Luftelektrizität und einen Beobachtungspunkt für Radiowellenausbreitung.

In diesem Winter wurden in der Montsche-Tundra zum ersten Male spektrophotometrische Untersuchungen des Polarlichts durchgeführt. In den nächsten Jahren soll ein nichtmagnetisches Schiff gebaut werden, speziell für Polarseefahrten geeignet.

Es ist das Projekt der komplexen Erforschung der Gesamtheit aller erdmagnetischen und elektrischen Erscheinungen auf dem Franz-Joseph-Land zur Vollendung gebracht.

Verschiedenes—Es werden jedes Jahr All-Union Kongresse der Forscher im Gebiete des Erdmagnetismus und der Elektrizität zusammengerufen.

Es ist die erste Nummer des "Informations-Archiv des Zentral-Observatoriums" erschienen—eine Vierteljahrschrift für Erdmagnetismus und Elektrizität (in russischer Sprache).

Das Zentralinstitut für Erdmagnetismus gibt aus: "Geomagnetisches und elektrisches Bulletin" (im Druck befindet sich No. 20); "Bulletin

für generale magnetische Aufnahme" (im Druck No. 2), "Arbeiten des Zentralinstituts für Erdmagnetismus" (im Druck No. 2).

Verzeichnis von Arbeiten, welche bereits veröffentlicht sind (kein volles)

(1) Allik, R. A., Zur Schätzung einiger Abweichungen bei der Messung elektrischer Leitfähigkeit der Luft nach Scherins Verfahren. Mitteilung des Geophysikalischen Zentral-Observatoriums, No. 2-3, 1934.

(2) Arkhangelsky, B. F., Leouschin, und Pabo, Beobachtung der Spannung des Feldes der Radiostationen in Slutzk. Technisch-Wiss. Sammlungen d. Volkskommissariats d. Post u. Telegraphie, Bd. I, 1931.

(3) ...., Materialien zum Studium der Ausbreitung der elektromagnetischen Wellen vom weitsprechenden Diapason. Arbeiten d. Energetischen Komitees.

(4) Arkhangelsky, B. F., und Leouschin, Einige Resultate des Studiums über die Ausbreitung der Radiowellen vom weitsprechenden Diapason während der Tageszeit. Technisch-Wiss. Sammlungen d. Elektro-Technischen Instituts der Kommunikation in Leningrad. No. 2-3, 1933.

(5) Gussev, P. J., Polarlicht und die oberen Schichten der Atmosphäre. Arbeiten d. Stratosphärenkonferenz, Leningrad, 1935.

(6) ...., On the new index of solar activity. Circular No. 10, 1934, Observatoire Poulkovo USSR.

(7) Janowsky, B. M., Ein Apparat zur Bestimmung des Induktions- und des Temperaturkoeffizienten permanenter Magnete und zur Messung der magnetischen Momente. Nachrichten des Geophysikalischen Zentral-Observatoriums, No 2-3, 1933.

(8) ...., Helmholtz' Spirale, als Etalon der Spannung des Magnetfeldes und des magnetischen Flusses. *Ebendaselbst*, No. 3, 1932.

(9) ...., Der Magnetismus und die Ausnutzung der Beobachtungsmaterialien magnetischer Observatorien. Arbeiten d. Stratosphärenkonferenz, Leningrad, 1935.

(10) Leouschin, N. I., Quelques résultats des enregistrements des atmosphériques à Sloutzk. Arbeiten der Konferenz der Französischen Association in Chambery, 1933.

(11) ...., Resultate der Registrierung der atmosphärischen Störungen von 1931 bis 1932 in Slutzk. Technisch-Wiss. Samml. d. Elektro-Techn. Instituts der Kommunikation, No. 6, Leningrad.

(12) ...., Ueber den stratosphärischen Ursprung der atmosphärischen Störungen. Tech.-Wiss. Samml. d. Elektro-Techn. Instituts der Kommunikation, Leningrad, 1935.

(13) ...., Gewitter als Quelle der atmosphärischen Störungen. Tech.-Wiss. Samml. d. Elektro-Techn. Instituts der Kommunikation in Leningrad, No. 7.

(14) ...., Zur Vervollkommnung des Elektrographs von Benndorf. Mitteilungen d. Geophys. Zentral-Observatoriums, No. 1, 1934.

(15) Malkin, N., Ueber die Veränderung der Elemente des Erdmagnetismus mit der Hohe (Näherungslösung des Derichlet-Problems für den Fall eines Halbraumes). Arbeiten d. Zentral-Observatoriums, Bd. 1, No. 3, 1934.

(16) ...., Ueber die Anwendung des Problems von Neumann auf die Untersuchung des Magnetfeldes über anomalen Gebieten. *Ebendaselbst.*

(17) ...., Ueber die Magnetisierung eines rechtwinkligen Prisma im homogenen Felde. *Ebendaselbst.*

(18) Puschkow, N. W., Die oberen Schichten der Atmosphäre und der Erdmagnetismus. Arbeiten d. Stratosphärenkonferenz, Leningrad, 1935.

(19) ...., Die neusten Theorien des Erdmagnetismus. Meteor. Westnik, No. 3-4, 1935.

(20) Rose, N. V., Allgemeine Magnetvermessung der USSR. Verhandlungen der VII. Tagung d. Baltischen Geodätischen Kommission, 1935.

(21) ...., N. N. Trubjatschensky, und B. M. Janowsky, Erdmagnetismus und magnetometrisches Verfahren bei der Aufsuchung von Erzlagerstätten, I. Teil, 1934.

(22) ...., und N. N. Trubjatschensky, Kurze Anweisung zur Durchführung magnetischer Aufnahmen, 2te Auflage, 1931.

(23) ...., Observations variométriques de la station à Hornsund. Libr. Glavsevmorput, Leningrad, 1935.

(24) Schlutschanowsky, A. S., Das Magnetfeld eines endlichen Kreiszylinders das homogen parallel seiner Achse magnetisiert ist. Arbeiten d. Zentral-Observatoriums, Bd. I, No. 3, 1934.
(25) ....,Ueber die Magnetisierung isotroper und homogener Körper. *Ebendaselbst.*
(26) ...., Ueber die Magnetisierung eines halbunendlichen Kreiszylinders in einem homogenen parallel seiner Achse gerichteten Felde. *Ebendaselbst.*
(27) Stroganow, W., und W. Swetlow, Ueber das ebene Magnetproblem. Arbeiten der Akademie der Wiss. d. U. d. S. S. R., No. 3, 1934.
(28) Trubjatschensky, N. N., Magnetometrisches Verfahren bei der Aufsuchung d. Erzlagerstätten im Nikopoler Rayon. Arbeiten der Vereinigung für angewandte Geologie, No. 166.
(29) ...., Geotektonik und Geomagnetismus. Verhandlungen d. VII. Tagung d. Baltischen Geodätischen Kommission, 1935.
(30) Uspensky, W. A., Die magnetische Generalaufnahme d. U. d. S. S. R. Klima und Wetter, 1934.
(31) Verigo, A. B., Untersuchung der Ausbreitung and Absorption der Höhenstrahlung in der Luft, Wasser, und Stahl. Mitteil. d. Geophysikalischen Zentralobservatoriums, No. 3-4, 1931, Leningrad.
(32) Weinberg, B. P., Preliminary summary of data on the present distribution of magnetic declination within the arctic zone. Terr. Mag., v. 36, 273-276, 1931.
(33) ...., Catalogue of magnetic determinations in USSR and adjacent countries from 1556 up to 1926, II. Leningrad, 217-297, 1932; Geophys. Journal, v. 2, No. 2 (4), 254-260, 1932, Leningrad.
(34) ...., Suggestion concerning field magnetic determinations in Polar Regions. Geophys. Journal, v. 2, No. 2 (4), 251-253, 1932.
(35) ...., Das Kriterium d. genügenden Kleinheit der Entfernung zwischen 2 Beobachtungspunkten bei einer magnetischen Lokalaufnahme. Journal für Geophysik, Bd. 2, No. 2 (4), 251-253, 1932.
(36) ...., Distribution of magnetic elements and of their annual changes. Terr. Mag., v. 37, 241-246, 1932; Geophys. Journal, v. 3, 242-246, 1932.
(37) ...., und Rogatschew, Catalogue of magnetic determinations in USSR and adjacent countries, III. Determinations made from 1926 up to 1932. Leningrad, I-VIII+297-392, 1933.
(38) ...., Catalogue of magnetic determinations in the polar regions. Moscow, 51 pp., 1933.
(39) ...., und M. S. Nodia, Resultate magnetischer Beobachtungen in Grusien und in der asiatischen Türkei in den Jahren 1915-1923. Tiflis, 5 Seiten.
(40) ...., Magnetische Beobachtungen in der Arktis. Natur 22, No. 5-6, 94-98, 1933, Leningrad.
(41) ...., Neues in der Methodik von magnetischen Aufnahmen. Soz. Rekonstruktion und Wissenschaft, No. 3, 99-103, 1934.
(42) ...., Das 4te Jahr der magnetischen Aufnahme in der U. d. S. S. R. *Ebendaselbst* No. 8, 74-77, 1934.
(43) ...., und J. M. Rogatschew, Zur Frage über ein schwimmendes magnetisches Observatorium für die Sowjetarktis. Nachrichten d. Hydrographischen Verwaltung der Polarseefahrt, No. 4.
(44) ...., Bemerkung über den täglichen Gang aperiodischer magnetischer Störungen. Mitteilungen d. Geophysikalischen Zentral-Observatoriums No. 2-3, 38-43, 1934.
(45) ...., Proposition regarding the publishing of the international catalogue of magnetic determinations. Bull. Union Géod. Géoph., Ass. Magn. Electr. Terr., No. 9, 260-263, 1934.
(46) Geophysikalisches Zentral-Observatorium, Bulletin der Magnetischen Generalaufnahme. Heft I, Leningrad 1934.

Verzeichnis der Arbeiten, die sich z. Z. im Druck befinden

(1) Allik, R. A., Ueber den Zusammenhang zwischen den Elementen der Luftelektrizität und Luftmassentypen.
(2) Anossow, S., Das Problem des Magnetfeldes einer Antiklinale.
(3) Arkhangelsky, B. F., Beobachtungen der Feldintensität der Radiostationen und der Störungen.
(4) Janowsky, B. M., und N. N. Kalitine, Untersuchung der Methode des Verteilungskoeffizienten beständiger Magnete mittels des Apparates von Schmidt.

(5) ...., Das Variometer für Registrierung der Vertikalkomponente nach der Lamont'schen Methode.
(6) ....,Neues System der Magnetographe.
(7) Malkin, N., Ueber die Magnetisierung zweier unendlicher nicht konzentrisch orientierter Zylinder in einem ebenen Magnetfelde.
(8) Merkulowa, E. S., Die Radioaktivität der Atmosphäre und die Luftmassen.
(9) Nikologorodsky, A., Das Magnetfeld einer homogenen magnetischen Linse.
(10) Puschkow, N. W., Die Eigentümlichkeit des normal täglichen Ganges im Polarlicht.
(11) Weinberg, B. P., und P. M. Kassatkin, Verzeichnis der magnetischen Anomalien der U. d. S. S. R.
(12) ...., Ueber die Abhängigkeit der optimalen Entfernung zwischen den Beobachtungspunkten bei einer magnetischen Lokalaufnahme von der Genauigkeit der Messungen.
(13) ...., Die Grösse der magnetischen Lokalstörungen in der U. d. S. S. R. und den angrenzenden Ländern.
(14) ...., Die Genauigkeit der Vorausberechnung von Werten magnetischer Elemente.
(15) ...., und J. M. Rogatschew, Isogonenkarte der Sowjetarktis für das Jahr 1935.
(16) ...., Karten der Deklination und Verzeichnis der Deklinationsanomalien von West-Europa.
(17) ...., Zur Frage der Ausscheidung des Normalfeldes.
(18) ...., On the principles of magnetic cartography.

ZENTRALINSTITUT FUER ERDMAGNETISMUS UND ATMOSPHAERISCHE ELEKTRIZITAET,
Slutzk, September 1935

REVIEWS AND ABSTRACTS

SCRASE, F. J.: *Some measurements of the variation of potential gradient with height near the ground at Kew Observatory.* London, Met. Office, Geophys. Mem., No. 67, 1935 (12 with 2 pls.).

According to Poisson's equation, the density of space-charge is proportional to the rate of change with height of the potential gradient. Various investigators, taking advantage of this fact, have determined the value and sign of space-charge near the ground through a determination of the variation of the potential gradient with height. For a similar purpose, the author measured the potential-difference between two collectors at a fixed vertical distance apart (1 meter). The actual height of the midpoint between the collectors was varied over five distances (1, 3, 5, 7, and 10 meters) above the ground. The author concludes, as a result of his measurements, that the space-charge at Kew is very nearly zero except during calm and can be accounted for by an excess of positive over negative small ions. During times of calm a charge of the order of +0.1 electrostatic unit per cubic meter occurs between heights five and ten meters above ground. This is greater than can be accounted for by an excess of positive small ions and it is therefore probable that the number of positive large ions exceeds that of negative large ions sufficiently to account for the space-charge. However, for most purposes, the positve and negative large ions may be considered equally numerous.

The author has computed the value of the space-charge ρ from the equation $\rho=-[(F_1-F_2)/4\pi\ (h_1-h_2)]$ where F_1 and F_2 are the potential gradients at heights h_1 and h_2, respectively.

It is customary to consider the gradient as given by the equation V/h, where V is the potential difference between ground and collector at a height h. In such cases, McNish[1] points out that, assuming a homogeneous distribution, the space-charge ρ is given by the equation

$$\rho=(V_1/h_1-V_2/h_2)/2\pi\ (h_2-h_1)$$

If the values of F_1 and F_2 were assumed to be equal to V_1/h_1 and V_2/h_2, respectively, as is customary, then the value of space-charge so computed, will be only one-half the correct value. The method of securing values of F_1 and F_2 is not specifically stated by the author.

G. R. WAIT

STAGG, J. M.: *Some general characteristics of aurora at Fort Rae, N. W. Canada, 1932-33.* London, British National Committee for the Polar Year 1932-33, 1935, 6 pp. 25 cm.

The visual auroral data at this station comprise an almost continuous series of observations at a place in the zone of maximum auroral frequency. Mr. Stagg's discussion is therefore of particular interest. The description of the general development of displays culminating in outbursts of increased activity associated with color, and followed by a period of degeneration into diffuse masses, is also typical of displays seen at Chesterfield, Northwest Territory.

Distinctive differences are observed between displays at Fort Rae and Chesterfield. At Fort Rae, low average intensity and lack of variety and definition in form were marked features. Bright displays and structured auroræ were relatively more frequent at Chesterfield which is situated well within the maximum auroral frequency-zone. The very high auroral frequency at Fort Rae in a year of low sunspot-activity, as contrasted with the surprisingly low average brightness of displays, led to the suggestion by Mr. Stagg that auroral frequency may not be influenced by the solar cycle, whereas vigor and definition of form of aurora probably are affected by it. It is to be hoped that all Polar-Year visual auroral observations will be published together with a discussion such as that given by Mr. Stagg.

F. T. DAVIES.

[1]Terr. Mag., 37, 439-446 (1932).

DER ZUSTAND DES NETZES DER MAGNETISCHEN OBSERVATORIEN DER U. d. S. S. R. UND WEITERE PERSPEKTIVE SEINER ENTWICKELUNG

VON N. ROSE

Bis 1917 zählte man auf dem Territorium der U. d. S. S. R. nur sechs magnetische Observatorien: Pavlovsk, Ekaterinburg, Sui (Irkutsk), Karsani (Tiflis), Kazan, und Odessa, wovon die letzteren zwei nur einige Jahre und zwar mit Unterbrechung arbeiteten; mit dem Beginn des Krieges 1914 hörte ihre Existenz auf.

Nach der Revolution, besonders in den letzten fünf Jahren, beginnt eine rasche Entwickelung des Observatoriumnetzes und zur jetzigen Zeit zählen wir dreizehn funktionierende Observatorien und ausserdem fünf im Zustande der Uebertragung oder der Reorganisation. Beifolgende Tabelle gibt erschöpfende Auskunft über die Lage der Observatorien und die Dauer ihrer Arbeit.

Bei der Organisation neuer Stationen wurde der nördliche Teil der U. d. S. S. R. besonders berücksichtigt, wo in den letzten Jahren mehrere Observatorien und zwar: auf Matotschkin Schar, Franz-Joseph-Land, Kandalakscha, Insel Dickson, Myss Tscheliuskin, Wellen, und Jakutsk eröffnet worden sind und in diesem Jahre im Rayon des Flusses Werchnjaja Kolyma ein Observatorium organisiert wird.

Im mittleren und südlichen Teile der U. d. S. S. R. sind in dieser Zeit neue Observatorien in Wladiwostok, Taschkent, Kutschino (in der Nähe von Moskau) eröffnet, in Kazan ein Observatorium restoriert, und in Odessa ein solches, das im Jahre 1912 seine Tätigkeit eingestellt hatte, wieder hergestellt worden.

Ausserdem mussten wegen der Elektrifikation der Eisenbahnlinien und wegen der elektrischen Strassenbahn das Observatorium Swerdlowsk (früher Ekaterinburg) gegen 30 km nach Süden und zwar nach der Wyssokaja Dubrawa, und das von Kutschino nach Nishnedewitzk übertragen werden. Im Zustande der Uebertragung, aus denselben Gründen, befinden sich zur Zeit auch die Observatorien in Kandalakscha, Taschkent, und Tiflis.

In der nächsten Zeit beabsichtigt man neue Observatorien im nördlichen Teil an den Flussmündungen, Lena und Kolyma, und in Petropavlovsk auf der Kamtschatka zu eröffnen.

Im europäischen Teil der U. d. S. S. R. werden keine Observatorien projektiert, dagegen gedenkt man solche im asiatischen Teil auf den Gebirgszweigen des Urals, in Kasaksstan, Tomsk, und Enisseisk zu eröffnen.

All die existierenden Observatorien, mit Ausnahme der Makeewka (Donbass), sind mit einer oder mehreren Serien der Deklination und der Apparatur für die D-, I-, und H-Messungen versehen worden. Die Polarobservatorien besitzen je eine Serie des Eschenhagen-Systems, einen Magnettheodolit, und den Induktionsinklinator der Firma Askania Werke. In Kandalakscha, Wellen, und Tscheliuskin sind Registrierapparate von la Cour aufgestellt worden. Mit einer Serie der Registrierapparate sind Kazan (System Edelmann), Wladiwostok, Taschkent, Jakutsk, und Nishnedewitzk (System Eschenhagen) versehen worden. Je zwei oder mehr Serien besitzen Slutzk, Wyssokaja Dubrawa, Sui, Karsani, und Stepanowka.

In Makeewka ist nur ein D-Variometer aufgestellt. In Slutzk sind

absolute Apparate aufgestellt worden und zwar drei von Wild-Freiberg und ein normaler Theodolit von Schmidt. In Wyssokaja Dubrawa, Karsani, und Stepanowka Wild-Freiberg's System, in Kazan, das System der Carnegie Institution und in Irkutsk, Wild-Dehring's System. In den übrigen Observatorien gibt es nur Apparate vom relativen Typus.

Vor dem Weltkriege wurden die Materialien der Observatorien in den Annalen des Zentral-Observatoriums oder in den Nachrichten entsprechender Observatorien veröffentlicht. Mit dem Beginn des Krieges wurde die Veröffentlichung derselben eingestellt und erst im Jahre 1925 wieder aufgenommen und zwar in besonderen Ausgaben unter dem Titel "Geomagnetisches Bulletin." Dem Observatorium Swerdlowsk gelang es aber die Veröffentlichungen ihrer Materialien bis zum Jahre 1917 fortzusetzen. Die Materialien der Observatorien Slutzk, Irkutsk, und Tiflis für die Jahre 1914-25 und solche des Observatoriums Swerdlowsk für die Jahre 1915-19, sind unveröffentlicht geblieben. In vollem Umfange sind bloss die Jahreswerte im "Terrestrial Magnetism" erschienen.

Für die Periode 1925 bis zum heutigen Tage ist es gelungen Materialien folgender Observatorien zu veröffentlichen: Slutzk 1924 bis 1931, Swerdlowsk 1926 und 1929, Irkutsk 1923 und 1929 im "Geomagnetischen Bulletin", Tiflis (Karsani) 1926 und 1927 in Veröffentlichungen des genannten Observatoriums und Taschkent 1928-30 in seinen Ausgaben. Gegenwärtig befinden sich im Druck Materialien des Observatoriums Slutzk für die Jahre 1932, 1933, und 1934 und Swerdlowsk für die Jahre 1930-33.

Die wissenschaftlich-methodischen Arbeiten werden im Geomagnetischen Observatorium in Slutzk, welches als Zentral-Observatorium für die Union gilt und wo der "Magnetische Standard" aufbewahrt wird, ausgeführt.

Von den Arbeiten, die das Observatorium Slutzk in den letzten Jahren ausgeführt hat, sind folgende zu verzeichnen:

(1) Die Untersuchung der Methode des Verteilungskoeffizienten beständiger Magnete mittels des Apparates von Schmidt (wird in den Arbeiten des Observatoriums, Bd. 2, gedruckt).

(2) Das Variometer für die Registrierung der Vertikalkomponente nach der Lamont'schen Methode (befindet sich im Druck in den Arbeiten des Observatoriums, Bd. 2).

(3) Neues System der Magnetographe (im Druck in den Arbeiten des Union-Instituts der Meteorologie und der Standarde, Bd. 8).

(4) Vierundzwanzigstündiger Gang magnetischer Pulsationen nach den Beobachtungen des Observatoriums Slutzk für die Zeitperiode 1923-33 (wird gedruckt in den Arbeiten des Observatoriums, Bd. 2).

(5) Gegenüberstellung der Aktivität des Magnetfeldes nach den verschiedenen Observatorien für die Periode des Polarjahres (wird gedruckt in den Materialien des zweiten Polarjahres).

Zur Zeit werden im Observatorium Slutzk folgende Arbeiten ausgeführt:

(1) Ueber Feststellung der "Magnetischen Standarde" für die U. d. S. S. R. und in Zusammenhang damit wird an der Vervollkommnung der absoluten magnetischen Beobachtungen gearbeitet.

(2) Es wird eine Methode für sofortige Mitteilung über magnetische Stürme ausgearbeitet.

(3) Ueber das Studium des vierundzwanzigstündigen Ganges der Empörung auf den Polarstationen.

Geomagnetische Untersuchungen in der Polarzone der U d. S. S. R.—Der Grund zu den systematischen geomagnetischen Untersuchungen in der Polarzone der U. d. S. S. R. wurde im Jahre 1923 mit der Organisation des beständigen Observatoriums Matotschkin Schar auf der Nowaja Zemlja gelegt. Dieses Observatorium wurde vollkommen mit aller für die absoluten und die Variationsbeobachtungen nötigen Apparatur ausgerüstet und hat während seiner Arbeit wertvolles Material für die Erklärung der Eigentümlichkeiten der geomagnetischen Vorgänge in der unmittelbaren Nähe von der Zone der maximalen Frequenz des Polarlichtes geliefert. Im Jahre 1928 wurde das Observatorium auf der Kandalakscha organisiert. Das für speziellen Zweck der Erforschung des Zusammenhangs der geomagnetischen Empörungen mit den Erdströmen erschaffene Observatorium war bis zum Jahre 1932 nur mit der Variations-Apparatur versehen. Im Jahre 1931 wurde das Observatorium in der Bucht Tichaja, auf der Franz-Josef-Insel eröffnet. Dieses Observatorium mit aller für die absoluten und Variationsbeobachtungen nötigen Apparatur versehen, stellt ein besonderes Interesse dar zur Erforschung der Eigentümlichkeiten der magnetischen Erscheinungen im Norden von der Zone der maximalen Frequenz des Polarlichtes.

Einen bedeutenden Stoss zur weitern Ausdehnung der geomagnetischen Untersuchungen in der Polarzone der U. d. S. S. R. erteilte das zweite Internationale Polarjahr. Die Teilnahme der U. d. S. S. R. an dieser Unternehmung drückte sich aus in der Organisation zweier neuen magnetischen, Observatorien auf der Insel Dickson und Jakutsk zur Ergänzung der Existierenden und in der Kompletierung der Ausrüstung des Observatoriums in Kandalakscha mit der Apparatur für absolute Messungen und einem Magnetograph grosser Geschwindigkeit. Die Bearbeitung der Beobachtungen der Stationen des zweiten Internanationalen Polarjahres naht ihrem Ende und wird im Jahre 1936 in vollem Umfange veröffentlicht werden.

Von den Beobachtungen der dem zweiten Internationalen Polarjahre vorausgehenden Jahre sind vorläufig nur solche des Observatoriums Matotschkin Schar für das Jahr 1934 und die Beobachtungsmaterialien für die Jahre 1899-1900 von Spitzbergen veröffentlicht worden.

Weitere Ausdehnung des Netzes der polarmagnetischen Observatorien erfolgte auch nach dem Abschluss des Internationalen Polarjahres. So wurde im Jahre 1933 das Observatorium in Wellen, im Jahre 1934 ein solches auf Myss Tscheliuskin organiziert; beide Observatorien sind mit der Apparatur für absolute und Variationsbeobachtungen versorgt worden. Für die nächsten Jahre wird die Organisation noch zweier beständigen magnetischen Observatorien im Ost-Sektor der Sowjetarktis projektiert.

Die allgemeine Leitung des existierenden Netzes der beständigen magnetischen Observatorien, mit Ausnahme des Kandalakscha Observatoriums, gehört dem Arktischen Institut der Union in dessen Programm ausser der Forschungen der geomagnetischen Erscheinungen auch das Studium des Polarlichtes, Erforschung der Ionensphäre mittels Radiowellen, atmosphärisch-elektrische Untersuchungen, Erforschung der Erdströme und Untersuchungen der kosmischen Strahlen inbegriffen sind.

Das Verzeichnis der Observationspunkte in der Union d. S. S. R. in denen Arbeiten der atmospherischen Elektrizität geführt werden

[Der Aufbewahrungsort des Materials ist mit Sternschen bezeichnet: (*) = Sektion der Atmosphärischen Elektrizität des Geomagnetischen Observatoriums in Slutzk (Leningrader Gebiet); (**) = An Ort und Stelle; (***) = Arktisches Institut.]

Namen des Punktes und des Ortes	Beginn Schluss der Arbeiten	Arbeitsprogramm
(1) Sektion der atmosphärischen Elektrizität des Magnetischen Observatoriums Slutzk (Leningrader Gebiet) (*)	1916 Beständ.	(1) Gradientsregistrierung des Potentials der positiven und negativen Leitfähigkeit; (2) Beobachtungen volum. Ladung der Luft (1932-34); (3) Beobachtungen über den Gehalt des radioaktiven Stoffes in der Atmosphäre mittels aktivierenden Drahtes und absoluter Methode (1933); (4) Systematische Messungen der Feldkraft der Radiostationen während der Tageszeit des weitsprechenden Diapasons (seit 1933); (5) Registrierung der Feldkraft der Radio-Station des weitsprechenden Diapasons während der Nachtstunden (1930); (6) Registrierung der atmosphärischen Störungen (1930); (7) Arbeiten der Vervollkommnung der Beobachtungsmethoden.
(2) Taschkent Geophysikalisches Observatorium (**)	1925 bis jetzt	(1) Registrierung des Potentialgradienten; (2) Registrierung der Leitfähigkeit (der positiven und negativen); (3) Beobachtungen über atmosphärische Störungen (1935).
(3) Kutschino (Moskau) (**)	1932 bis jetzt	(1) Registrierung der Störungen; (2) Registrierung der positiven und negativen Leitfähigkeit; (3) Beobachtungen über radioaktive Regime der Atmosphäre.
(4) Irkutsk [Ostsibirisches Geophysikalisches Institut (Sui)] (**)	1927-29 1933 bis jetzt	(1) Registrierung der Störungen; (2) Radioaktive Messungen nach der aktivierenden Methode.
(5) Wladiwostok, Observatorium des Fernen Ostens (**)	1930	(1) Feldregistrierungen.
(6) Woronesh (**)	1935	(1) Feldregistrierungen; (2) Registrierungen der Leitfähigkeit.
(7) Poljarnoe (*)	1932-34	(1) Registrierungen des Feldes; (2) Messungen der Leitfähigkeit; (3) Radioaktivitätsmessungen.
(8) Insel Dickson (*)	1932-33 bis jetzt	(1) Radioaktivitätsmessungen; (2) Feldregistrierungen (von 1933-34 ab).
(9) Jakutsk (*)	1933	(1) Feldregistrierungen.
(10) Kandalakscha (*)	1930-32	(1) Feldregistrierungen.
(11) Myss Tscheliuskin (***)	1934-35	(1) Feldregistrierungen; (2) Messungen der Leitfähigkeit; (3) Volumeladung der Nebel; (4) Vertikaler Strom.

Verzeichnis magnetischer Observatorien der Union d. S. S. R. zum 1. September 1935

Namen der Observatorien	Geogr. Koordin. Breite N	Geogr. Koordin. L. nach E Grw.	Arbeitszeit Beginn	Arbeitszeit Schluss	Elemente registr.	Familiename des Chefs
	° ′	° ′				
(1) Buchta Tichaja (Fr.-Josef-Land)	80 20	52 48	1931	....	alle	Nikolskij
(2) Myss Tscheliuskin	77 17	104 17	1934	....	"	Fedorow
(3) Insel Dickson	73 30	80 25	1932	....	"	Noroshnych
(4) Matotschkin Schar (**)	73 16	56 24	1923	1935	"	
(5) Kandalakscha (*)	67 08	32 26	1928	1935	"	Kopylow
(6) Wellen	66 10	190 09	1933	....	"	Miljaew
(7) Jakutsk	62 01	129 43	1932	....	"	Fomenko
(8) Seimtchany (***)	63 51	118 30	1935	....	"	Issajew
(9) Slutzk (Zentral-Observatorium)	59 41	30 29	1878	....	"	Janowsky
(10) Wyssokaja Doubrawa	56 44	61 04	1930	....	"	Abels
(11) Swerdlowsk (gew. Ekaterinburg) (**)	56 30	60 38	1837	1930	"	
(12) Saimischtsche (Kazan)	55 50	48 51	1887	....	"	Puschkin
(13) Kutschino (Moskau) (**)	55 46	37 58	1919	1934	"	
(14) Irkutsk	52 16	104 19	1887	1914	"	
(15) Sui (Irkutsk)	52 28	104 02	1914	....	"	Dobroljubsky
(16) Nishnedewitsk	51 33	38 21	1934	....	"	Gussew
(17) Makeewka	48 01	37 54		....		Ssemiletow
(18) Odessa	46 26	30 46	1896	1910		
(19) Stepanowka (Odessa) (***)	46 47	30 53	....	....	"	Aganin
(20) Maitun (Wladiwostok)	43 15	132 20	1933	....	"	Vakanz
(21) Karsani (Tiflis) (*)	41 50	44 42	1905 (1844)	....	"	Ormozadse
(22) Taschkent	41 20	69 18	1926	1934	"	Michalkow

Bemerkung: Das eine Sternchen (*) bedeutet, dass das Observatorium sich in der Uebertragungsperiode befindet; zwei Sternchen (**), dass das Observatorium geschlossen ist; drei Sternchen (***), dass das Observatorium neu organisiert wird.

Verzeichnis der gedruckten Arbeiten

(1) Resultate der Feldregistrierung in Irkutsk (Leuschin). Met. Westnik, 1929, No. 7, Juli.

(2) Beobachtung der Spannung des Feldes der Radiostationen in Slutzk (Arkhangelsky und Leouschin). Technisch-Wiss. Sammlungen d. Volks-Kommissariats d. Post und d. Telegraphie, No. 2-3, 1930, Bd. 4.

(3) Beobachtung der Spannung des Feldes der Radiostationen in Slutzk (Arkhangelsky, Leouschin) . . . daselbst, Bd. 1, 1931.

(4) Materialien zum Studium der Verbreitung der elektromagnetischen Wellen d. radioweitsprechenden Diapasons. (Arkhangelsky, Leouschin, und Pabo). Arbeiten d. Energetischen Komitees.

(5) Einige Resultate des Studiums über das Passieren der Radiowellen d. weitsprechenden Diapasons während der Tageszeit. (Arkhangelsky und Leouschin). Technisch-wissenschaftliche Samml. d. Elektro-Techn. Instituts der Kommunikation in Leningrad, No 2-3, 1933.

(6) Quelques résultats des enregistrements des atmosphériques à Sloutzk. (Leouschin). Arbeiten der Konferenz der Französischen Association in Chambery, 1933.

(7) Resultate der Registrierung der atmosphärischen Störungen von 1931 bis 1932 in Slutzk (Leouschin). Technisch-wissenschaftl. Samml. d. Elektro-Technischen Instituts der Kommunikation No. 6, Leningrad.
(8) Ueber den stratosphärischen Ursprung der atmosphärischen Störungen (Leouschin). Sammlungen der Konferenz der Erforschung der Stratosphäre an der Akademie der Wissenschaften der U. d. S. S. R.
(9) Gewitter als Quelle der atmosphärischen Störungen (Leouschin). Techn.-Wiss. Samml. d. Elektro-Techn. Instituts der Kommunikation in Leningrad, No. 7.
(10) Artikel über durchdringende Radiation (A. B. Verigo). Mitteilungen des Geophysikalischen Zentral-Observatoriums, 1931-33.
(11) Artikel von W. I. Baranow über die Methodik radioaktiver Messungen.
(12) Zur Schätzung einiger Abweichungen bei der Messung elektrischer Leitfähigkeit der Luft nach dem Schering's Verfahren (Allik). Mitteilungen des Geophysikalischen Zentral-Observatoriums, No. 2-3, 1934.
(13) Zur Vervollkommnung des Elektrographs von Benndorf (Leouschin). Mitteilungen des Geophysikalischen Zentral-Observatoriums, No. 1, 1934.

Zentralinstitut des Erdmagnetismus und der atmosphaerischen Elektrizitaet,
Slutzk, U. S. S. R.

THE WARSAW MEETING OF THE COMMISSION OF TERRESTRIAL MAGNETISM AND ATMOSPHERIC ELECTRICITY SEPTEMBER 1935

By V. Laursen

The meeting opened September 2 but most of the members present, together with several others interested, had met September 1 for which day an excursion to the Swider Magnetic Observatory had been arranged. The Observatory with its installations and fairly complete instrumental equipment made a very favorable impression as did also the hospitable reception accorded the visitors by Professor Kalinowski and his family.

The official meeting was opened by the President, Professor Ch. Maurain, the following countries being represented by 17 members of the Commission, and other interested persons who attended the sessions and took part in the deliberations: Austria, Canada, China, Czechoslovakia, Denmark, East Africa, Finland, France, Germany, Great Britain, Holland, Italy, Japan, New Zealand, Norway, Poland, Spain, Sweden, and Switzerland.

It should also be mentioned that several important proposals for deliberation had been received from the United States and the U. S. S. R., and that a very interesting report on the progress of Russian magnetic work had been prepared for the Warsaw meeting.

At the opening session the President presented a report on the progress of the different undertakings for which the Commission has been responsible or to which it has given its support. He mentioned that the number of observatories contributing to the De Bilt publication of magnetic character-figures is now 52 and that the International Association of Terrestrial Magnetism and Electricity has been able to realize its publication of "numerical" character-figures.

It was further pointed out that the activity of the Magnetic Commission, in support of the Polar-Year Commission in its efforts to secure the best possible magnetic, electric, and auroral results from the second International Polar Year appeared to have been an important factor in the incontestable success which has attended this great undertaking.

The 37 points of the agenda were discussed during four sessions.

In connection with the report, prepared by Dr. van Dijk, on the publication of magnetic character-figures, two proposals made by Dr. Fanselau were discussed, namely: A proposal that the magnetic character should be given for every 12-hour period (two figures for each day) could not be generally adopted even if it were recognized that a subdivision of the day might prove useful in several respects. It was also found undesirable that the numerical character-figures should be deduced from the mean hourly values instead of the instantaneous values. In accordance with a proposal of Drs. Fleming and Heck, it was recommended in a resolution that the magnetic character-figures from all the Polar-Year stations should be printed as a supplement to the present De Bilt publications.

A memorandum on frequent international comparisons through transmission of instruments by mail was presented by Dr. la Cour and gave rise to a long discussion. The meeting was much in favor of the project and recommended it with the single reservation that such comparisons should not diminish the personal intercourse between the different observatories. It was decided that a subcommission representing both the Commission and the Association should care for the organization of the work.

A proposal of Sir George Simpson that all diurnal inequalities should be published in terms of zonal mean time rather than of Greenwich mean time was discussed in detail. It was generally recognized that a publication using zonal time would be useful for several special researches, but bearing in mind that the use of Greenwich mean time seemed to be indispensable for many other purposes, there was not found sufficient reason for changing the generally adopted rule that all magnetic data should be published in terms of Greenwich mean time.

A proposal of Dr. la Cour that continuous magnetic quick-run records should be obtained in two networks of stations—one well distributed over the whole Earth and the other, more dense, covering a certain region—was supported by a resolution but no action was taken as to the selection of the stations.

Professor Nippoldt and Dr. Sucksdorff were elected members of a joint committee of the Commission and of the Association to consider the question of uniform methods and codes for adequately describing magnetic disturbances and perturbations. The appointment of such a committee was proposed by Dr. Fleming who was asked to act as its chairman.

A proposal of Dr. Fleming that observatories in or near the auroral zone should be asked to publish hourly ranges in declination gave rise to a discussion which showed that several of the stations in question had already measured these hourly ranges. It was decided that the Secretary of the Commission should prepare a list containing the stations from which such values would be of special interest, and further to see that the values were made available for investigators.

Dr. Fleming also presented some remarks concerning research in the ionosphere which gave rise to an interesting discussion on the question as to whether the ionosphere belongs to the domain of the magneticians.

The desirability of further magnetic and atmospheric-electric observations at sea was discussed. The Commission adopted a resolution congratulating the British authorities upon the acceptance of their plans for the construction of a new non-magnetic vessel.

A subcommission consisting of Professor Chapman (chairman), Professor Bartels, and Dr. van Dijk was nominated to consider the question of publishing magnetic character-figures derived from records older than the De Bilt publications (proposal of Professor Chapman).

In an interesting memorandum Professor Chapman underlined the desirability of increasing the number of magnetic recording stations in the arctic regions. The commission supported Professor Chapman's proposal by a resolution.

The need of a new list of all magnetic and electric observatories was discussed and Professor Nippoldt was asked to cooperate in this matter

with the Central Bureau of the Association which has already been charged with the preparation of such a list.

A subcommission consisting of Professor Keränen (chairman), Dr. Ljungdahl, and Professor Rose will consider the question of an international service for magnetic charts (proposed by Professors Rose and Weinberg), and another subcommission consisting of O. H. Gish (chairman), Professor Nippoldt, and Dr. Romaña is to consider the question of a definite convention for indicating the sense of earth-current components.

In addition to the four sessions devoted to discussion, a session on September 4 was reserved for the following special communications: (1) G. van Dijk on measures of terrestrial-magnetic activity (presented by Professor van Everdingen); (2) F. Link on tables d'éclairements crepusculaires de la haute atmosphère; (3) J. Lugeon on Vorläufige magnetische Ergebnisse an der polnischen Polarstation Bären Insel; and (4) St. Kalinowski on État actuel du travail de l'Observatoire Magnétique de Swider.

The Commission[1] passed the following resolutions which were presented to the subsequent Conference of Directors for final adoption or approval:

(I)* The Commission finds it desirable that the magnetic character of each day be published for all the Polar Year stations as a supplement to the present publication and recommends that the data from all the magnetic stations which have made registrations during the Polar Year and which have not yet sent this information to De Bilt, be supplied as soon as possible and not later than March 1, 1936.

The Commission is of the opinion that it would be desirable to do the same in connection with the publication of the numerical magnetic character of the days.

(II) The Commission considers the study of the influence of the moon on geophysical phenomena to be of great importance for the advancement of science and recommends that long series of hourly values of the barometric pressure, if not printed, and especially from the South American States and West and Northwest Africa, should be communicated to the Commission for Professor Chapman and other investigators.

(III) The Commission having noted the relatively small number of magnetic observatories which publish diurnal inequalities for the internationally selected disturbed days, recommends that all observatories be asked to publish the mean diurnal-inequalities for such days for the months, seasons, and year.

(IV) The Commission recommends that whenever a set of diurnal inequalities is published, the magnitude of the non-cyclic change per 24 hours should also be published, but considers that the correction for the non-cyclic change should not be incorporated in the inequalities.

(V)* Resolution XLI of Innsbruck indicates the form of the publications of the Polar Year without however requiring a change in the form of the existing year books: the Commission is of the opinion that if some of the institutes have adopted the form recommended, they should continue to use that form provided that it contains all the data which were found in the previous form.

(VI)* The Commission considers as extremely important frequent comparisons between the determinations of the value of the force of the magnetic field at the various observatories. It is of the opinion that the comparisons of the measurements made at the different observatories should include an examination of the homogeneity of the field around the piers used for the determination of the absolute values in each observatory as well as an examination of the magnetic properties of the instruments used by the observatories for their measurements.

The Commission is glad to learn that the Association of Terrestrial Magnetism and Electricity has given consideration to the question of intercomparisons by instruments circulating by mail. It is of the opinion that this procedure is at present the only manner of obtaining comparisons in sufficient number to assure the reciprocal and

[1]The resolutions indicated by asterisks as adopted are in French and have been translated by H. D. Harradon.

satisfactory control of the normal values at all the observatories in the world and at their secular stations.

The Commission offers its moral support to the inauguration of such a circulation of instruments and requests all the directors of magnetic observatories kindly to cooperate therein.

The Commission recommends that comparisons between certain selected observatories well distributed over the Globe take place at short intervals and that these assure comparisons with the other observatories.

The Commission finds it desirable to charge a permanent subcommission composed of persons who are members of the Commission and also of the Association with organizing a service of comparisons of determinations of the absolute values.

(VII)* In view of the importance of the study of rapid or sudden magnetic variations which appear at unforeseen epochs, the Commission is of the opinion that it would be very desirable to assure, also after the Polar Year, continuous quick-run magnetic registrations at stations belonging (1) to a world-net of stations well distributed over the Earth, and (2) to a more dense regional net.

The cooperation of other stations, either continuously or on chosen days, will be appreciated.

(VIII) It is quite obvious from what was obtained during the Polar Year that magnetic registerings are needed from even more stations in the arctic in order to meet all needs of the study of the magnetic phenomena in those regions, and the Commission strongly recommends that magnetic recorders with low sensitivity be mounted in the artic regions at as many supplementary stations as possible.

The Commission is of the opinion that in case suitable instruments are available these supplementary stations could, if absolutely necessary, be attended to by non-professional people. In that case, however, determinations of the absolute values of the field should be carried out by a professional observer whenever circumstances would permit, while the base-line values should be tested through comparisons of the results obtained on magnetic quiet days with those found at the nearest permanent observatories.

The Commission invites designers and manufacturers of magnetic recording-instruments to pay special attention to the possibility of constructing instruments suitable for installation in places where professional attention could be given at rare intervals only.

(IX) The Commission regards as desirable the publication of a revised edition of the "Liste des Observatoires magnétiques et des Observatoires séismologiques" by Merlin and Somville published in 1910 giving a complete list of all magnetic and electric observatories, both operating and non-operating, with brief statements of geographical coordinates, elevations, instrumental equipment, data published regularly, data available upon special request, and special remarks covering operation and investigations.

(X) The Commission recommends the appointment of a joint committee of the Commission and of the International Association of Terrestrial Magnetism and Electricity to consider and to propose suggestions at the Edinburgh Assembly of the International Association for uniform methods and codes to adequately describe magnetic disturbances and perturbations.

(XI) The Commission expresses the hope that the hydrographic services of maritime nations will lose no opportunity of urging upon their governments the desirability of undertaking geophysical observations over the oceans on vessels either specially constructed for the purpose or already carrying out hydrographic work.

The Commission desires particularly to emphasize the need of (*a*) frequent observations in all oceans in order to record the secular variation of the Earth's magnetic field and (*b*) of additional observations of terrestrial electricity.

(XII) The Commission having noted the decision of the British Admiralty to construct a non-magnetic vessel especially designed for continuing the accurate magnetic survey of the oceans which was conducted by the Carnegie Institution of Washington from 1905 to the destruction of the *Carnegie* in 1929, records its appreciation of this decision of such vital importance to maritime interests and particularly to continued theoretical interpretation of the Earth's magnetism and the enhancement of the data already obtained; the Commission congratulates the British Admiralty, the Astronomer Royal, and the Chairman of the British National Committee of Geodesy and Geophysics upon the acceptance of their plans for this work.

(XIII) The Commission is of opinion that a non-magnetic vessel especially constructed for carrying out magnetic and electric measurements in the arctic sea would be of extreme value for geophysical investigations of those regions.

(XIV) The Commission has heard with great satisfaction of the plans that have been prepared by Canada for placing the temporary geophysical station established at Chesterfield Inlet during the International Polar Year on a permanent basis. It is of opinion that the station being near the North Magnetic Pole and within the zone of maximum auroral activity will be of the greatest importance in the study of terrestrial magnetism and allied geophysical phenomena and trusts that it may be permanently established at a very early date.

(XV)* The Commission notes with great satisfaction that much progress has been made on the magnetic survey of the Baltic during the last decade and expresses the desire that the observations be continued so as to have as soon as possible a chart of the magnetic anomalies of the whole Baltic.

(XVI)* The Commission is glad to have observed, on the occasion of its meeting at Warsaw, the excellent installation of the magnetic observatory so well situated at Swider. The Commission expresses the wish that Mr. Kalinowski may find the funds necessary for increasing the equipment of his Observatory so that it may fulfill its mission of a first-class observatory in the eastern part of Central Europe.

Det Danske Meteorologiske Institut,
Copenhagen, Denmark

NOTES

(See also pages 432 and 448)

31. *Andrew Carnegie Centennial Celebration*—The program of the Andrew Carnegie Centennial Celebration, was held November 25, 26, 27, 1935, in New York, Pittsburgh, Washington, and other cities and communities throughout the country, under the direction of Dr. F. P. Keppel, President of the Carnegie Corporation of New York, the largest of the six Carnegie foundations in the United States.

The different Carnegie trusts in the United States which participated in the Centennial Celebration, in the order of their establishment, were: Carnegie Institute of Pittsburgh, 1896, which conducts an institute of technology, a museum of fine arts, a music hall, a museum of natural history, a public library and a library school; Carnegie Institution of Washington, 1902, devoted to scientific research; Carnegie Hero Fund Commission, 1904, to recognize heroic acts performed in the peaceful walks of life; the Carnegie Foundation for the Advancement of Teaching, 1905, to provide retiring pensions for teachers and to advance higher education; the Carnegie Endowment for International Peace, 1910, to serve the purpose indicated by its name; Carnegie Corporation of New York, 1911, for the advancement and diffusion of knowledge and understanding among the people of the United States and the British dominions and colonies.

While each of these organizations observed the centennial with its individual program, all joined in the three principal events in New York, New York, consisting of a special choral-orchestral performance in Carnegie Hall November 25, a formal assembly at the New York Academy of Medicine November 26, and a dinner at the Waldorf-Astoria Hotel November 27.

On November 26, the formal assembly at the New York Academy of Medicine was held as a memorial to Andrew Carnegie for his many benefactions in different fields for the advancement of mankind. Dr. Nicholas Murray Butler, President of the Carnegie Endowment for International Peace and of Columbia University, presided at this assembly, which was addressed by Sir James Irvine, Principal and Vice-chancellor of St. Andrews University, Scotland, which with the other Scottish universities is the beneficiary of a Carnegie trust. Sir James Irvine was the official representative at the American Carnegie Centennial Celebration for the four British Carnegie trusts, namely, Carnegie United Kingdom Trust, Carnegie Dunfermline Trust, Carnegie Trust for the Universities of Scotland, and Carnegie Hero Fund Trust.

The final event on the program in New York was the dinner on the evening of November 27 at the Waldorf-Astoria Hotel, at which Dr. Henry S. Pritchett, President Emeritus of the Carnegie Foundation for the Advancement of Teaching presided and Dr. Keppel and President James Bryant Conant of Harvard University were the principal speakers.

32. *Magnetic disturbance, South Indian Ocean*—From the *Marine Observer* of October 1935 we note the following extract from the Meteorological Record of S.S. *Berwickshire* (Captain E. H. Evers, Observer J. C. Robertson, Second Officer) en route from Cape Town to Fremantle.

"October 25 to 30, 1934—Running the easting down in latitude 39° south steering 090° (E.S.E.'ly courses by compass), normal deviation 2° west, the deviation of the compass was found to increase westerly between the longitudes of 50° and 70° east, reaching maximum 6° west in longitude 70° and decreasing from there on, until normal in longitude 85° east. On successive voyages over a period of some years this identical change has taken place in varying latitudes from 38° to 43° south always commencing about longitude 50° east reaching maximum around 70° and returning to normal again from 85° to 90° east longitude. The depth of water, in this vicinity, surely precludes any possibility of local magnetic attraction, and it is suggested that the variation-curves may take a much sharper curve to the northward than is charted. It would be interesting to note if other ships have experienced a similar change."

THE LOCAL VARIATION OF THE EARTH'S ELECTRIC FIELD

By Joseph G. Brown

Abstract—Assuming that local diurnal-variations are small over the oceans, and absent over the polar ice during the winter night, a curve has been adopted which represents the unitary diurnal variation for a mean ocean-field of 129 volts per meter. The unitary variation at each land-station has been found by multiplying the ocean variation by the ratio of the mean land-gradient at each station to the mean ocean-gradient. By subtracting these unitary variations from the observed mean curves the local diurnal-variations have been obtained for 22 stations. Each local variation curve has been resolved into a 24 hour component and a daytime depression component. The stations fall into groups according to latitude, and the mean latitude-stations are of two types depending upon the amount of pollution. A theory is proposed which shows the significance of the components and accounts for the local diurnal variation by the cycle of turbulence, convection, and subsidence which is present at all land stations on all clear days.

The Earth's electric field is generally considered to be a spherical-condenser field with the surface of the Earth as the negative plate, the Kenelly-Heaviside Layer as the positive plate, and the lower atmosphere as the dielectric. Since the lower atmosphere is ionized, the condenser leaks thus giving an air-earth current. The measured field at the surface varies but the average field remains constant, hence some source maintains the potential. The thunderstorm, as suggested by C. T. R. Wilson, is the most probable source[1], providing an earth-air current which balances the leak.

The field has been measured continuously for various periods at many land stations and at sea. The most noticeable feature is a diurnal variation which differs with the station and with the time of year. From observations on ocean and polar ice it has been shown that a part of this variation is in the field as a whole, that is, the potential of the condenser varies in a simple manner with a 24-hour period. This was called by Mauchly[2], who made the discovery, "variation on universal time," but it can quite properly be called unitary variation. Whipple[3] has pointed out that such an oscillaton of the field can be accounted for on Wilson's theory of maintenance by a variation in the number and intensity of thunderstorms which exist.

Elimination of the unitary variation—If we assume that the small local variations over the oceans are cancelled out by plotting all values on Greenwich mean time, and that local variations do not occur over the ice in polar regions during the winter night, we can get the form and magnitude of the unitary diurnal-variation over the oceans. Using all suitable data a curve has been computed which has a mean gradient of 129 volts per meter.

Following a suggestion of Scrase[4] we can compute the unitary variation at land-stations from the variation over the ocean. Let R be the total resistance between the conducting layer and the surface, and let V be the potential of the conducting layer. Then i, the air-earth current, is V/R. Let F be the potential gradient at the surface, and let λ_+ be the positive conductivity at the surface. Then $i = F\lambda_+$.

[1]Phil. Trans. R. Soc., **221**, A, 73-115 (1920); J. Frank. Inst., **208**, 1-12 (1929).

[2]Terr. Mag., **28**, 61-81 (1923).

[3]Q. J. R. Met. Soc., **55**, 1-17 (1929).

[4]F. J. Scrase, London, Met. Office, Geophys. Mem. No. 60 (1934).

Omitting the effects of diffusion, turbulence, and convection upon the current, we can write $V/R = F\lambda_+$. Over the open ocean, where radioactive ionizers do not exist, and where meteorological processes are not subject to regular or extreme changes, the conductivity and total resistance may be considered approximately constant. Using the subscript o for ocean-values, we may then write $V/R_o = F_o\lambda_{+o}$, or $V/F_o = R_o\lambda_{+o}$ = a constant. This result explains why observations of potential gradient over the ocean give only the unitary variation in the field.

Using the subscript l for land-stations, we may write $V/R_l = F_l\lambda_{+l}$. While V is variable it presumably has nearly the same value everywhere at any one time, hence eliminating V from the two equations we obtain the relation $F_l = F_o\ (R_o\lambda_{+o}/R_l\lambda_{+l})$. This relation indicates that to obtain the hourly values of F_l it is necessary to multiply each hourly value of F_o by a factor $(R_o\lambda_{+o}/R_l\lambda_{+l})$, which would be different for each hourly value used. To reduce the amount of computation involved an almost equivalent process has been adopted. Let $\triangle V$, $\triangle F_o$, and $\triangle F_l$ represent departures from mean at any time; then $\triangle V = \triangle F_o R_o\lambda_{+o} = \triangle F_l R_l\lambda_{+l}$, or $\triangle F_l = \triangle F_o\ (R_o\lambda_{+o}/R_l\lambda_{+l})$. Since $\triangle V$ is small as compared with V, we can use mean values of $(R_o\lambda_{+o}/R_l\lambda_{+l}) = F_{lm}/F_{om}$, the subscript m signifying mean values. Taking the departure from the mean for each of the 24 hours over the ocean and multiplying each departure by the mean value of F_{lm}/F_{om} for any land-station gives the approximate departure for each hour at the land-station due to variation of V.

By this process unitary variation-curves have been computed and subtracted in proper phase from the mean monthly, seasonal, and annual curves at all land-stations for which data are available. The residual curves then give the true local variations of the field at these stations. The interesting result is that all local curves are much nearer the same general form than are the observed curves. Figure 1 shows the observed variation, the computed unitary-variation, and the residual local varia-

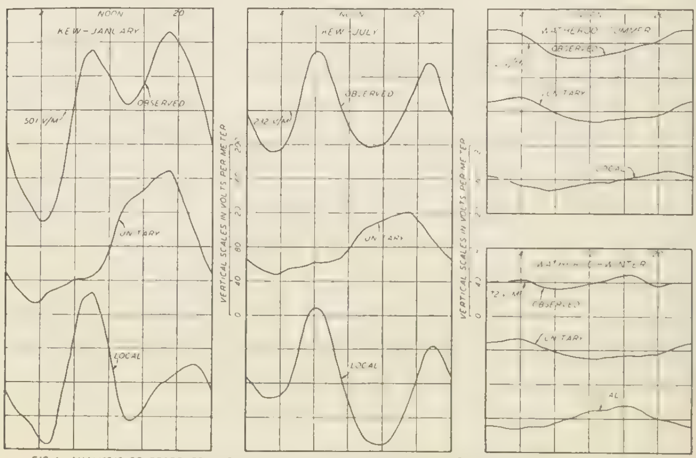


FIG. 1—ANALYSIS OF OBSERVED DIURNAL-VARIATIONS IN ATMOSPHERIC POTENTIAL-GRADIENT AT KEW AND WATHEROO

tion for January and July at Kew Observatory, and for summer and winter at Watheroo, Western Australia.

A great deal of work has been done, chiefly by harmonic analysis, in trying to relate the observed curves to various local factors such as temperature, pressure, wind, pollution, etc. It is evident that such work has been seriously handicapped by the presence of the unitary component. Especially is this true at those stations like Watheroo and Mount Stromlo in Australia where the local variation is in opposite phase to the unitary variation. More success along this line should be expected by dealing with the true local-variation curves than with the observed curves. Also a direct analysis seems better adapted to the problem than harmonic analysis.

Analysis of the local variation If we examine the local diurnal-variations of the gradient at the various land-stations we find that the portions of the curves extending from about 8 P. M. to 8 A. M. are very similar at all stations at all times of year. Moreover, during this part of the day the curves approximate roughly a portion of a simple harmonic graph. At some high-latitude stations during the winter months, and at some mean-latitude stations during the entire year, the local variation-curves as a whole approximate a simple harmonic graph. These facts indicate that some factors exist which, if acting alone, would produce an approximate simple harmonic variation in the gradient with a 24-hour period at all times, while some other factors usually exist which produce a depression in the gradient during the daytime. This depression varies in magnitude and time of occurrence at the different stations and varies with the time of year at each station. If we assume that the factors which produce the approximate simple harmonic variation are active at all stations on all clear days, then we can draw in hypothetical daytime parts of the 24-hour curves and thus get some notion of the character of the daytime depression.

The exact form given to the hypothetical parts of these 24 hour curves is somewhat arbitrary. However, there is always a decided change in slope during the morning increase, coming from one to two hours after sunrise, and a similar, though less definite, change in slope after sunset. By continuing the curves along the slopes indicated by the morning increase and the evening decrease, there is very little choice in completing a smooth curve. The method followed has been to complete the monthly curves first and then to use the annual mean of the monthly curves as typical for the station.

By taking the difference between the hypothetical part of the curve and the observed variation during the daytime, a curve giving the form, magnitude, and time of occurrence of the depression has been obtained for each station. Figure 2 shows this analysis for the monthly local curves at Ebro, and Figure 3 for the seasonal local curves at Potsdam, Stanford, and Mount Stromlo. The changes in time of occurrence and magnitude of the depression with the season is clearly evident at these stations, as it is at all stations well away from the equator.

Using the method illustrated by these curves, the mean local variations at 22 stations have been analyzed into 24-hour and depression-components. On comparing the form of the 24 hour components it appears that they fall into groups according to latitude, and that the mean-latitude curves are of two types Figure 4 shows the analysis

for nine mean-latitude stations in which the maximum of the 24-hour component is relatively narrow. This type of curve is seen to be characteristic of stations where the mean gradient is low and the air is pre-

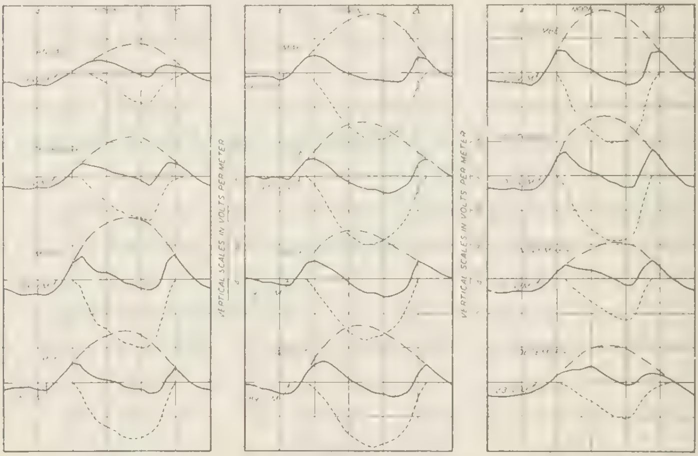


FIG 2 ANALYSIS OF LOCAL DIURNAL VARIATIONS IN ATMOSPHERIC POTENTIAL-GRADIENT AT
—·—· 24-HOUR COMPONENT ---- DEPRESSION COMPONENT

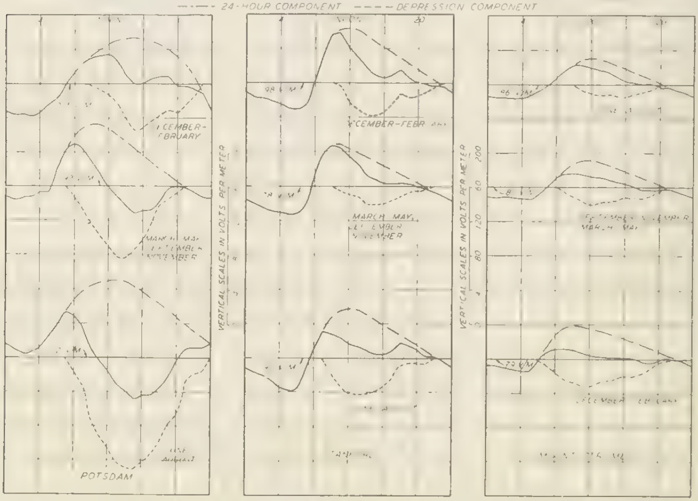


FIG 3—ANALYSIS OF LOCAL DIURNAL-VARIATIONS IN ATMOSPHERIC POTENTIAL-GRADIENT AT POTSDAM, STANFORD, AND MOUNT STROMLO
—·—· 24-HOUR COMPONENT ---- DEPRESSION-COMPONENT

sumably quite free from pollution. Figure 5 shows the analysis for six mean-latitude stations in which the maximum of the 24-hour component is relatively broad. All except one of these stations have a high mean gradient and all are located in or near large cities where pollution of the air is known to exist. Figure 6 shows the analysis of four high-latitude

stations and three tropical stations. The difference in the slope of the morning increase of the 24-hour components and the difference in form of the depressions are quite evident.

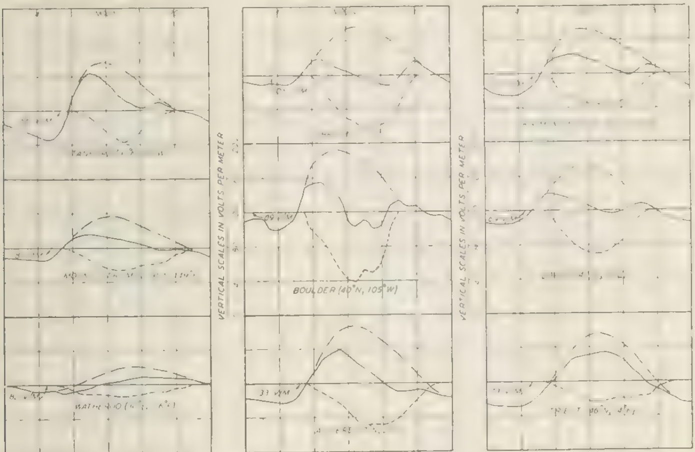


FIG. 4—MEAN LOCAL DIURNAL-VARIATIONS IN ATMOSPHERIC POTENTIAL-GRADIENT FOR MEAN LATITUDES, NARROW TYPE

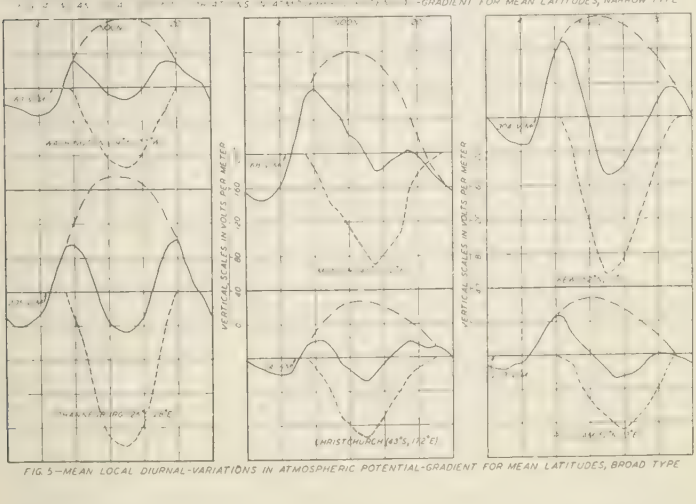


FIG. 5—MEAN LOCAL DIURNAL-VARIATIONS IN ATMOSPHERIC POTENTIAL-GRADIENT FOR MEAN LATITUDES, BROAD TYPE

In order to bring out the similarity in form of the components in a given group, and the differences in form between the several groups, the components have all been reduced to the same amplitude. Figure 7 shows the 24-hour components and Figure 8 the depression-components, thus reduced. If we consider the observed variation in the gradient to be the resultant of these two hypothetical components together with the variation of field as a unit, it is seen that the great variety of observed

variations can all be accounted for by relatively small differences in the components, which are characteristic of the location of the station and the amount of pollution in the vicinity. The effect of altitude has not been considered, but it is undoubtedly of some importance.

A theory of the local variation—The chief differences between the earth-condenser and an ordinary air-condenser are in the nature of the outer plate, gaseous instead of solid, and in the variation of conductivity with height, decreasing slightly then increasing from the surface upward. There is some question as to the height at which the conductivity becomes large enough so that the air may serve as a condenser-plate. While it has been common to refer to the Kennelly-Heaviside Layer as the positive plate, it has been suggested[5] that the conductivity is

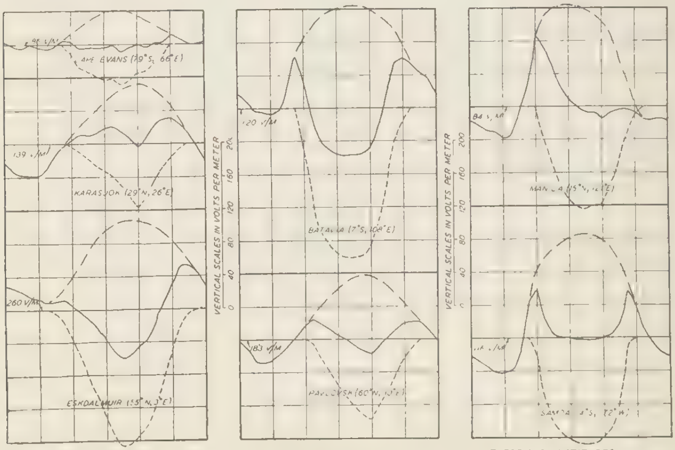


FIG. 6—MEAN LOCAL DIURNAL-VARIATIONS IN ATMOSPHERIC POTENTIAL-GRADIENT FOR HIGH LATITUDES AND TROPICAL LATITUDES

sufficient even at 14 km. It is certain that somewhere between these limits the conductivity is large enough to equalize potentials in a very short time, so that the condenser-analogy is warranted. Except for the unitary variation, therefore, it seems proper to assume that the difference of potential between the surface and a conducting layer at some height remains constant everywhere over the Earth. Local changes in the field-strength at the surface may then be attributed to one of two causes: First, the conductivity of the upper air may vary, thus changing the effective plate-distance of the condenser; and second, the conductivity of the lower air may vary, thus modifying the relative resistance between the plates, since saturation-conditions do not exist here. It has been inferred[6] that the conductivity at the upper plate-level is constant so that changes in field-strength are due to the second cause alone. The fact that the field-strength at some land-stations is nearly inversely proportional to the conductivity as measured at the surface, and that there is evidence for very small variations in conductivity and

[5] B. F. J. Schonland, Atmospheric electricity, p. 61 (1932).

[6] B. F. J. Schonland, *ibid.*, p. 60.

field over the oceans, lends support to this inference. However, the results of observations on the reflection and absorption of radio waves indicate decided changes in conductivity at levels as low as 50 km and

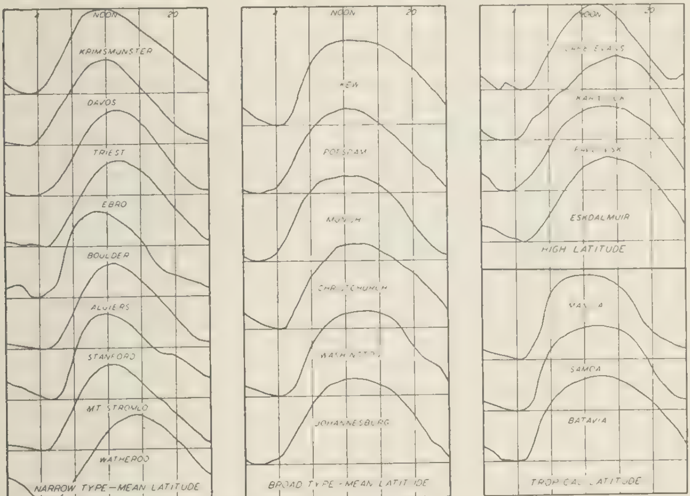


FIG. 7—MEAN LOCAL DIURNAL-VARIATIONS IN ATMOSPHERIC POTENTIAL-GRADIENT—24-HOUR COMPONENTS REDUCED TO SAME AMPLITUDE

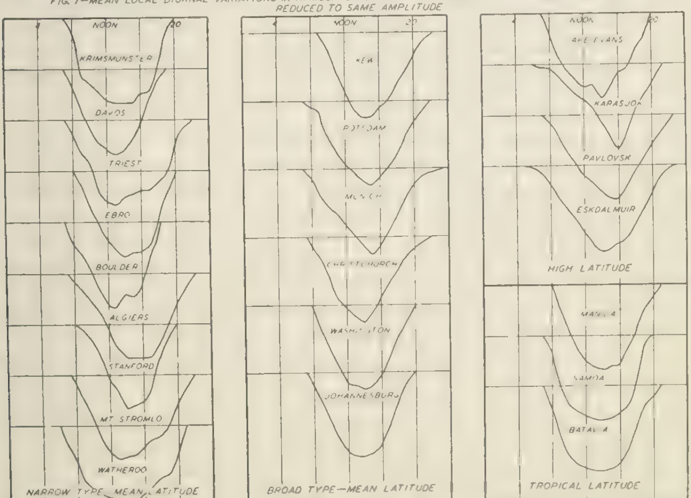


FIG. 8—MEAN LOCAL DIURNAL-VARIATIONS IN ATMOSPHERIC POTENTIAL-GRADIENT—DEPRESSION-COMPONENTS REDUCED TO SAME AMPLITUDE

hence make it likely that the first cause may be effective to some extent.

While this way of considering the field is attractive, it is somewhat

misleading. We know that the gradient falls to very small values at 8 km above the surface so that nearly all of the lines of force from the Earth terminate upon positive charge below that level. In fact, it has been estimated that the positive space-charge is so dense near the surface when the air is quiet that half of the lines of force terminate in the first 50 meters. The existence of this positive space-charge is largely a direct consequence of the air-earth current and the increase in conductivity of the atmosphere with height. In addition there is the electrode-effect which results from the fact that negative ions do not leave the surface under the small gradients which exist. In a certain sense the positive plate of the condenser is very close to the surface and only a small part of the field is due to lines terminating in the highly conducting upper atmosphere. On this account it would seem that even considerable changes in the conductivity of the upper atmosphere would have little effect upon the field at the surface as compared with changes taking place in the lower atmosphere.

It is hence quite evident that the cause for local variations in the gradient must be changes in the atmosphere immediately surrounding the station. This has long been recognized as true for the sudden, short-period changes which are present in every daily record, and quite generally accepted as true for diurnal and seasonal changes. Because of the approximate inverse relation between the gradient and the conductivity near the surface, both for diurnal and seasonal changes, it is also agreed that conductivity is the chief factor in determining the gradient.[7] An explanation of the variation in conductivity, therefore, becomes the essential part of the problem. Now conductivity of the atmosphere depends primarily upon the number of small ions present, and the number of small ions depends upon the rate at which molecular ions are formed and the number of suspensions upon which the small ions are absorbed to form large or intermediate ions.

There are three sources of small-ion production definitely recognized in the lower atmosphere, namely cosmic radiation, earth-radiation, and atmospheric radiation. Over the oceans the cosmic radiation alone is important, but at land-stations all three sources must be considered. Hogg[8] has recently estimated the relative importance at Canberra, Australia, as 7, 28, and 65 per cent respectively. Cosmic radiation may be considered as constant at any given level everywhere, and as increasing with height. Earth-radiation can only be effective in a very thin layer of air next to the surface and is dependent upon the radioactive content of the surface-material; at any one station, however, it must be fairly constant. Atmospheric radiation is due to radon and thoron and their decomposition-products. These gases get into the air from the surface-material by diffusion and transpiration. While the rate of transpiration is undoubtedly variable, this variation can affect only the air in a thin layer at the surface. Changes in conductivity due to varying amounts of radon and thoron would take place near the surface, but motions of the air would be the controlling factors in determining the amounts of these gases, and hence the rate of formation of small ions, at higher levels.

[7]B. F. J. Schonland, *loc. cit.*, pp. 49 and 59.

[8]A. R. Hogg, Beitr. Geophysik, **43**, 372 (1935).

[9]H. L. Wright, Proc. Phys. Soc., **45**, 152-171 (1933).

It seems probable from the work of Wright[9] and P. J. Nolan[10] that the suspensions most likely to be effective in the adsorption of small ions are the hygroscopic particles in the air which serve as nuclei of condensation. The gathering of water upon these particles and its subsequent evaporation may produce some variation in the rate of adsorption, but this would seem to be of much less importance than the number of nuclei. The sources of these nuclei are varied, but they probably come chiefly from the surface. Over the oceans they are undoubtedly fine particles of salt, while over the land they are largely the products of combustion and decaying organic matter. In general the number in the air is large near the surface and falls off with height. It is quite certain that seasonal variations in conductivity and gradient are partly explained by the rate of formation of nuclei. The mean gradient near large cities in mean latitudes is much higher in winter than in summer on account of the pollution from fires, but very little change in mean gradient occurs with the season in regions far removed from such pollution.

It is quite probable that the mean conductivity at any land-station is determined fully by the rates of formation of small ions and nuclei, and that seasonal variations are determined by changes in these rates. But it is difficult to conceive of any way by which the rates of formation can be changed each day so as to account for the diurnal variation in conductivity which takes place locally at every land-station. It is believed that the cycle of turbulence, convection, and subsidence which takes place over land on every clear day is a much more important factor in this case.

It should be noted first of all that in determining the potential gradient, variations in conductivity at all levels must be taken into consideration. Moreover, it can not be assumed that observed variations at any one level mean the same variation at other levels. In the present discussion it is contended that the conductivity in any given level is the result of the rate of formation of small ions and the number of nuclei in that level at any time.

While it must be admitted that changes in rate of formation of small ions do take place in the lower atmosphere on account of varying amounts of radon present, it is believed that this is a much smaller cause for variation in conductivity than the number of nuclei present. More information is necessary to decide this point but it will be assumed as a basis for the remaining argument.

The first measurements of number of nuclei in the atmosphere were made by Aitken[11] with his "dust-counter." Aitken showed very definitely that movement of the air is the primary factor in determining the number of nuclei at any time or place. That entirely different vertical distributions of nuclei exist under cyclonic and anticyclonic conditions, was shown in fourteen balloon-flights by Wigand[12] in 1919. With anticyclonic conditions the number of nuclei falls off approximately logarithmically with height, but with cyclonic conditions a very marked inversion in number of nuclei occurs at a height of about half a kilometer. That similar changes take place daily has been shown recently by Landsberg[13] making nuclear counts on mountains and adjoining plains and

[10]P. J. Nolan, Proc. R. Irish Acad., A, 41, 61-69 (1933).
[11]J. Aitken, Collected scientific papers (1923).
[12]A. Wigand, Ann. Physik, 59, 689-742 (1919).
[13]H. Landsberg, Mon. Weath. Rev., 62, 442-445 (1934).

valleys. During the daytime very great increases in the number of nuclei occur on the mountains while actual decreases take place in the valleys or on the plains. There can be little doubt that these effects are the result of turbulence and convection. Wait[14] has published curves showing the diurnal variation of nuclei and space-charge for a few days at Washington, D. C. In a general way the two curves are quite similar, which indicates that both quantities are dependent upon the same process for their value. No nuclei-measurements have been made at Stanford, but the results of two years of space-charge measurements at one meter and 15 meters above the surface show very nicely the details of the changes which occur. Figure 9 shows the mean diurnal-variation of space-charge at the two levels, together with the components of the gradient and the wind-speed for the four seasons.

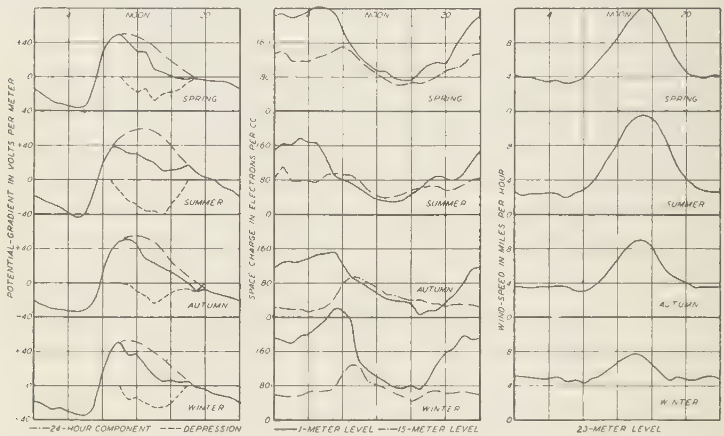


FIG. 9—LOCAL DIURNAL-VARIATIONS—STANFORD, 1932-34

During the quiet part of the night, from 9 P. M. to 5 A. M. the space-charge increases because nothing interferes with the electrode-effect. But during the same period the nuclei settle onto the surface, the conductivity increases and the gradient decreases. It is quite likely that condensation upon the nuclei is a very important factor in producing the settling. From 5 A. M. to 9 A. M. turbulence mixes the surface air with the higher air and produces an almost uniform space-charge up to at least 15 meters, and probably much higher. During this period nuclei are carried up from the surface, the conductivity decreases and the gradient increases. From 9 A. M. to 2 P. M. vertical convection is increasing, and if intense enough produces an inversion in both the space-charge and the number of nuclei. Such an inversion is found in the space-charge records during the summer and autumn at Stanford in the first 15 meters, and has been observed in the number of nuclei at higher elevations elsewhere.

It is inferred that the 24-hour component in the gradient is the change which might occur if the inversion did not take place. The depression-component is then the lowering in the gradient which results from the

[14]Terr. Mag., **36**, 120 (1931).

inversion. During the period of increasing convection the space-charge and the number of nuclei near the surface decrease but presumably increase at higher levels. From 2 P. M to 9 P. M. the convection and turbulence are both decreasing, that is, a period of subsidence exists. This condition allows the space-charge at the surface to increase but the number of nuclei again increases by settling from the higher levels and results in a corresponding decrease in the conductivity and increase in the gradient. The time required for this settling will depend greatly upon the number and kind of nuclei present, and it is believed that this is one of the factors which accounts for the different types of components and therefore of the modifications in the form of the gradient-curves.

An additional confirmation of the theory that the depression in the gradient is a result of vertical convection is found in the rather close inverse relation between the depression-components and the wind-speeds for the different seasons at Stanford. Still another correlation is found in the per cent of cumulus clouds at Blue Hill Observatory[15] and the depression at Washington, D. C., as shown in Figure 10.

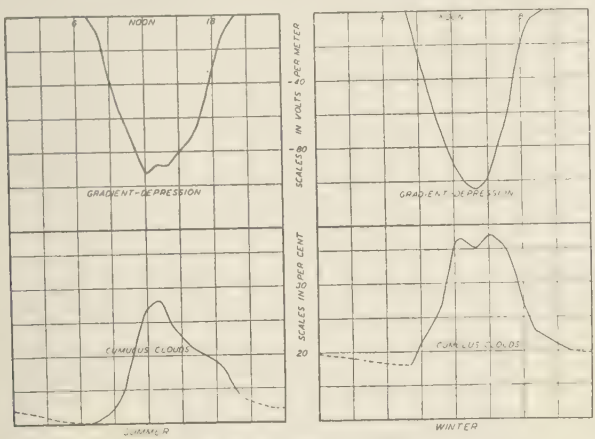


FIG 10—LOCAL DIURNAL-VARIATIONS

While the foregoing explanation is based upon the assumption that number of nuclei is the controlling factor in determining the conductivity, it cannot be maintained that the rate of formation of ions is not a factor. In fact it is quite likely that at some stations it may be the controlling factor. The rate of formation of ions at Mount Stromlo[16] varies almost inversely as the local component of the potential gradient, and the variation is of the same order of magnitude. This certainly indicates the importance of ion-formation at this station. It is to be expected that the distribution of radon by turbulence and convection will be similar to that of nuclei, but it is not likely that either radon or its decomposition-products will settle like nuclei during subsidence. The general conclusion is that the cycle of turbulence, convection, and subsidence is primarily

[15]H. H. Clayton, Ann. Astr. Obs. Harvard Coll., 30, Pt. 4.

[16]A. R. Hogg, Beitr. Geophysik, 43, 359-378 (1935).

responsible for the diurnal variation in conductivity, but the exact form of the variation will depend upon the relative importance of nuclei and radon at the place and time of the observation.

While this theory of the local variation has not yet been subjected to any severe tests, it involves processes which occur at all land-stations and it can be checked where sufficient electric and meteorological records are both available.

Physics Department, Stanford University,
Stanford University, California

DIURNAL VARIATION OF INTERMEDIATE AND LARGE IONS OF THE ATMOSPHERE AT WASHINGTON, D. C.

By G. R. Wait and O. W. Torreson

Abstract—Continuous records of the intermediate-ion and the large-ion contents of the atmosphere at Washington, D. C., were secured for identical samples of air, during the latter part of May and the first half of June 1933. The results reveal diurnal variations in the two sets of ions that are similar in character. The most prominent features of each is a pronounced maximum centering on about 21^h, local time. For the 12-hour interval, from 4^h to 16^h, both ion-contents are low and undergo but minor variations. The variation in the number of intermediate ions from day to day is proportional to the variation in the number of large ions present, but the variation is correspondingly less during times of the higher than during times of the lower large-ion content of the atmosphere. Intermediate ions are generally numerous during times of thunderstorms, not as a consequence of the Lenard effect, as surmised by Schachl, but probably because of the increased number of small ions present at such times. The increase in number of small ions is apparently due, according to previous results, to radioactive matter carried down by the rain after having been carried upwards from the Earth's surface through convection-currents during the formation of the thunderstorm. Without further investigational work, the full significance of the similarity in character of the diurnal-variation curves for intermediate and large ions is not fully apparent, but it appears that particles are coming into the atmosphere, assorted into two definite sizes, from a common source.

A type of ion in the air, having a mobility intermediate between that of the small gas-ion and the large or Langevin ion, was discovered in 1909 by J. A. Pollock[1] of the University of Sydney. He found that the average mobility of this ion was about one-fiftieth cm per sec per volt per cm, but that its mobility varied with the vapor-pressure of the atmosphere, decreasing as the vapor-pressure increased. He also secured results which he interpreted to mean that this type of ion was composed of water in the vaporous condition, but which at a critical vapor-pressure of 15 mm condensed into a liquid droplet, the droplet being the large ion of the atmosphere. Only a few scattered sets of observations appear to have been made since Pollock's discovery on this type of ion. No report seems ever to have been made regarding the character of diurnal variation of this ion, or of any attempt having been made to secure a continuous record of the intermediate-ion content of the atmosphere. A knowledge of the diurnal variation of the intermediate ion, especially in connection with the large-ion diurnal-variation curve, should be of assistance in interpreting the nature of both types of ions. Continuous records of intermediate and large ions, therefore, were attempted during a portion of May and June 1933.

Intermediate and large-ion records were secured from the same samples of air, the air-stream passing first through the small-ion counter where the intermediate ions were counted, then through the large-ion counter for a large ion-count. In order not to include the small ions in the count with the small-ion counter, they were removed from the air-stream by an electric filter consisting of a cylindrical condenser-system similar to that used for a small-ion counter. A potential, slightly in excess of the saturating potential for the small ions, was applied to this filter

[1]Phil. Mag., **29**, 636-646 (1915).

in order to ensure that none of the small ions would pass through the filter into the small-ion counter. A brief description of the apparatus employed for recording the small and the large ions of the atmosphere at the Department of Terrestrial Magnetism, has already been published,[2] and only a few additional details need be given at this time.

The system employed for recording the large ions was such that the electrometer assumed a constant deflection, which was maintained unless a change occurred in the number of large ions. The magnitude of deflection was proportional to the number of large ions caught by the counter, that is, to the large-ion content of the atmosphere. This was accomplished by passing the electric charges, caught by the condenser-system of the counter, to Earth through a high resistance (about 10^{12} ohms) connected in parallel with the electrometer. This system of recording had the advantage of eliminating errors due to insulation-leak, providing the leak was the same with air-flow off and on, for the leak would add to the current passing through the high resistance during the hourly zeros as well as during intervals between zeros and the deflection from zero would be unaffected by the leak. This system of recording could not be used in the case of intermediate ions, because they were not sufficiently numerous to furnish a large current, which such a system demands. In recording the intermediate ions, the electric charge was allowed to accumulate on the central cylindrical electrode of the small-ion counter. The electrometer, which was connected between this cylinder and Earth, showed a more or less gradually increasing deflection from the beginning to the end of the hour, in response to the accumulating charge on the central electrode. The number of ions per unit-volume in the atmosphere was deduced from the rate of increase in deflection, that is, from the slopes of the recorded hourly record.

Records of the small ions in the atmosphere at Washington were secured from October 1932 to September 1933, the results of which have already been published[2] by the authors. During the last half of May and the first half of June, in this period, the intermediate ions instead of the small ions were recorded. These recorded values have furnished data for the present analysis. Only those days that were complete in both intermediate and large ions were used in deriving hourly means, from which diurnal-variation curves for these two elements were drawn. The data were placed into two groups, the first containing days free from thunderstorms and the second days when thunderstorms occurred but were of short duration and the record was either complete or could be made so through interpolation over two hours or less. Only four days during the month of May and seven days during the month of June 1933, are available for the first group. By combining groups one and two, there were available twelve days for the month of June. In Figure 1 are shown curves for the various groups, curves 1 and 5 for the intermediate ions and curves 2 and 6 for the large ions for group one for May and June, respectively. Curves 3 and 4 are curves for intermediate and large ions, respectively, for June, obtained by combining data for groups one and two thus securing twelve days as a total. These curves represent diurnal-variation curves for intermediate and large ions. It is obvious that the inclusion of data for times of thunderstorms has not greatly affected the results. In general, the intermediate-ion content of the atmosphere

[2]Terr. Mag., 39, 47-64 and 111-119 (1934).

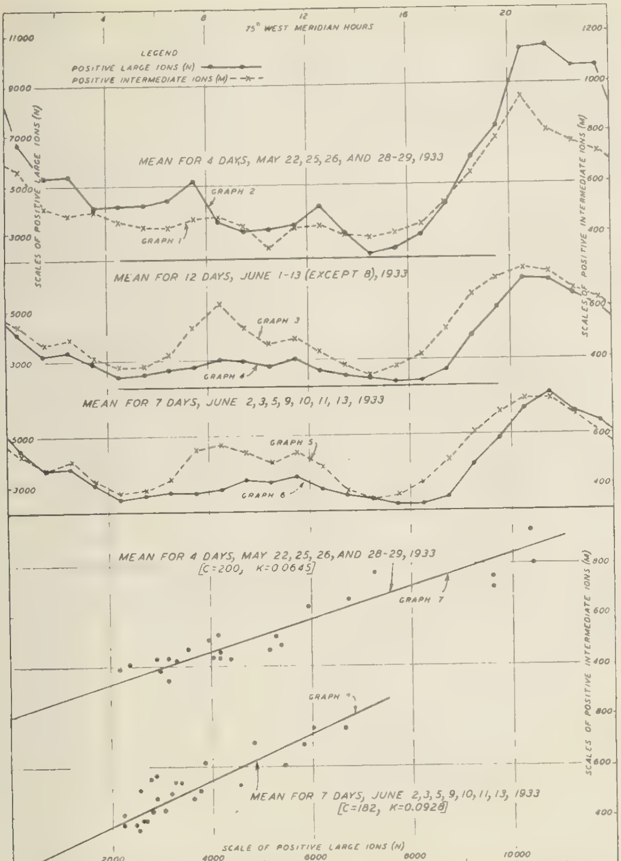


FIG. 1—DIURNAL VARIATIONS OF POSITIVE INTERMEDIATE IONS (M) AND OF POSITIVE LARGE IONS (N), GRAPHS OF MEAN HOURLY VALUES OF M AGAINST N, AND RESULTING SLOPES AND INTERCEPTS DETERMINED BY LEAST SUMS ASSUMING RELATIONSHIP M=C+KN FROM DATA DURING MAY AND JUNE, 1933, AT WASHINGTON, D.C.

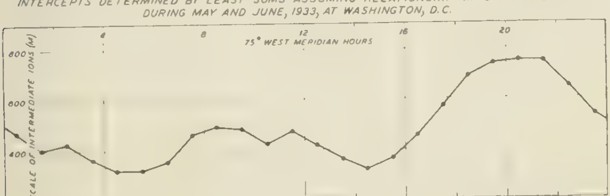


FIG. 2—DIURNAL VARIATION OF INTERMEDIATE IONS (M) AFTER APPLYING CORRECTION (BASED ON MOBILITY VARYING WITH VAPOR-PRESSURE) TO DATA, FOR GRAPH 5 OF FIG 1, JUNE, 1933 (7 DAYS)

is high during times of thunderstorms. However, meteorological disturbances were not sufficiently frequent to greatly modify the hourly means. A comparison between the two sets of curves for June emphasizes

another fact, namely, that as few as seven days are sufficient to give as representative a mean as twelve days. From an inspection of these curves it is apparent that the general character of diurnal variation for the two types of ions is similar for this particular time of year. The similarity is still further emphasized by the curves of Figure 2, where intermediate ions are plotted as ordinates and the large ions as abscissae; a general linear relationship is indicated between the number of ions in the atmosphere of the two types indicated. The relationship is expressible by an equation of the general form, $M=C+KN$, where M represents the number of intermediate ions and N the number of large ions per cc in the atmosphere, while C and K are constants as determined by method of least sums. The mean values of M, N, C, and K for the two months concerned are summarized in Table 1. It is of interest to note from

TABLE 1—*Mean values of intermediate-ion and large-ion contents of the air at Washington, D. C., and of the constants C and K (as determined by method of least sums)*

Month	Mean value per cc		Mean value of constant	
	M	N	C	K
May 1933	534	5180	200	0.0645
June 1933	516	3590	182	0.0928

Table 1 that the value of the constant K increases by about 50 per cent from May to June while the value of the constant C remains almost unchanged. The significance of the constant C has not been fully determined. The possibility of accounting for its existence through the behavior of the instruments has been considered. The persistence of a large insulation-leak in the small-ion counter would be of such a nature as to bring it about. Such an explanation, however, would require the existence of such a large insulation-leak as to be detectable during the six-minute test automatically carried out each hour. In addition, such a large insulation-leak would produce noticeable change in slope in the hourly record. No such evidence of leak is present and therefore such explanation of the occurrence of the constant C must be dismissed. Consideration has also been given to the possibility of accounting for its occurrence through ionization inside the collector-system of the small-ion counter, due to accumulation of radioactive matter on exposed parts of air-flow system subjected to a field. The production of ions in such numbers as required to account for the value of C would be easily detectable during the leak-test. The records of leak-tests show no evidence of a high rate of production of ions inside the condenser-system of the small ion counter, and consequently indicate the need for seeking other explanations for the existence of the constant C. The value of C, as revealed by the June data, was found to vary considerably through the day, being considerably less during the daylight interval from 7^h to 8^h and 18^h to 19^h than for the remaining 12 hourly intervals of the day. The results may be interpreted as indicating that in general, while the variation in the number of intermediate ions is proportional to the variation in the number of large ions, the variation is correspondingly less for the higher than for the lower values of the large-ion content of the atmosphere.

The number of intermediate ions as derived for this discussion was computed on the assumption that the mobility of these ions is constant. The necessity of assuming a value for the mobility arose because their conductivity rather than their number was recorded. The instrument functioned as a conductivity-apparatus rather than as an ion-counter since the potential applied to the condenser-system was below that required for the saturation of intermediate ions. The mobility of the intermediate ions was assumed to remain constant at 0.07 cm per sec per volt per cm—a value indicated from later tests to be about its mean. Actually, the evidence indicates that the mobility varies with the vapor-pressure of the atmosphere, diminishing as the vapor-pressure increases. This is evidenced since the current, i_M, in the ion-counter due to the collection of intermediate ions, was found to vary with the vapor-pressure, P, according to the equation: $1/i_M = A + BP$, where A and B are constants. This relationship immediately follows if one assumes that the mobility of the intermediate ions varies with vapor-pressure according to Blanc's law, just as in the case of the negative small ions of the air. If later work bears out this indicated relationship between mobility and vapor-pressure, considerable evidence regarding the true nature of the intermediate ion will result. For the present, however, it need be pointed out that if the mobility of the intermediate ions varies, instead of remaining constant as assumed, the calculated number of ions will be proportionally in error. The curves in Figure 1 are based on data derived on the assumption that the mobility remains constant, whereas the curve shown in Figure 2 is constructed from intermediate-ion numbers calculated on the assumption that the mobility varies with vapor-pressure in accordance with the equation indicated above. Comparing this curve with curve 5 of Figure 1, it is apparent that no large error has been introduced by assuming that the mobility is constant, the general character of the curve remaining but little changed.

By plotting each mean hourly value, for the entire group of days, of intermediate-ion number to large-ion number, as in curves 7 and 8 of Figure 1, a linear relationship between the two elements is indicated. It is possible, however, for two elements to appear to be related in a linear

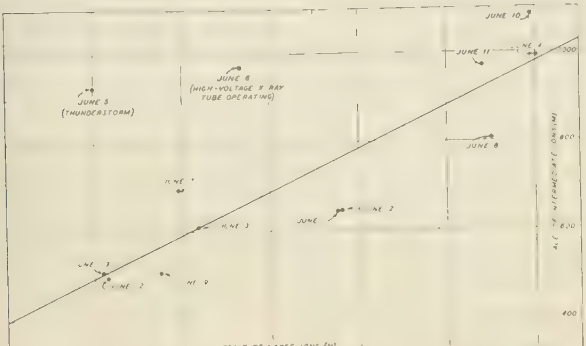


FIG. 3—GRAPH SHOWING MEAN HOURLY VALUES OF LARGE IONS AND OF INTERMEDIATE IONS FROM 20h TO 21h, JUNE 1-13, 1933, AT WASHINGTON, D. C. (SLOPE AND INTERCEPT DETERMINED BY METHOD OF LEAST SUMS)

fashion, only because of a similarity or dissimilarity in their diurnal-variation curves. To overcome this objection in the case of the intermediate and large ions, the hourly value of M for each day was plotted to the value of N for the corresponding hourly interval. Thus, 24 different curves were obtained, one for each hourly interval, from which the relationship between M and N can be studied for each hour of the day. The results of this study have already been stated, namely, that the value of the intercept varies throughout the day, being less for the daylight interval than for the night interval. In addition, it was found that the slope of the curves changed through the day, being greater during the daylight hours and less for the night hours. A curve for the interval of 20^h to 21^h is shown as Figure 3, which includes 12 days (June 1 to 13, excluding June 8). The points all fall reasonably close to a straight line, drawn on the basis of slope and intercept as determined by method of least sums, except the values for June 5 and June 6; on June 5 a thunderstorm was in progress and on June 6 a high voltage X-ray tube was in operation between 50 and 100 yards away. The values of intermediate-ion number on these two occasions were proportionately greater than on other occasions. An increase in the number of intermediate ions during times of thunderstorm (time of Böen) was noted by Schachl,[3] who believes this to be an indication that the ions are formed through the Lenard effect. Such an explanation could not be advanced for their increased number during the time the X-ray tube was in operation, and appears not to be the explanation of the increased number during times of thunderstorms, in view of the evidence obtained on this point. From measurements with a thin-walled ionization-chamber,[4] it was observed that the rate of small-ion formation was considerably increased at the times of thunderstorms originating over land areas. This increase in the rate is apparently due to the decay-products of radium, principally radium B and C in equilibrium with radium B, being carried to earth with the rain and in their disintegration producing the ionization. As a consequence of this increased rate of small-ion production, the number of small ions in the atmosphere at such times is greatly increased, the number decreasing immediately following the thunderstorm in a manner similar to the decreased rate of small-ion production. In a similar manner the number of intermediate ions were found to diminish, and it is believed that the decrease is to be attributed to the decrease in number of small ions. This is also in agreement with other observations made, which showed that when the small ions were caused to decrease in number through the introduction of a large number of large ions into the air of a closed room, the intermediate ions likewise increased in number, then as the number of small ions gradually increased the number of intermediate ions also increased in a similar manner. These facts, together with the information that the recombination-coefficient for intermediate ions has been found to be quite large,[5] make it seem possible to account for the increase in number of intermediate ions coincident with the time when the X-ray tube was in operation and during times of thunderstorms, on the basis of a large combination-coefficient between small

[3]Beitr. Geophysik, **38**, 202-219 (1933).
[4]Mon. Weath. Rev. **62**, 1-4 (1934).
[5]Phys. Rev., **48**, 383 (1935).

ions and intermediate ions as compared with the combination-coefficient between small ions and uncharged intermediate-ion particles.

One might be tempted, as a result of the close similarity between the diurnal-variation curves for intermediate and large ions, to conclude that one set of ions alone was responsible for the current in the two ion-counters, that is, that there were not two independent sets of ions present in the atmosphere where the observations were made. However, such a conclusion is not justified in view of the results of various tests. Thus by varying the potential on the small-ion counter, the intermediate ions were found to have an average mobility of about 0.07 cm per sec per volt per cm, the range in values being quite small. The results also indicated that practically no other group of ions exists between the intermediate and the large ions. These results were also obtained by keeping the potential on the ion-counter constant but varying the potential on the ion-filter. Additional tests have established the existence of the large-ion group in the atmosphere at this station. The full significance of the fact that these two groups of ions should exist without other groups being present, and especially that they should be related in such an intimate manner as found, is not at once apparent. It seems quite probable that two sets of particles come into the air, already graded as to size through some process, and from a common source. It is hoped that future investigations may be of assistance in throwing light on such matters and in providing more information regarding the precise processes at work. There is real need for additional observation on these two elements at other stations where conditions may be appreciably different from those in Washington, in order to determine if the results thus far obtained are peculiar to this locality, or if they may be accepted as more or less representative of other stations.

Department of Terrestrial Magnetism,
Carnegie Institution of Washington,
Washington, D. C.

NOTES

(See also pages 412 and 448)

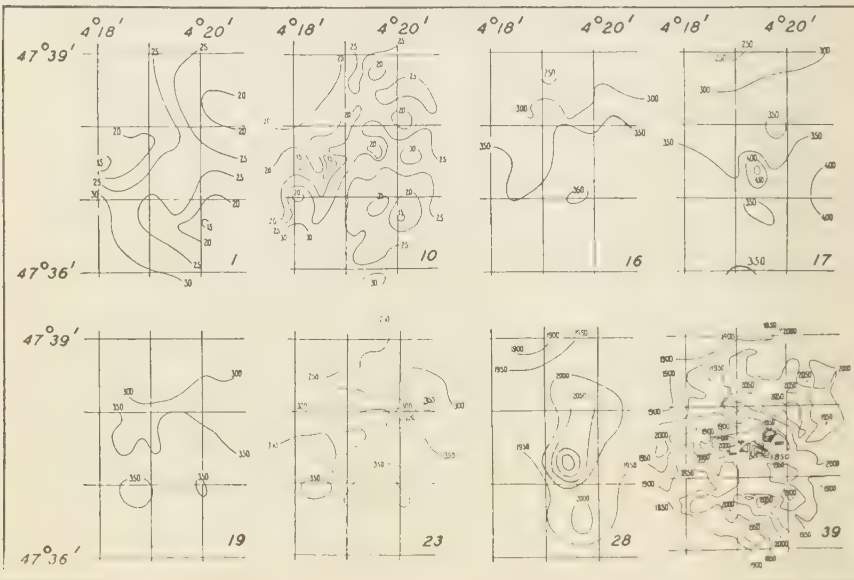
33. *Aurora of September 24, 1935*—A letter has been received from Mr. H. G. Dyar, describing an auroral display seen at Skyland, Virginia, on the night of September 24, 1935. The observation was made from a point 4,000 feet above sea-level. The display, seen along the horizon from north to northwest, continued from 10:30 p. m. until midnight. Magnetic records for this period at the Cheltenham Magnetic Observatory show considerable disturbance, while those from the Huancayo and Watheroo magnetic observatories are moderately disturbed. The occurrence of aurora at low latitudes is accompanied, as in this case, by increased magnetic activity. It is not often that aurora is seen as far south as on this occasion, and for this reason Mr. Dyar's communication is particularly interesting.

34. *Errata*—In the September number of the JOURNAL the following errata and author's corrections are to be noted.

Article by P. Robert Zeilinger: page 281 in fifth last line of Abstract for "may be due to" read "may be partly due to"; page 282 in sixth line of second paragraph for "curie qcm sec" read "curie/qcm/sec"; page 283 for subheading "Messvorgänge" read "Messvorgang"; page 286 in last sentence of first paragraph under §4 for "Dies letztere scheint" read "Diese letztere Vermütung scheint"; page 288 in title of Table 8 for "*deiser*" read "dieser"; page 291 in sixth line of third paragraph for "gestalten" read "gestatten"; page 294 in last sentence of third paragraph for "(Jänner)" read "(Dezember-Jänner)."

In article by B. W. Currie in formula near bottom of page 320 insert "sin" to follow "C_n".

In article by Boris Weinberg the author desires the following corrections to be made: Page 325 in sixth line of third paragraph for "we reproduce the" read "we give here different" and in eleventh line for ", 14 to 22," read ", 14, 17 and 20, 15, 17 and 21, 16, 19 and 22," in sentence before equation (1) for "whose values" read "the number of which (in the list of observations)," in equations (1) and (2) replace $=$ by $\equiv$, in sentence before equation (2) for "10, 11, 12, 23, 24, 36, 37, and 38" read "10, 11 and 12, 23, 24 and 25, and 36, 37 and 38"; page 328 the last sentence of second paragraph should read "Some of the variations from this general method are listed below.", in last sentence of third paragraph insert "some of" before "which are"; page 329 in fifth line of sixth paragraph insert "[methods (2) and (6)]" after "isolines." The author also reports errors in his original copy for Figures 1, 10, 16, 17, 19, 23, 28, and 39 for which corrected versions are indicated in cut below. In Figure 24 as printed in the September number cancel the isolines 400, 450, 500, and 550 surrounding the point 47°36′.4 north, 4°19′.2 east.



THE PROBLEM OF DIRICHLET FOR AN ELLIPSOID

By I. S. and E. S. Sokolnikoff

(1) *Introduction*—In a group of important problems in potential theory it is required to determine a harmonic function which takes on preassigned continuous values on the boundaries of some region R. Under the proper limitations on the geometrical characteristics of the region R, it is known that the solution of the problem of Dirichlet exists and is unique, but the actual task of determining such a function often presents enormous analytical difficulties even for relatively simple geometrical configurations.

An important type of Dirichlet's problem appears in a number of investigations in electrodynamics and geophysics, where the boundaries of the region are conductors and the boundary-values assumed by the harmonic functions are constants. Recently C. M. Rigby[1] determined the electrical field produced by a distribution of charge in equilibrium on a conducting sphere lying midway between two parallel conducting planes at zero-potential. The present paper treats of a more general problem in which the sphere is replaced by either an oblate or prolate ellipsoid of revolution, and thus enables one to calculate the field due to a charged circular disc and thin rod lying between the infinite parallel earthed planes.

The method employed in the solution is a classical one. It consists of constructing a class of harmonic functions which satisfy the boundary-conditions on the planes, and of suitably combining these functions so as to satisfy the boundary-conditions on the surface of the ellipsoid. The calculation of the potential function V is reduced to the solution of an infinite system of linear algebraic equations in infinitely many unknowns. The present state of knowledge of the difficulties involved in the treatment of infinite systems of algebraic equations makes it futile to attempt anything but an approximate solution of the system appearing in this problem.[2] Accordingly, a scheme of successive approximations is outlined which may prove to be of value in treating similar problems. The paper also contains the formulas for the calculation of the capacity of the system.

(2) *Mathematical formulation of the problem*—Consider a charged conducting ellipsoid of revolution placed with its center at the origin of the coordinate system and midway between the two parallel earthed planes $z=\pm h$. The analytical formulation of the problem is: To find the solution of Laplace's equation

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}=0 \tag{1}$$

subject to the boundary-conditions

$$V=0, \text{ on } z=\pm h \tag{2}$$

$$V=\text{constant, on the surface of } (x^2+y^2)/a^2+z^2/b^2=1 \tag{3}$$

[1]London, Proc. Math. Soc., Ser. 2, 33, 525 (1932).

[2]Encyklopädie Math. Wiss., 2, 9, p. 1417; A. Wintner, Spektraltheorie der unendlichen Matrizen. p. 139, Leipzig, 1929; F. Riesz, Les systèmes d'équations linéaires, Paris, 1913.

A harmonic function satisfying (1) and (2) can be easily constructed by utilizing an infinite set of images. In fact, consider a unit-charge placed at the point $(0, 0, \zeta)$, $/\zeta/ < h$. Then the solution of (1) valid at every point within the planes, except at the point occupied by the charge, and satisfying (2) is given by

$$
\begin{aligned}
V_0\,(x, y, z, \zeta) &= [x^2+y^2+(z-\zeta)^2]^{-1/2} \\
&+ \sum_{k=1}^{\infty} \{x^2+y^2+(z-4kh-\zeta)^2\}^{-1/2} - \{x^2+y^2+[z-2h(2k-1)+\zeta]^2\}^{-1/2} \\
&+ \sum_{k=1}^{\infty} \{x^2+y^2+(z+4kh-\zeta)^2\}^{-1/2} - \{x^2+y^2+[z+2h(2k-1)+\zeta]^2\}^{-1/2} \qquad (4)
\end{aligned}
$$

This is a familiar expression for the potential of positive unit-charge at $(0, 0, \zeta)$ and an infinite train of positive unit-charges placed at $(0, 0, \pm 4kh+\zeta)$, and negative unit-charges at $[0, 0, \pm 2h\,(2k-1)-\zeta]$, $(k=1, 2, \ldots)$. Now, the reciprocal of the distance between the point $(0, 0, a)$ and (x, y, z) is given by[3]

$$
\pm\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{du}{(z-a)+ix\cos u+iy\sin u} = \pm\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{dt}{(z-a)+i\sqrt{x^2+y^2}\cos t}\ \begin{cases} +\text{if } z>a \\ -\text{if } z<a \end{cases}
$$

and it is readily verified that (4) can be written as

$$
\begin{aligned}
V_0(\zeta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} dt\Bigg[\frac{1}{\rho-\zeta} - \sum_{k=1}^{\infty}\Bigg\{&\frac{1}{\rho-\zeta-4kh} - \frac{1}{\rho-\zeta+4kh} \\
&+ \frac{1}{\rho+\zeta-2\,(2k-1)\,h} - \frac{1}{\rho+\zeta+2(2k-1)h}\Bigg\}\Bigg] \qquad (5)
\end{aligned}
$$

where $\rho \equiv z+i\sqrt{x^2+y^2}\cos t$, and which is valid if $\zeta < z < h$. Expanding each term of the sum in (5) in negative powers of h gives[4] for $/\rho-\zeta/ < 4h$ and $/\rho+\zeta/ < 2h$

$$
V_0(\zeta) = \frac{1}{2\pi}\left\{\int_{-\pi}^{\pi}\frac{dt}{\rho-\zeta} - \int_{-\pi}^{\pi} dt\sum_{q=0}^{\infty}\,[A_q(\rho-\zeta)^q+B_q(\rho+\zeta)^q]\right\} \qquad (6)
$$

where

$$
A_0 - B_0 = \frac{1}{h}\sum_{n=1}^{\infty}\frac{(-1)^n}{n} = -\frac{\log 2}{h}
$$

$$
A_q = \frac{1+(-1)^q}{(4h)^{q+1}}S_{q+1},\quad B_q = \frac{1+(-1)^q}{(2h)^{q+1}}\left(1-\frac{1}{2^{q+1}}\right)S_{q+1},\quad (q=1,\ 2,\ \ldots)
$$

and S_q is the Riemann-zeta function $S_q \sum_{r=1}^{\infty} = 1/r_q$.

[3] H. E. Heine, Handbuch der Kugelfunktionen, 2, 99, Berlin (1878); E. T. Whittaker, Math. Ann., 57, 332 (1903).

[4] Cf. C. M. Rigby, London, Proc. Math. Soc., Ser. 2, 33, 526 (1932).

(3) *Prolate ellipsoid*—The boundary-condition (3) is unwieldy, and it is convenient to change the variables x, y, z according to the scheme

$$\left.\begin{aligned} z&=c\cosh\eta\cos\theta\\ y&=c\sinh\eta\sin\theta\sin\phi\\ x&=c\sinh\eta\sin\theta\cos\phi \end{aligned}\right\} \tag{7}$$

where c is a constant.

It is clear that the surfaces for which η is constant are confocal prolate ellipsoids

$$\frac{x^2+y^2}{c^2\sinh^2\eta}+\frac{z^2}{c^2\cosh^2\eta}=1$$

while $\theta=$constant gives a family of hyperboloids of revolution, and $\phi=$ constant represents a family of planes. If the correspondence of points of the (x, y, z)-space be one-to-one with the points of the (η, θ, ϕ)-space, it is necessary to restrict the range of values η, θ, ϕ as follows: $0\leq\eta<\infty$; $0\leq\theta\leq\pi$; $0\leq\phi<2\pi$.

The advantage of the curvilinear coordinate system η, θ, ϕ lies in the fact that the boundary-condition (3) is now simply expressed as $V=$constant when $\eta=\cosh^{-1}b/c$, where $c^2=b^2-a^2$.

If the independent variables in (1) are changed with the aid of the equations of transformation (7) and the solution of (1) is assumed to be given by

$$V(\eta, \theta, \phi)\equiv \mathrm{H}(\eta)\,\Theta(\theta)\,\Phi(\phi)$$

where H, Θ, and Φ are respectively functions of η, θ, and ϕ alone, a familiar argument leads to the well-known ordinary differential equations whose linearly independent solutions are

$$\mathrm{H}=\begin{cases} P_n^m(\cosh\eta)\\ Q_n^m(\cosh\eta)\end{cases},\quad \Theta=\begin{cases} P_n^m(\cos\theta)\\ Q_n^m(\cos\theta)\end{cases},\quad \Phi=\begin{cases}\cos m\phi\\ \sin m\phi\end{cases}$$

The functions $P_n^m(\mu)$ and $Q_n^m(\mu)$ are the associated Legendre functions of the first and second kinds respectively.

We proceed to express (6) in terms of these normal solutions. The first integral in (6) represents the reciprocal of the distance between the points $(0, 0, \zeta)$ and (x, y, z) and may[5] be expanded in a uniformly convergent series of Legendre functions to give

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{dt}{\rho-\zeta}=\frac{1}{c}\sum_{n=0}^{\infty}(2n+1)P_n(\mu)Q_n(r)P_n\left(\frac{\zeta}{c}\right)$$

where $\mu=\cos\theta$, $r=\cosh\eta$, and which is valid if $0<\eta<\cosh^{-1}\dfrac{2h-\zeta}{c}$.

The harmonic function $V_0(\zeta)$ was so constructed that it vanishes on the planes $z=\pm h$, and obviously any derivative of it with respect to ζ is harmonic and also vanishes on $z=\pm h$. Substituting the expansion for

[5] H. E. Heine, Handbuch der Kugelfunktionen, 2, 103 (1878).

the reciprocal of the distance in the first term of (6), differentiating m times with respect to ζ, and setting $\zeta=0$, gives

$$V_m(0)=\frac{1}{c}\sum_{n=0}^{\infty}(2n+1)P_n(\mu)Q_n(r)\left[\frac{d^m P_n(\frac{\zeta}{c})}{d\zeta^m}\right]_{\zeta=0}$$

$$+\frac{1}{2\pi}\int_{-\pi}^{\pi}dt\left\{\sum_{q=m}^{\infty}\frac{q!}{(q-m)!}\rho^{q-m}[(-1)^m A_q-B_q]\right\}$$

$$=\frac{1}{c}\sum_{n=0}^{\infty}(2n+1)P_n(\mu)Q_n(r)(-\frac{1}{c})^m P_n{}^m(0)$$

$$+\frac{1}{2\pi}\sum_{q=m}^{\infty}\frac{q!}{(q-m)!}[(-1)^m A_q-B_q]\int_{-\pi}^{\pi}\rho^{q-m}dt \tag{8}$$

since

$$\frac{d^m P_n(\mu)}{d\mu^m}=(-1)^m(1-\mu^2)^{-\frac{m}{2}}P_n{}^m(\mu)$$

The expansion (8) is valid so long as $0<\eta<\beta$, where $\beta\equiv\cos^{-1}\dfrac{2h}{c}$.

Now

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\rho^n dt=\frac{1}{2\pi}\int_{-\pi}^{\pi}(z+i\sqrt{x^2+y^2}\cos t)^n dt$$

$$=\frac{c^n}{2\pi}\int_{-\pi}^{\pi}(r\mu+i\sqrt{r^2-1}\sqrt{1-\mu^2}\cos t)^n dt$$

where the last step results from replacing the values of x, y, z in terms of η, θ, ϕ.

Consider next

$$I(m)=\int_{-\pi}^{\pi}(z+ix\cos u+iy\sin u)^n\cos mu\,du$$

where m and n are integers such that $0\leq m\leq n$.

If the parameters x, y, z in this integral are transformed to spheroidal coordinates with the aid of (7) there results

$$I(m)=c^n\int_{-\pi}^{\pi}[r\mu+i\sqrt{r^2-1}\sqrt{1-\mu^2}\cos(\phi-u)]^n\cos mu\,du$$

$$=c^n\int_{-\pi}^{\pi}(r\mu+i\sqrt{r^2-1}\sqrt{1-\mu^2}\cos t)^n\cos m(\phi+t)dt$$

$$=c^n\int_{-\pi}^{\pi}\rho^n\cos m(\phi+t)dt$$

$$=c^n\cos m\phi\int_{-\pi}^{\pi}\rho^n\cos mt\,dt$$

since the integrand is a periodic function of t and $\rho^n\sin mt$ is an odd function of t. This last integral can be readily evaluated by expanding in a series of Legendre polynomials and integrating term-by-term. Indeed, $\rho^n=\sum_{t=0}^{n}a_{t,n}P_t(\rho)$, where the coefficients $a_{t,n}$ are as follows

$$\left.\begin{aligned} a_{i,n}&=0 && \text{when } (n-i) \text{ is odd or negative} \\ a_{i,n}&=\frac{(2i+1)2^i n!(n/2+i/2)!}{(n/2-i/2)!(n+i+1)!} && \text{when } (n-i) \text{ is even and non-negative} \end{aligned}\right\} \quad (9)$$

Thus

$$I(m)=c^n \cos m\phi \int_{-\pi}^{\pi} \sum_{i=0}^{n} a_{i,n} P_i(\rho) \cos mt dt$$

$$=2\pi c_n \cos m\phi \sum_{i=0}^{n} a_{i,n} (-1)^m \frac{(i-m)!}{(i+m)!} P_i^m(r) P_i^m(\mu)$$

since[6]

$$\int_{-\pi}^{\pi} P_i(\rho) \cos mt dt = 2\pi (-1)^m \frac{(i-m)!}{(i+m)!} P_i^m(r) P_i^m(\mu)$$

Setting $m=0$

$$I(0)=\int_{-\pi}^{\pi} \rho^n dt = 2\pi c^n \sum_{i=0}^{n} a_{i,n} P_i(r) P_i(\mu)$$

and substituting this value in (8) gives

$$V_m(0)=\frac{1}{c}\sum_{n=0}^{\infty}(2n+1)P_n(\mu)Q_n(r)(-\frac{1}{c})^m P_n^m(0)$$

$$+\sum_{q=m}^{\infty}\frac{q!}{(q-m)!}[(-1)^m A_q - B_q]c^{q-m}\sum_{i=0}^{q-m} a_{i,q-m} P_i(r) P_i(\mu)$$

$$=\frac{1}{c}\sum_{n=0}^{\infty}(2n+1)P_n(\mu)Q_n(r)(-\frac{1}{c})^m P_n^m(0)$$

$$+\sum_{p=0}^{\infty}\frac{(m+p)!}{p!}[(-1)^m A_{m+p} - B_{m+p}]c^p \sum_{i=0}^{p} a_{i,p} P_i(r) P_i(\mu)$$

But the potential function $V_m(0)$ is to be symmetrical about the plane $z=0$ and hence m must be even. Moreover, $P_n^{2m}(0)=0$ if n is odd, and $A_q=B_q=0$ if q is odd, while

$$A_{2q}-B_{2q}=\frac{2}{(2h)^{2q+1}}\mu_{2q+1}, \text{ where } \mu_n=\sum_{j=1}^{\infty}\frac{(-1)^j}{j^n}$$

Therefore a set of harmonic functions vanishing on the planes $z=\pm h$ is given by

$$V_{2m}(0)=\frac{1}{c}\sum_{n=0}^{\infty}(4n+1)P_{2n}(\mu)Q_{2n}(r)\frac{1}{c^{2m}}P_{2n}^{2m}(0)$$

$$+\sum_{p=0}^{\infty}\frac{(2m+2p)!}{(2p)!}\ \frac{2c^{2p}}{(2h)^{2m+2p+1}}\ \mu_{2m+2p+1}\sum_{i=0}^{2p} a_{i,2p} P_i(r) P_i(\mu) \quad (10)$$

[6] E. T. Whittaker and G. N. Watson, A course of modern analysis, 327, Cambridge, 1927; A. M. Legendre, Calcul intégral, 2, 262-269, Paris, 1817.

We next form a series each term of which satisfies (1) and (2)

$$V=\sum_{m=0}^{\infty} C_m/(2m)!\cdot V_{2m}(0) \tag{11}$$

and it remains to be shown that a set of constants C_m can be chosen so as to satisfy (3).

Substituting the expansion (10) in (11) gives

$$V=\sum_{m=0}^{\infty}\frac{C_m}{c^{2m+1}}\Big[\frac{1}{(2m)!}\sum_{n=0}^{\infty}(4n+1)P_{2n}(\mu)Q_{2n}(r)P_{2n}^{2m}(0)$$
$$+\sum_{p=0}^{\infty}\frac{(2m+2p)!2c^{2p+2m+1}}{(2p)!(2m)!(2h)^{2m+2p+1}}\mu_{2m+2p+1}\sum_{t=0}^{2p}a_{t,2p}P_t(r)P_t(\mu)\Big] \tag{12}$$

On the surface of the ellipsoid $(x^2+y^2)/a^2+z^2/b^2=1$, $r=\cosh\eta_1=b/c\equiv\beta$, and V must reduce to a constant, say unity. Setting $r=\beta$ in (12) and collecting the coefficients of $P_{2n}(\mu)$ gives

$$1=\sum_{m=0}^{\infty}\frac{C_m}{c^{2m+1}}\Big[\frac{1}{(2m)!}\sum_{n=0}^{\infty}(4n+1)P_{2n}(\mu)Q_{2n}(\beta)P_{2n}{}^{2m}(0)$$
$$+2\sum_{n=0}^{\infty}P_{2n}(\mu)P_{2n}(\beta)\sum_{q=n}^{\infty}a_{2n,2q}\frac{(2m+2q)!}{(2m)!(2q)!}\left(\frac{c}{(2h)}\right)^{2m+2q+1}\mu_{2m+2q+1}\Big] \tag{13}$$

Inasmuch as the $P_n(\mu)$ are orthogonal functions and the expansion of unity in a series of Legendre polynomials is unique, we have, on equating the coefficients of $P_{2n}(\mu)$ on both sides of (13)

$$\delta_s{}^0=\sum_{m=0}^{\infty}\frac{C_m}{c^{2m+1}}\Big[\frac{(4s+1)Q_{2s}(\beta)P_{2s}{}^{2m}(0)}{(2m)!}+$$
$$2P_{2s}(\beta)\sum_{q=s}^{\infty}\frac{(2m+2q)!}{(2m)!(2q)!}a_{2s,2q}\mu_{2m+2q+1}\,\alpha^{2m+2q+1}\Big],\quad s=0,1,2,\ldots) \tag{14}$$

where $a_{2s,2q}$ are given by (9), $\alpha=c/(2h)$, and $\delta_0{}^0=1$, $\delta_s{}^0=0$, $(s=1,2,\ldots)$. Thus equation (13) furnishes a set of infinitely many linear algebraic equations in infinitely many unknowns C_m.

Dividing both members of (14) by $(4s+1)P_{2s}(\beta)\ P_{2s}{}^{2s}(0)$ enables one to rewrite (14) in the form

$$\delta_s{}^0=\sum_{m=0}^{\infty}(A_{s,m}+B_{s,m})x_m,\quad (s=0,\ 1,\ 2,\ldots) \tag{15}$$

where

$$A_{sm}=\frac{Q_{2s}(\beta)}{P_{2s}(\beta)}\,\frac{P_{2s}{}^{2m}(0)}{P_{2s}{}^{2s}(0)}\,\frac{1}{(2m)!},\quad A_{sm}=0,\ \text{if}\ m>s,$$

$$B_{sm}=\frac{2}{P_{2s}{}^{2s}(0)}\sum_{q=s}^{\infty}\frac{(2m+2q)!}{(2m)!(2q)!}\,\frac{2^{2s}(2q)!(q+s)!}{(q-s)!(2q+2s+1)!}\mu_{2m+2q+1}\,\alpha^{2m+2q+1}$$

$$x_m=C_m/c^{2m+1}$$

Equations (15) are preferable to (14) because the coefficients of the unknowns x_m form null-sequences with respect to either s or m.

(4) *Oblate ellipsoid*---The development of the expression for potential due to an oblate ellipsoid is entirely similar to that of the prolate. In fact, all one needs to do is to replace the equations of transformation (7) by

$$\left.\begin{aligned} z&=c\sinh\eta\cos\theta \\ y&=c\cosh\eta\sin\theta\sin\phi \\ x&=c\cosh\eta\sin\theta\cos\phi \end{aligned}\right\} \qquad (7')$$

where $0\leq\theta\leq\pi$, $0\leq\phi<2\pi$, $0\leq\eta<\infty$.

The parameter $\eta=$constant gives a family of confocal oblate ellipsoids

$$\frac{x^2+y^2}{c^2\cosh^2\eta}+\frac{z^2}{c^2\sinh^2\eta}=1$$

so that the boundary-condition (3) is now given by

$$V=\text{constant when } \eta=\sinh^{-1} b/c, \text{ where } c\equiv\sqrt{a^2-b^2}$$

The normal solutions of Laplace's equation in this coordinate system become

$$\mathrm{H}=\begin{cases}P_n^m(i\sinh\eta)\\ Q_n^m(i\sinh\eta)\end{cases}, \quad \Theta=\begin{cases}P_n^m(\cos\theta)\\ Q_n^m(\cos\theta)\end{cases}, \quad \Phi=\begin{cases}\cos m\phi\\ \sin m\phi\end{cases}$$

An analysis entirely analogous to that of the preceding section leads to the solution

$$V=\sum_{m=0}^{\infty}C_m\left(\frac{i}{c}\right)^{2m+1}\Big[\frac{1}{(2m)!}\sum_{n=0}^{\infty}(4n+1)P_{2n}(\mu)Q_{2n}(r)P_{2n}^{2m}(0)$$
$$+2\sum_{p=0}^{\infty}\frac{(2m+2p)!}{(2m)!(2p)!}\left(\frac{c}{2hi}\right)^{2m+2p+1}\mu_{2m+2p+1}\sum_{t=0}^{2p}a_{t,2p}P_t(r)P_t(\mu)\Big] \qquad (12')$$

in which $r\equiv i\sinh\eta$. On the surface of the oblate ellipsoid of semi-axes a and b, $(a>b)$, r becomes $r=b/\sqrt{b^2-a^2}\equiv i\beta$

The coefficients C_m are now obtained from the solutions of a set of linear algebraic equations

$$\delta_s^0=\sum_{m=0}^{\infty}(\bar{A}_{s,m}+\bar{B}_{s,m})x_m, \quad (s=0,\ 1,\ 2,\ \ldots) \qquad (15')$$

where $x_m=C_m/c^{2m+1}$,

$$\bar{A}_{s,m}=\frac{(i)^{2m+1}Q_{2s}(i\beta)}{(2m)!P_{2s}(i\beta)}\,\frac{P_{2s}^{2m}(0)}{P_{2s}^{2s}(0)}, \quad \bar{A}_{s,m}=0 \text{ if } m>s$$

and

$$\bar{B}_{s,m}=\frac{2}{P_{2s}^{2s}(0)}\sum_{q=s}^{\infty}\frac{(-1)^q(2m+2q)!}{(2m)!(2q)!}\,\frac{2^{2s}(2q)!(q+s)!}{(q-s)!(2q+2s+1)!}\mu_{2m+2q+1}a^{2m+2q+1}$$

It may be remarked that a direct comparison of (15) with (15′) shows that for any fixed β and a, $|\bar{A}_{s,m}|<|A_{s,m}|$ and $|\bar{B}_{s,m}|<|B_{s,m}|$. The discussion in the following sections will be confined to the system (15) inasmuch as the treatment of (15′) is identical.

(5) *Solution of equations*—It will be observed that the coefficients of the unknowns in (15) consist of the sum of two terms $A_{s,m}$ and $B_{s,m}$, the first of which is a function of β, whereas the second is a power series in $\alpha = c/2h$, where c is the focal distance of the ellipsoid. The system of equations (15) can be written in matrix form as

$$(P+Q)\xi = \delta \tag{16}$$

where P and Q are infinite matrices whose elements are respectively $A_{s,m}(\beta)$ and $B_{s,m}(\alpha)$, ξ is the vector whose components are $(x_0, x_1, x_2, \ldots\ldots)$, and δ is the vector $(1, 0, 0, 0, \ldots\ldots)$.

Inasmuch as the elements $B_{s,m}$ of Q are infinite series in odd powers of α, the matrix Q can be expressed as an infinite series of matrices

$$Q = \sum_{n=0}^{\infty} Q_n \alpha^{2n+1}$$

where Q_i is the infinite matrix whose element in the s-th row and m-th column is

$$b_{s,m}{}^{i-m} \text{ if } i-m \geq s \text{ and } 0 \text{ if } i-m < s$$

$$b_{s,m}{}^{q} \frac{2}{P_{2s}{}^{2s}(0)} \frac{(2m+2q)!2^{2s}(q+s)!}{(2m)!(q-s)!(2q+2s+1)!} \mu_{2m+2q+1}$$

Now consider the family of systems of linear equations

$$\left.\begin{aligned} &P\xi^{(0)} = \delta \\ &(P+Q_0\alpha)\xi^{(1)} = \delta \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ &(P+Q_0\alpha+Q_1\alpha^3+ \ldots\ldots +Q_r\alpha^{2r+1})\xi^{r+1} = \delta \end{aligned}\right\} \tag{17}$$

where $\xi^{(r)}$ is the vector $(x_0{}^{(r)}, x_1{}^{(r)}, x_2{}^{(r)}, \ldots\ldots)$. It is clear that the system of equations (17) becomes identical with (16) when r is allowed to increase indefinitely. But for any finite value of r the system can be solved. In fact, if the distance $2h$ between the planes is made very large, the elements $B_{s,m}$ in Q contribute little to the coefficients of x_m and the system (16) can be approximated by $P\xi^{(0)} = \delta$. Since $A_{s,m} = 0$ when $m > s$, the first equation of (17) can be written as

$$\sum_{m=0}^{s} A_{s,m} x_m{}^{(0)} = \delta_s{}^0, \quad (s = 0, 1, 2, \ldots)$$

so that the solution of the problem of a charged ellipsoid located in free space can be written down at once. The solutions for this case are

$$x_0{}^{(0)} = \frac{1}{A_{00}}, \quad x_s{}^{(0)} = -\frac{1}{A_{ss}} \sum_{i=0}^{s-1} A_{s,i} x_i{}^{(0)}, \quad (s = 1, 2, \ldots)$$

If the separation of the planes is not large enough to permit neglecting all powers of α, one can improve the solutions $x_m{}^{(0)}$ by including higher powers of α in the coefficients of (15). The important feature of this

scheme of approximations lies in the fact that it enables one to write down explicitly all the values $x_m^{(r)}$ at each stage of approximation. Thus the solutions which include the third powers of α are

$$x_0^{(2)}=[A_{00}+b_{00}{}^0\alpha+b_{00}{}^1\alpha^3-\frac{b_{10}{}^0\alpha^3}{A_{11}}(A_{10}+b_{10}{}^1\alpha^3)]^{-1}$$

$$x_1^{(2)}=-\frac{A_{10}+b_{10}{}^1\alpha^3}{A_{11}}x_0^{(2)}$$

$$x_s^{(2)}=-\frac{1}{A_{ss}}\sum_{t=0}^{s-1}A_{s,t}x_t^{(2)}, \quad (s=2,\ 3,\ \ldots)$$

(6) *Capacity of an ellipsoid*—It will be shown next that in order to calculate the capacity of the ellipsoid of revolution in the presence of the grounded conducting sheets of infinite extent, it is merely necessary to compute the coefficient C_0 in the expansions (12) and (12′). Inasmuch as the analysis and the final results are identical for the two types of ellipsoids under consideration, the calculations will be performed for the prolate ellipsoid only.

Since the potential of the ellipsoid is taken to be unity, the capacity of the system is equal to the total charge on the surface of the ellipsoid, i. e.

$$C=-\frac{1}{4\pi}\int_\Sigma\epsilon\left(\frac{\partial V}{\partial n}\right)_\Sigma d\sigma \tag{18}$$

where ϵ is the dielectric constant of the material in which the ellipsoid is immersed, and n denotes the normal exterior to the surface Σ of the ellipsoid.

It is easily checked, with the aid of the equations of transformation (7), that the metrical coefficients g_ϕ, g_η, g_θ are given by

$$g_\phi=c^2\sinh^2\eta\sin^2\theta$$
$$g_\eta=g_\theta=c^2(\sinh^2\eta+\sin^2\theta)$$

so that (18) becomes

$$C=-\frac{\epsilon}{4\pi}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}c\sinh\eta\sin\theta\left[\frac{\partial V}{\partial\eta}\right]_{\eta=\cosh^{-1}\beta}d\phi d\theta \tag{19}$$

Substituting for $\frac{\partial V}{\partial\eta}$, and recalling that on the surface of the conducting ellipsoid $\sinh\eta=\sqrt{\beta^2-1}$, (19) becomes

$$C=-\frac{\epsilon}{2}\int_0^\pi d\theta c(\beta^2-1)\sin\theta\sum_{m=0}^{\infty}\frac{C_m}{c^{2m+1}}\left\{\sum_{n=0}^{\infty}(4n+1)P_{2n}(\mu)\frac{dQ_{2n}(\beta)}{d\beta}P_{2n}{}^{2m}(0)\right.$$
$$\left.+2\sum_{p=0}^{\infty}\frac{(2m+2p)!}{(2m)!(2p)!}\alpha^{2m+2p+1}\mu_{2p+2m+1}\sum_{t=0}^{2p}a_{t,2p}\frac{dP_t(\beta)}{d\beta}P_t(\mu)\right|$$

But

$$\int_0^\pi P_n(\cos\theta)\sin\theta d\theta=0,\text{ if }n\neq0,\text{ and }=2,\text{ if }n=0$$

so that the foregoing expression reduces to

$$C = -\epsilon(\beta^2 - 1)C_0 \frac{dQ_0(\beta)}{d\beta}$$

and, since

$$Q_0(\beta) = \tfrac{1}{2} \log \frac{\beta+1}{\beta-1}$$

$$C = \epsilon C_0 \tag{20}$$

In order to indicate the practicability of the method of solution outlined in Section 5, the results of calculation of the capacity of a prolate ellipsoid whose major axis is 3 and minor axis is 1.5, and which is placed between planes four units apart, are given below. In this case $x_0^{(0)} = 0.760$, $x_0^{(1)} = 1.155$, $x_0^{(2)} = 1.216$, $x_0^{(3)} = 1.229$, so that $C = 1.299\ \epsilon x_0$, since $c = 1.299$

It is interesting to note that the first approximation, $x_0^{(0)}$, determines the capacity of the ellipsoid located in an infinite dielectric, so that the capacity of the ellipsoid under consideration is increased by the presence of the planes by nearly 62 per cent. A partial check on the foregoing calculations is afforded by the well-known exact formula[7] for the capacity of the ellipsoid in free space, which agrees to three places of decimals with the result given above.

[7] M. Mason and W. Weaver, The electromagnetic field, 132, Chicago, 1932.

University of Wisconsin,
Madison, Wisconsin

COULOMB WAVE-FUNCTIONS

By F. L. Yost, John A. Wheeler,[1] and G. Breit

It is well known that magnetism is closely related to the fundamental problems of physics and that even ferromagnetism cannot be understood without a rather thorough study of atomic phenomena. Nuclear studies are throwing a new and deeper light into the constitution of matter and one may hope that through a study of nuclear phenomena a proper understanding of magnetism and electricty will be reached.

One of the ways of learning the nature of nuclear binding-forces consists in studying the rate of reactions caused by the bombardment of nuclei with charged projectiles such as protons, deuterons, and alpha particles. In order to interpret the experimental results in terms of nuclear binding-forces it is necessary to take into account rather accurately the effect of the Coulombian repulsion between the nucleus and the incident projectile. It is usual to treat this part of the problem by approximate methods which may at times lead to misleading results because the effect of the Coulombian repulsion is large and a small error in its estimate may lead to a considerable error in conclusions about the nuclear structure. The object of the computations presented below is to remove these uncertainties and to provide accurate values for the wave-functions which enter the calculations.

The radial equation for a particle in an inverse-square field of force with a definite angular momentum L, is reducible to a well-known type known as the confluent hypergeometric equation. By standard mathematical methods one obtains solutions of this equation which have known asymptotic forms for large distances of the projectile from the nucleus.[2] Expressions obtained in this manner are long and the power-series which they contain often converge rather slowly. For this reason it is often desirable to have numerical tables of these functions. Such tables are provided below for $L=0$ and $L=1$.

It is convenient to standardize the possible solutions by introducing proper normalization and by using as one of the two linearly independent solutions, the solution which is regular at the center of the nucleus. This solution will be called F_L/r where r is the distance between the projectile and the nucleus. In the notation summarized below, the asymptotic form of F_L for large r is

$$F_L \sim \sin\ (\rho - L\pi/2 - \eta \ln\ 2\rho + \sigma_L)$$

It is convenient to choose for the other solution a function G_L such that G_L/r is a solution of the radial equation and such that for large r

$$G_L \sim \cos\ (\rho - L\pi/2 - \eta \ln\ 2\rho + \sigma_L)$$

Thus F and G are defined by making both have unit-amplitude for large r and by making the phase of G lead the phase of F by 90°. The index L will be omitted in the following discussion in the symbols F_L, G_L. A linear combination of F and G can be used to represent the general solu-

[1]National Research Fellow.

[2]F. L. Yost, John A. Wheeler, and G. Breit, Phys. Rev., in press.

tion. Such a linear combination can be used to give the proper solution when the Coulombian field is modified inside a distance r into some other field of force. In such cases the regular solution of the wave-equation in the interval $0<r<r_0$ is determined to within an arbitrary constant factor by the potential energy in this interval. If this solution is F_i then F'_i/F_i is known for $r\leq r_0$. Here the prime (′) stands for differentiation with respect to 0. Introducing

$$\delta=[F'/F-F'_i/F_i]_{r=r_0} \tag{1}$$

one obtains the solution for $r>r_0$ which corresponds to a sum of F and a divergent wave. This solution is

$$\overline{F}=F+e^{+iK}(\sin K)\ (G+iF) \tag{2}$$

where the phase K is given by

$$\tan K=[F^2\delta/(1-FG\delta)]_{r=r_0} \tag{3}$$

The asymptotic form for large r is

$$\overline{F}\sim e^{iK}\sin\left(\rho-L\pi/2-\eta\ln 2\rho+\sigma_L+K\right) \tag{4}$$

For $r=r_0$

$$\overline{F}=[F/(1-FG\delta-iF^2\delta)]_{r=r_0} \tag{5}$$

This determines the heretofore arbitrary factor of the solution for $r<r_0$. The functions F, G thus determine the solution everywhere in all cases. The phase-constant K is the usual constant entering calculations on anomalous scattering.

In order to obtain an idea of the magnitude of attractive forces acting on a particle inside the nucleus it is frequently supposed that the potential is constant for $r<r_0$. Such approximations are useful because they enable fairly rapid estimates of average potentials without entering in detail into more difficult questions. The functions F are in this case multiples of Bessel functions of half integral order multiplied by $r^{1/2}$. Tables are given below also for these functions and $\rho F'_i/F_i$.

In applying the tables to special problems it is convenient to deal with $\rho F'/F$ and $\rho F'_i/F_i$ because $\rho d/d\rho$ does not change with a change of scale of ρ. It is safer to compute $\rho\delta$ and (FG/ρ) rather than δ and FG. Otherwise it is easy to make the mistake of differentiating F_i and F with respect to different arguments.

The external functions F, G are not tabulated directly. The quantities tablulated (Φ, Φ^*, Ψ, Ψ^*) were chosen so as to make interpolation easy. The other quantities which enter F and G can be computed with relatively little trouble. Among them C, D vary too rapidly with the energy to make interpolation safe. It is usually more practical to interpolate Θ and Θ^* than Ψ and Ψ^*.

Summary of notations and solutions

m, m' and Ze, $Z'e$, masses and charges of the particles
  μ, reduced mass: $\mu^{-1}=m^{-1}+(m')^{-1}$

v, relative velocity of particles
  $\mu v=\hbar k$, momentum

$L\hbar$, angular momentum
ψ_L, wave-function for definite value of L

$E=\mu v^2/2=\hbar^2k^2/2\mu$, energy reduced to center of gravity at rest

$V=V(r)$, potential energy

$a=\hbar^2/\mu ZZ'e^2$, characteristic radius
$\eta=1/ak=(ZZ'/137)\ (c/v)=2\pi\lambda/a$; reduced wave-length
$\rho=kr$; $x=(8\eta\rho)^{1/2}=(8r/a)^{1/2}$

$$\left\{\frac{\mathrm{d}^2}{\mathrm{d}r^2}+\frac{2\mu}{\hbar^2}\ [E-V(r)]-\frac{L\ (L+1)}{r^2}\right\}\ (r\psi_L)=0$$

Regular solution, $r\psi_L=F_L$; irregular solution, $r\psi_L=G_L$. [Multiplicative constant so chosen that in all cases $F'G-FG'=1$ (independent of r). F' means $\mathrm{d}F/\mathrm{d}r$ in cases I and II; $\mathrm{d}F/\mathrm{d}\rho$ in cases III and IV.]

Case I: $E=0$, $V=0$; no normalization at infinity:
$$F_L=r^{L+1}\qquad G_L=r^{-L}/(2L+1)$$

Case II: $E=0$, $V=ZZ'e^2/r=(\hbar^2/2\mu)(2/ar)$; no normalization at infinity:

$$F_L=xI_{2L+1}\ (x)\qquad G_L=-(a/4)\ xK_{2L+1}(x)$$

Case III, internal functions: $E-V=\overline{E}$, $2\mu\overline{E}/\hbar^2=k^2$, $\rho=kr$; functions normalized at infinity:

$$\begin{aligned}F_L&=(-1)^L\rho^{L+1}(\rho^{-1}\mathrm{d}/\mathrm{d}\rho)^L\ (\sin\ \rho/\rho)\\&=(\rho^{L+1}/2^{L+1}L!)\int_{-1}^{1}\exp\ (i\rho z)\ (1-z^2)^L\mathrm{d}z\\&=(-i)^L(\rho/2)\int_{-1}^{1}\exp\ (i\rho z)\ P_L(z)\mathrm{d}z\\&=[\rho^{L+1}/1\cdot3\cdot5\ \ldots\ (2L+1)]\ [1-\rho^2/2(2L+3)\ \ldots]\\&=(\pi\rho/2)^{1/2}\ J_{L+1/2}(\rho)\backsim\sin\ (\rho-L\pi/2)\end{aligned}$$

$$\exp\ (i\rho\cos\ \theta)=\sum_0^\infty\ (2L+1)\ i^L\ P_L\ (\cos\ \theta)\ (F_L/\rho)$$

G_L may be obtained from the first expression for F_L by replacing $\sin\ \rho$ by $\cos\ \rho$, and $\cos\ \rho$ by $-\sin\ \rho$; similarly for G'_L. $G_L\backsim\cos\ (\rho-L\pi/2)$. The following relations for F and F' hold also for G and G':

$$\begin{aligned}&F_{L+1}-(2L+1)\ \rho^{-1}F_L+F_{L-1}=0\\&F'_L=(L+1)\ \rho^{-1}\ F_L-F_{L+1}=-L\rho^{-1}\ F_L+F_{L-1}\\&F_0=\sin\ \rho\\&F_1=\rho^{-1}\ \sin\ \rho-\cos\ \rho\\&F_2=(3\rho^{-2}-1)\ \sin\ \rho-3\rho^{-1}\ \cos\ \rho\\&F_3=(15\rho^{-3}-6\rho^{-1})\ \sin\ \rho+(-15\rho^{-2}+1)\ \cos\ \rho\\&F_4=(105\rho^{-4}-45\rho^{-2}+1)\ \sin\ \rho+(-105\rho^{-3}+10\rho^{-1})\ \cos\ \rho\end{aligned}$$

For determining the chance of penetrating a nuclear well, the following integral is useful

$$\int_0^\rho F_L^2 d\rho = (\rho/2)\ [F_L^2 - F_{L-1}F_{L+1}]$$

where for $L=0$ one should replace F_{L-1} by G_0.

Case IV, external functions: $E=\hbar^2k^2/2\mu$, $V=ZZ'e^2/r=(\hbar^2k^2/2\mu)(2\eta/\rho)$; functions are normalized at infinity:

$$\left\{\frac{d^2}{d\rho^2}+1-2\eta/\rho-L(L+1)/\rho^2\right\} \text{ operating on } G \text{ or } F \text{ gives } 0$$

$$F_L=[\rho^{L+1}/2^{L+1}\exp[(1/2)\pi\eta]\,|\,\Gamma(L+1+i\eta)|$$

$$\int_{-1}^{1}\exp\ (i\rho z)(1+z)^{L-i\eta}(1-z)^{L+i\eta}dz$$

$$\sim \sin\ (\rho-L\pi/2-\eta\ln 2\rho+\sigma_L)$$

$$G_L\sim \cos\ (\rho-L\pi/2-\eta\ln 2\rho+\sigma_L)$$

$$\sigma_L=arg\Gamma(L+1+i\eta)$$

$$F_L=C_L\rho^{L+1}\Phi_L\quad G_L=D_L\rho^{-L}\Theta_L$$

$$F'_L=C_L\rho^L\Phi^*_L\quad G'_L=D_L\rho^{-L-1}\Theta^*_L$$

$$(2L+1)C_LD_L=1$$

$$C_L=[1/1\cdot3\cdot5\ \ldots\ (2L+1)]\,[1+\eta^2/L^2]^{1/2}\,[1+\eta^2/(L-1)^2]^{1/2}\ \ldots\ [1+\eta^2/1^2]^{1/2}[2\pi\eta]^{1/2}[\exp\ (2\pi\eta)-1]^{-1/2}$$

$$\Theta_L=\Psi_L+\rho^{2L+1}(p\ln_e 2\rho+q)\Phi_L$$

$$\Theta^*_L=\Psi_L^*+\rho^{2L+1}(p\ln_e 2\rho+q)\Phi^*_L+\rho^{2L+1}p\Phi_L$$

$$p=\frac{2^{2L+1}}{(2L)!(2L+1)!}[L^2+\eta^2]\,[(L-1)^2+\eta^2]\ \ldots\ [1+\eta^2]\eta$$

$$q=\frac{2^L}{(2L)!}\left\{\frac{2^{L+1}}{(2L+1)!}\eta(L^2+\eta^2)\ \ldots\ (1+\eta^2)\right.$$

$$\left[\frac{L}{L^2+\eta^2}+\frac{L-1}{(L-1)^2+\eta^2}+\ \ldots\ +\frac{1}{1+\eta^2}\right.$$

$$\left.-\frac{1}{2L+1}-\frac{1}{2L}-\ \ldots\ -\frac{1}{1}+\text{R.P.}\frac{\Gamma'(1+i\eta)}{\Gamma(1+i\eta)}-2\frac{\Gamma'(1)}{\Gamma(1)}\right]$$

$$\left.+(-1)^{L+1}\sum_{n=-L}^{L}\frac{2^n}{(L+n)!(L+1-n)}\text{I.P.}(i\eta+n-1)\ \ldots\ (i\eta-L)\right\}$$

In the attached Tables A the values of Φ_L, Ψ_L, Φ_L^*, Ψ_L^* ($L=0$, 1) are tabulated for values of η, ρ spaced at equal intervals on a logarithmic scale. The Tables A are subdivided into A_1, A_2, A_3, A_4, A_5, A_6, and A_7 corresponding to the values $L=0$, 1, to the quantities tabulated, and to different ranges of the parameters ρ and ka. In these tables a diagonal line slanting from the left upper side to the right lower side corresponds to a fixed ρ/ka and therefore to a fixed distance between particles. Increasing the energy at a fixed radius thus corresponds to moving down

Tables A to I, Pages 1–11, To Accompany
COULOMB WAVE-FUNCTIONS
By F. L. Yost, John A. Wheeler, and G. Breit

Table A--Values of Φ_L, Φ_L^*, Ψ_L, and Ψ_L^*

(Notes: In this Table ln η^{-1} = ln ak; probable percentages of error for the first and last values in a row are given in the columns headed "o/o error"--in some cases the probable percentage of error in a value is indicated by number in parentheses directly following the value; the values of $-\Psi_L^*$ are tabulated rather than those of Ψ_L^*; value of ρ corresponding to each column is given in first row of each column.)

Subdivision A_1

o/o error	ln η^{-1}	Values of Φ_0 for ρ										o/o error
		.01000	.01259	.01585	.01995	.02512	.03162	.03981	.05012	.06310	.07944	
.004	-2.0	2.3947	2.9132	3.6823	4.8641	6.7583	9.9429	1.56141·10	2.63858·10	4.8441·10	9.7612·10	.012
	-1.8	1.7783	2.0347	2.3943	2.9123	3.6817	4.8629	6.7558	9.9396	1.56058·10	2.6371·10	
.007	-1.6	1.4545	1.5921	1.7783	2.0345	2.3946	2.9128	3.6816	4.8632	6.7557	9.9381	.010
	-1.4	1.2730	1.3512	1.4543	1.5920	1.7782	2.0344	2.3941	2.9121	3.6803	4.8611	
.008	-1.2	1.1671	1.2131	1.2729	1.3512	1.4543	1.5920	1.7780	2.0341	2.3935	2.9111	.025
	-1.0	1.1034	1.1313	1.1670	1.2131	1.2729	1.3511	1.4540	1.5916	1.7774	2.0330	
.0016	-0.8	1.06442	1.08156	1.10335	1.13120	1.16698	1.21302	1.27281	1.35085	1.45371	1.59082	.0051
	-0.7	1.05093	1.06439	1.08150	1.10327	1.13113	1.16683	1.21285	1.27255	1.35047	1.45312	
.0014	-0.4	1.02531	1.03192	1.04029	1.05089	1.06433	1.08139	1.10311	1.13088	1.16636	1.21206	.0067
.0014	0.0	1.01002	1.01259	1.01589	1.02002	1.02523	1.03178	1.04007	1.05053	1.06377	1.08046	.0075
.0002	0.1	1.007947	1.010007	1.012519	1.015863	1.019980	1.025158	1.031700	1.039917	1.050278	1.063342	.0003
.0002	0.5	1.003149	1.003970	1.004979	1.006256	1.007860	1.009864	1.012375	1.015510	1.019414	1.024263	.0003

o/o error	ln η^{-1}	Values of Φ_0^* for ρ										o/o error
		.01000	.01259	.01585	.01995	.02512	.03162	.03981	.05012	.06310	.07944	
.029	-2.0	4.2512	5.6217	7.7631	1.1250·10	1.7209·10	2.7955·10	4.8585·10	9.104·10	1.8561·10²	4.1589·10²	.020
	-1.8	2.7193	3.3408	4.2490	5.6179	7.7571	1.1240·10	1.7192·10	2.7926·10	4.8524·10	9.0918·10	
.014	-1.6	1.9691	2.2830	2.7199	3.3415	4.2510	5.6206	7.7616	1.1248·10	1.7202·10	2.7939·10	.020
	-1.4	1.5686	1.7392	1.9687	2.2820	2.7187	3.3400	4.2485	5.6171	7.7551	1.12364·10	
.015	-1.2	1.3430	1.4405	1.5690	1.7395	1.9691	2.2826	2.7180	3.3405	4.2480	5.6152	.0087
	-1.0	1.2101	1.2680	1.3427	1.4403	1.5687	1.7390	1.9682	2.2813	2.7167	3.3362	
.0009	-0.8	1.13017	1.16520	1.21008	1.26780	1.34260	1.43995	1.56796	1.73789	1.96629	2.27797	.027
	-0.6	1.08114	1.10267	1.13007	1.16501	1.20983	1.26743	1.34198	1.43906	1.56651	1.73547	
.0058	-0.4	1.05078	1.06413	1.08104	1.10248	1.12980	1.16461	1.20920	1.26649	1.34050	1.43671	.032
	-0.2	1.03190	1.04023	1.05075	1.06400	1.08090	1.10226	1.12940	1.16395	1.20809	1.26460	
.0014	0.0	1.02003	1.02524	1.03182	1.04008	1.05054	1.06371	1.08039	1.10146	1.12812	1.16184	.0017
.0002	0.1	1.015899	1.020018	1.025208	1.031742	1.039983	[illegible]	[illegible]	1.030677	1.038281	1.047649	.0004
											1.126901	.0005

Table A--Continued

Subdivision A_2

o/o error	$\ln \eta^{-1}$	Values of Ψ_0 for ρ .01000	.01259	.01585	.01995	.02512	.03162	.03981	.05012	.06310	.07944	o/o error
.033	-2.0	-3.9193	-7.8011	$-1.51924\cdot 10$	$-2.98072\cdot 10$	$-6.[illegible]007\cdot 10$	$-1.25658\cdot 10^2$	$-[illegible]44\cdot 10^2$	$-6.5[illegible]05\cdot 10^2$	$-1.641[illegible]\cdot 10^{[illegible]}$	$-4.5[illegible]\cdot 10^{[illegible]}$	[illegible]
	-1.8	-0.6398	-1.8132	-3.9185	-7.7998	$-1.51900\cdot 10$	$-2.97994\cdot 10$	$-[illegible]5\cdot 10$	$-1.2[illegible]640\cdot 10^2$	$-2.76956\cdot 10^{[illegible]}$	-6.49[illegible]	
.048	-1.6	0.4177	0.0288	-0.6407	-1.8145	-3.9206	-7.8024	$-1.[illegible]954\cdot 10$	$-2.9[illegible]089\cdot 10$	$-6.[illegible]000\cdot 10$	$-1.[illegible]\cdot 10^{[illegible]}$	[illegible]
	-1.4	0.7846	0.6473	0.4176	0.0286	-0.6409	-1.8145	-3.9203	-7.8017	$-1.51890\cdot 10$	$-2.97915\cdot 10$	
.019	-1.2	0.9182	0.8677	0.7846	0.6473	0.4175	0.0286	-0.6409	-1.8143	-3.9187	-7.7974	[illegible]
	-1.0	0.9684	0.9492	0.9181	0.8676	0.7845	0.6471	0.4172	0.0282	-0.6412	-1.8144	
.0018	-0.8	0.98761	0.98021	0.96830	0.94909	0.91796	0.86726	0.7[illegible]08	0.64647	0.41650	0.0271[illegible]	2.1
.0014	-0.5	0.99690	0.99507	0.99216	0.98747	0.97998	0.96793	0.9[illegible]51	0.91706	0.86584	0.78193	.008[illegible]
.0014	-0.3	0.99875	0.99801	0.99683	0.99495	0.99197	0.98718	0.97[illegible]51	0.96720	0.94737	0.91[illegible]	.00[illegible]7
.0014	-0.1	0.99948	0.99916	0.99867	0.99789	0.99664	0.99466	0.9[illegible]49	0.98643	0.97834	0.96535	[illegible]
.0001	0.1	0.999760	0.999620	0.999396	0.999042	0.998479	0.997585	0.996163	0.9939[illegible]4	0.990302	0.984560	[illegible]
.0001	0.3	0.999875	0.999801	0.999684	0.999500	0.999207	0.998742	0.99[illegible]004	0.996674	0.994976	0.992025	[illegible]
.0001	0.5	0.999920	0.999873	0.999799	0.999682	0.999495	0.999200	0.99[illegible]731	0.997989	0.996813	0.994949	[illegible]

o/o error	$\ln \eta^{-1}$	Values of $-\Psi_0^*$ for ρ .01000	.01259	.01585	.01995	.02512	.03162	.03981	.05012	.06310	.07944	o/o error
.029	-2.0	$1.21656\cdot 10$	$2.27344\cdot 10$	$4.3980\cdot 10$	$8.8580\cdot 10$	$1.86949\cdot 10^2$	$4.1638\cdot 10^2$	$9.8647\cdot 10^2$	$2.5060\cdot 10^3$	$6.886\cdot 10^3$	$2.063\cdot 10^4$	1.0
	-1.8	3.7849	6.7022	$1.21640\cdot 10$	$2.27328\cdot 10$	$4.3975\cdot 10$	$8.8559\cdot 10$	$1.86919\cdot 10^2$	$4.1631\cdot 10^2$	$9.8614\cdot 10^2$	$2.5047\cdot 10^3$	
.021	-1.6	1.2801	2.1831	3.7868	6.7052	$1.21693\cdot 10$	$2.27388\cdot 10$	$4.39368\cdot 10$	$8.8579\cdot 10$	$1.86910\cdot 10^2$	$4.16193\cdot 10^2$	.01[illegible]
	-1.4	0.4580	0.7614	1.2803	2.1832	3.7867	6.7038	$1.21655\cdot 10$	$2.27297\cdot 10$	$4.39534\cdot 10$	$8.84864\cdot 10$	
.10	-1.2	0.1702	0.2781	0.4581	0.7614	1.2803	2.1827	3.7856	6.7013	$1.21572\cdot 10$	$2.27086\cdot 10$	.[illegible]7
	-1.0	0.0649	0.1049	0.1703	0.2784	0.4584	0.7618	1.2809	2.1833	3.7854	6.6994	
.089	-0.8	0.02517	0.04039	0.06503	0.10508	0.17066	0.27883	0.45906	0.76275	1.28176	2.18421	.053
	-0.6	0.00990	0.01582	0.02533	0.04064	0.06535	0.10567	0.17157	0.28026	0.46110	0.76570	
.37	-0.4	0.00396	0.00631	0.01005	0.01606	0.02570	0.04121	0.06630	0.10708	0.17374	0.28347	.047
	-0.2	0.00163	0.00259	0.00410	0.00653	0.01042	0.01663	0.02661	0.04264	0.06852	0.11054	
1.5	0.0	0.00070	0.00112	0.00178	0.00283	0.00449	0.00714	0.01136	0.01813	0.02895	0.04632	.053
.21	0.1	0.000481	0.000763	0.001210	0.001921	0.003053	0.004851	0.007715	0.012279	0.019564	0.031212	.045
.40	0.3	0.000251	0.000398	0.000632	0.001001	0.001589	0.002520	0.004000	0.006348	0.010088	0.016011	.039
.63	0.5	0.000160	0.000254	0.000402	0.000637	0.001010	0.001601	0.002538	0.004022	0.006375	0.010104	.061

o/o error	$\ln \eta^{-1}$	Values of Φ_0 for ρ .1000	.1259	.1585	.1995	.2512	.3162	.3981	.5012	.6310	.7944	o/o error
.038	-2.0	$2.1840\cdot 10^2$	$5.493\cdot 10^2$	$1.572\cdot 10^3$	$5.177\cdot 10^3$ (2)	$1.97\cdot 10^4$ (5)	$8.64\cdot 10^4$					14.
.027	-1.8	$4.8405\cdot 10$	$9.7524\cdot 10$	$2.1817\cdot 10^2$	$5.485\cdot 10^2$	$1.569\cdot 10^3$	$5.164\cdot 10^3$	$1.964\cdot 10^4$ (2)	$8.61\cdot 10^4$ (5)	$4.3\cdot 10^5$ (14)	$2.4\cdot 10^6$	34.
.010	-1.6	$1.5601\cdot 10$	$2.6356\cdot 10$	$4.8358\cdot 10$	$9.7411\cdot 10$	$2.1765\cdot 10^2$	$5.4696\cdot 10^2$	$1.[illegible]62\cdot 10^3$	$5.133\cdot 10^3$ (2)	$1.948\cdot 10^4$ (6)	$8.5\cdot 10^4$	16.
.019	-1.4	6.751	9.929	$1.5589\cdot 10$	$2.6304\cdot 10$	$4.8225\cdot 10$	$9.6978\cdot 10$	$2.1638\cdot 10^2$	$5.420\cdot 10^2$	$1.543\cdot 10^3$	$5.045\cdot 10^3$	1.7
.0061	-1.2	3.6781	4.8561	6.7412	9.9050	$1.55288\cdot 10$	$2.61605\cdot 10$	$4.[illegible]8759\cdot 10$	$9.[illegible]97[illegible]\cdot 10$	$2.13142\cdot 10^2$	$5.305\cdot 10^2$	.11
.009	-1.0	2.3915	2.9076	3.6714	4.8426	6.7149	9.8490	$1.5[illegible]973\cdot 10$	$2.58540\cdot 10$	$4.70249\cdot 10$	$9.[illegible]50[illegible]\cdot 10$	.[illegible]10
.011	-0.8	1.7759	2.0303	2.3866	2.8980	3.6545	4.8092	5.6465	9.7036	$1.50654\cdot 10$	$2.5047[illegible]\cdot 10$	[illegible]
.014	-0.6	1.4523	1.5884	1.7720	2.0238	2.3752	2.8772	3.[illegible]147	4.7323	6.4897	9.3[illegible]	.[illegible]11
.016	-0.4	1.2711	1.3481	1.4491	1.5831	1.7629	2.0074	2.3462	2.8243	3.5150	4.[illegible]37[illegible]	.[illegible]1
.012	-0.2	1.1652	1.2101	1.2681	1.3432	1.4410	1.5694	1.7[illegible]95	1.9668	2.2739	2.6932	.[illegible]7
.0013	0.0	1.10161	1.12843	1.16245	1.20567	1.26078	1.331[illegible]0	1.42067	1.53522	1.66129	1.82157	[illegible]
.0003	0.1	1.079806	1.100573	1.120749	1.159709	1.201[illegible]99	1.253547	1.[illegible]19158	1.401075	1.502473	1.626044	.00[illegible]
.0003	0.2	1.062695	1.078778	1.098918	1.124064	1.150443	1.194[illegible]04	1.[illegible]42203	1.300643	1.370354	1.451449	[illegible]
.0004	0.3	1.049242	1.061690	1.077164	1.096305	1.119906	1.145[illegible]	1.[illegible]93372	1.224400	1.271450	1.[illegible]625	[illegible]
	0.4	1.038631	1.048237	1.060081	1.074575	1.092[illegible]33	1.113257	1.177994	1.166115	1.196453	1.[illegible]26[illegible]	
	0.5	1.030255	1.037630	1.046641	1.057519	1.0705[illegible]2	1.085687	1.1[illegible]2798	1.12[illegible]184	1.1[illegible]9076	1.[illegible]943	

Table A--Continued

Subdivision A_3

o/o error	ln η^{-1}	Values of Φ_0^* for ρ .1000	.1259	.1585	.1995	.2512	.3162	.3981	.5012	.6310	.7944	o/o error
.089	-2.0	$1.0356\cdot 10^{3}$	$2.899\cdot 10^{3}$	$9.22\cdot 10^{3}$ (1)	$3.36\cdot 10^{4}$ (3)	$1.40\cdot 10^{5}$ (8)	$6.7\cdot 10^{5}$					[illegible]
.058	-1.8	$1.8534\cdot 10^{2}$	$4.1524\cdot 10^{2}$	$1.0338\cdot 10^{3}$	$2.8927\cdot 10^{3}$	$9.20\cdot 10^{3}$ (1)	$3.35\cdot 10^{4}$ (3)	$1.40\cdot 10^{5}$ (8)	$6.6\cdot 10^{5}$			21.
.022	-1.6	$4.8533\cdot 10$	$9.0894\cdot 10$	$1.85189\cdot 10^{2}$	$4.1456\cdot 10^{2}$	$1.0309\cdot 10^{3}$	$2.881\cdot 10^{3}$	$9.144\cdot 10^{3}$ (2)	$3.32\cdot 10^{4}$	$1.38\cdot 10^{5}$		16.
.036	-1.4	$1.71789\cdot 10$	$2.7892\cdot 10$	$4.8428\cdot 10$	$9.0627\cdot 10$	$1.84437\cdot 10^{2}$	$4.1218\cdot 10^{2}$	$1.0228\cdot 10^{3}$	$2.8494\cdot 10^{3}$	$9.01\cdot 10^{3}$	$3.26\cdot 10^{4}$	[illegible]
.010	-1.2	7.7488	$1.12206\cdot 10$	$1.71415\cdot 10$	$2.77967\cdot 10$	$4.8186\cdot 10$	$8.9968\cdot 10$	$1.82556\cdot 10^{2}$	$4.0632\cdot 10^{2}$	$1.0025\cdot 10^{3}$	$2.771\cdot 10^{3}$	[illegible]
.024	-1.0	4.2402	5.6008	7.7218	$1.1165\cdot 10$	$1.7025\cdot 10$	$2.7534\cdot 10$	$4.7555\cdot 10$	$8.8338\cdot 10$	$1.7792\cdot 10^{2}$	$3.9194\cdot 10^{2}$	[illegible]
.037	-0.8	2.7109	3.3260	4.2216	5.5654	7.6536	$1.1026\cdot 10$	$1.6731\cdot 10$	$2.6875\cdot 10$	$4.5968\cdot 10$	$8.425\cdot 10$	.095
	-0.6	1.96214	2.27098	2.69891	3.3040	4.1819	5.4898	7.5052	$1.0725\cdot 10$	$1.6089\cdot 10$	$2.5457\cdot 10$	
.0065	-0.4	1.56224	1.72882	1.95110	2.2519	2.6661	3.2459	4.0756	5.2904	7.116	9.936	.064
	-0.2	1.33729	1.43131	1.55361	1.71371	1.92556	2.2078	2.5887	3.1078	3.8232	4.8181	
.0025	0.0	1.20476	1.25925	1.32862	1.41699	1.53010	1.6746	1.8596	2.0956	2.3945	2.7670	.046
.0002	0.1	1.159923	1.201590	1.254099	1.320166	1.403358	1.507449	1.637153	1.79681	1.98937	2.21307	.0030
	0.2	1.124934	1.156795	1.196535	1.245874	1.306944	1.381625	1.471869	1.57829	1.69883	1.82532	
	0.3	1.097558	1.121872	1.151861	1.188544	1.233061	1.286038	1.347644	1.41623	1.48681	1.54741	
	0.4	1.076044	1.094511	1.116987	1.143992	1.175971	1.21270	1.25317	1.29434	1.32964	1.34615	
.0003	0.5	1.059109	1.073022	1.089676	1.109231	1.13163	1.15606	1.18073	1.20169	1.21145	1.19641	.015

o/o error	ln η^{-1}	Values of Ψ_0 for ρ .1000	.1259	.1585	.1995	.2512	.3162	.3981	.5012	.6310	.7944	o/o error
	-2.0	$-1.356\cdot 10^{4}$	$-4.51\cdot 10^{4}$	$-1.66\cdot 10^{5}$	$-6.7\cdot 10^{5}$							
	-1.8	$-1.6401\cdot 10^{3}$	$-4.500\cdot 10^{3}$	$-1.355\cdot 10^{4}$	$-4.50\cdot 10^{4}$	$-1.66\cdot 10^{5}$	$-6.7\cdot 10^{5}$					
.018	-1.6	$-2.7687\cdot 10^{2}$	$-6.494\cdot 10^{2}$	$-1.6386\cdot 10^{3}$	$-4.494\cdot 10^{3}$	$-1.352\cdot 10^{4}$	$-4.49\cdot 10^{4}$ (4)	$-1.65\cdot 10^{5}$ (9)	$-6.7\cdot 10^{5}$			20.
.026	-1.4	$-5.9948\cdot 10$	$-1.2547\cdot 10^{2}$	$-2.7642\cdot 10^{2}$	$-6.479\cdot 10^{2}$	$-1.6334\cdot 10^{3}$	$-4.474\cdot 10^{3}$	$-1.344\cdot 10^{4}$	$-4.46\cdot 10^{4}$ (4)	$-1.63\cdot 10^{5}$ (9)	$-6.6\cdot 10^{5}$	19.
.011	-1.2	$-1.51772\cdot 10$	$-2.97568\cdot 10$	$-5.9855\cdot 10$	$-1.25195\cdot 10^{2}$	$-2.7550\cdot 10^{2}$	$-6.448\cdot 10^{2}$	$-1.6222\cdot 10^{3}$	$-4.431\cdot 10^{3}$	$-1.326\cdot 10^{4}$	$-4.38\cdot 10^{4}$	3.3
.033	-1.0	-3.9175	-7.7912	$-1.51571\cdot 10$	$-2.96933\cdot 10$	$-5.9649\cdot 10$	$-1.24522\cdot 10^{2}$	$-2.7332\cdot 10^{2}$	$-6.373\cdot 10^{2}$	$-1.5946\cdot 10^{3}$	$-4.323\cdot 10^{3}$	.35
.14	-0.8	-0.6422	-1.8146	-3.9143	-7.7756	$-1.51024\cdot 10$	$-2.95160\cdot 10$	$-5.9104\cdot 10$	$-1.22817\cdot 10^{2}$	$-2.67727\cdot 10^{2}$	$-6.1816\cdot 10^{2}$	.022
	-0.6	0.41397	0.02371	-0.64629	-1.8174	-3.9093	-7.7406	$-1.4974\cdot 10$	$-2.90967\cdot 10$	$-5.77732\cdot 10$	$-1.18609\cdot 10^{2}$	
.012	-0.4	0.78028	0.64076	0.40824	0.0148	-0.6549	-1.8216	-3.8916	-7.6467	$-1.46366\cdot 10$	$-2.80262\cdot 10$	.029
	-0.2	0.91359	0.86048	0.77374	0.63072	0.39419	-0.00255	-0.67539	-1.83095	-3.84442	-7.40898	
.0018	0.0	0.96357	0.94167	0.90641	0.84940	0.75682	0.60580	0.35801	-0.05064	-0.72899	-1.8558	.039
.0009	0.1	0.975396	0.960735	0.937244	0.89952	0.83875	0.74064	0.58172	0.32378	-0.09536	-0.77580	.013
	0.2	0.982754	0.972549	0.956268	0.930260	0.88865	0.82203	0.71525	0.54426	0.27102	-0.16327	
.0004	0.3	0.987336	0.979880	0.968029	0.949176	0.919170	0.87145	0.79561	0.67541	0.48588	0.18972	.137
	0.4	0.990201	0.984454	0.975341	0.960892	0.937980	0.901726	0.844445	0.75437	0.61371	0.39661	
.0002	0.5	0.991995	0.987314	0.979899	0.968169	0.949615	0.920349	0.874299	0.80224	0.69040	0.51914	.011

o/o error	ln η^{-1}	Values of $-\Psi_0^*$ for ρ .1000	.1259	.1585	.1995	.2512	.3162	.3981	.5012	.6310	.7944	o/o error
	-2.0	$6.78\cdot 10^{4}$	$2.44\cdot 10^{5}$	$9.7\cdot 10^{5}$	$4.2\cdot 10^{6}$							
	-1.8	$6.88\cdot 10^{3}$	$2.061\cdot 10^{4}$	$6.77\cdot 10^{4}$	$2.45\cdot 10^{5}$	$9.7\cdot 10^{5}$	$4.2\cdot 10^{6}$					
.041	-1.6	$9.8557\cdot 10^{2}$	$2.502\cdot 10^{3}$	$6.871\cdot 10^{3}$	$2.057\cdot 10^{4}$	$6.75\cdot 10^{4}$	$2.44\cdot 10^{5}$ (6)	$9.7\cdot 10^{5}$ (14)	$4.2\cdot 10^{6}$			30.
.026	-1.4	$1.8665\cdot 10^{2}$	$4.1541\cdot 10^{2}$	$9.8320\cdot 10^{2}$	$2.495\cdot 10^{3}$	$6.842\cdot 10^{3}$	$2.045\cdot 10^{4}$ (1)	$6.70\cdot 10^{4}$	$2.42\cdot 10^{5}$ (6)	$9.6\cdot 10^{5}$	$4.1\cdot 10^{6}$	29.
.020	-1.2	$4.3897\cdot 10$	$8.8325\cdot 10$	$1.8619\cdot 10^{2}$	$4.1400\cdot 10^{2}$	$9.7841\cdot 10^{2}$	$2.477\cdot 10^{3}$	$6.778\cdot 10^{3}$	$2.019\cdot 10^{4}$	$6.59\cdot 10^{4}$	$2.36\cdot 10^{5}$	5.8
.028	-1.0	$1.21479\cdot 10$	$2.2676\cdot 10$	$4.3796\cdot 10$	$8.8011\cdot 10$	$1.8517\cdot 10^{2}$	$4.1062\cdot 10^{2}$	$9.6695\cdot 10^{2}$	$2.436\cdot 10^{3}$	$6.619\cdot 10^{3}$	$1.955\cdot 10^{4}$	.71
.048	-0.8	3.7846	6.6914	$1.21195\cdot 10$	$2.2585\cdot 10$	$4.3514\cdot 10$	$8.7139\cdot 10$	$1.8248\cdot 10^{2}$	$4.0199\cdot 10^{2}$	$9.3775\cdot 10^{2}$	$2.332\cdot 10^{3}$	.042
	-0.6	1.2856	2.1880	3.7850	6.6770	$1.2055\cdot 10$	$2.2363\cdot 10$	$4.2824\cdot 10$	$8.5021\cdot 10$	$1.7586\cdot 10^{2}$	$3.8075\cdot 10^{2}$	
.039	-0.4	0.46593	0.77241	1.2941	2.1957	3.7828	6.6343	$1.18833\cdot 10$	$2.1800\cdot 10$	$4.1084\cdot 10$	$7.9754\cdot 10$	.029
	-0.2	0.17903	0.29152	0.47769	0.78873	1.31433	2.21339	3.77431	6.52491	$1.14498\cdot 10$	$2.04039\cdot 10$	
	0.0	0.07428	0.11945	0.19275	0.31220	0.50804	0.83083	1.36640	2.25934	3.75340	6.25242	
.038	0.1	0.049865	0.07982	0.12804	0.20589	0.33200	0.53682	0.8705	1.4141	2.2979	3.7228	.037
	0.2	0.034781	0.055464	0.08855	0.14153	0.22653	0.36286	0.58152	0.9309	1.4847	2.3490	
.040	0.3	0.025442	0.040451	0.06434	0.09935	0.16287	0.25898	0.41121	0.6506	1.0224	1.5871	.041
	0.4	0.019633	0.031148	0.04941	0.07833	0.12411	0.19629	0.30956	0.48560	0.7549	1.1548	
.057	0.5	0.016009	0.025362	0.04016	0.06353	0.10037	0.15818	0.24836	0.38746	0.5982	0.9079	.063

Table A--Continued

Subdivision A_4

o/o error	ln η^{-1}	Values of Φ_1 for ρ: .0100	.01259	.01585	.01995	.02512	.03162	.03981	.05012	.06310	.07944	o/o error
.011	-2.0	1.6118	1.8122	2.0933	2.4978	3.0986	4.0254	5.5213	8.0681	1.26894·10	2.17243·10	.034
	-1.8	1.3582	1.4662	1.6120	1.8120	2.0932	2.4976	3.0986	4.0253	5.5203	8.0659	
.014	-1.6	1.2155	1.2771	1.3581	1.4660	1.6118	1.8118	2.0928	2.4972	3.0977	4.0238	.024
	-1.5	1.1685	1.2155	1.2771	1.3581	1.4659	1.6117	1.8120	2.0928	2.4970	3.0973	
.013	-1.3	1.1039	1.1321	1.1683	1.2155	1.2771	1.3579	1.4658	1.6114	1.8113	2.0919	.011
	-1.1	1.0646	1.0818	1.1039	1.1321	1.1683	1.2155	1.2770	1.3579	1.4656	1.6112	
.014	-0.9	1.0403	1.0509	1.0645	1.0816	1.1037	1.1318	1.1680	1.2149	1.2763	1.3569	.015
	-0.7	1.02529	1.03192	1.04031	1.05094	1.06447	1.08166	1.10362	1.13176	1.16798	1.21483	
.0014	-0.5	1.01590	1.02004	1.02529	1.03190	1.04028	1.05090	1.06438	1.08153	1.10340	1.13138	.0063
	-0.3	1.01001	1.01261	1.01588	1.02002	1.02525	1.03184	1.04018	1.05074	1.06413	1.08113	
.0014	-0.1	1.00631	1.00793	1.00999	1.01258	1.01585	1.01996	1.02515	1.03170	1.03994	1.05037	.0017
	0.0	1.00500	1.00630	1.00793	1.00998	1.01256	1.01581	1.01991	1.02506	1.03154	1.03971	
.0002	0.1	1.003968	1.004994	1.006286	1.007908	1.009953	1.012520	1.015850	1.019809	1.024907	1.031307	.0009
.0002	0.3	1.002499	1.003143	1.003953	1.004969	1.006248	1.007848	1.009856	1.012370	1.015511	1.019426	.0009
.0002	0.5	1.001592	1.001976	1.002483	1.003118	1.003914	1.004909	1.006150	1.007697	1.009615	1.011985	.0009

o/o error	ln η^{-1}	Values of Φ_1^* for ρ: .0100	.01259	.01585	.01995	.02512	.03162	.03981	.05012	.06310	.07944	o/o error
.011	-2.0	3.9606	4.6461	5.6362	7.1108	9.3919	1.30744·10	1.93394·10	3.0653·10	5.2561·10	9.8552·10	.033
	-1.9	3.4722	3.9589	4.6438	5.6329	7.1077	9.3866	1.30693·10	1.9332·10	3.0640·10	5.2541·10	
.012	-1.7	2.8587	3.1193	3.4723	3.9583	4.6434	5.6323	7.1060	9.3845	1.30637·10	1.9320·10	.055
	-1.5	2.5158	2.6637	2.8587	3.1193	3.4723	3.9587	4.6433	5.6323	7.1054	9.3821	
	-1.4	2.4021	2.5151	2.6629	2.8577	3.1181	3.4706	3.9565	4.6404	5.6282	7.099	
.035	-1.3	2.3154	2.4026	2.5157	2.6637	2.8590	3.1194	3.4722	3.9583	4.6426	5.628	.11
	-1.2	2.2479	2.3155	2.4028	2.5157	2.6638	2.8588	3.1193	3.4719	3.9577	4.6410	
	-1.1	2.1952	2.2478	2.3154	2.4026	2.5157	2.6634	2.8586	3.1188	3.4712	3.9564	
.0067	-0.9	2.1215	2.1537	2.1949	2.2475	2.3150	2.4018	2.5148	2.6622	2.8566	3.1159	.011
	-0.7	2.0761	2.0961	2.1215	2.1537	2.1948	2.2473	2.3146	2.4014	2.5138	2.6605	
.030	-0.5	2.0478	2.0603	2.0760	2.0960	2.1214	2.1535	2.1945	2.2468	2.3137	2.3998	.19
	-0.3	2.03004	2.03786	2.04772	2.06016	2.07591	2.09580	2.12100	2.15296	2.19356	2.24523	
.0007	-0.1	2.01890	2.02381	2.02998	2.03776	2.04758	2.05993	2.07552	2.09520	2.12003	2.15139	.0029
	0.0	2.01500	2.01889	2.02376	2.02993	2.03768	2.04743	2.05971	2.07516	2.09462	2.11909	
.0003	0.1	2.011900	2.014977	2.018846	2.023709	2.029834	2.037522	2.047190	2.059328	2.074560	2.093654	.0006
	0.3	2.007485	2.009418	2.011841	2.014878	2.018695	2.023469	2.029448	2.036918	2.046224	2.057788	
.0001	0.5	2.004707	2.005914	2.007428	2.009319	2.011686	2.014635	2.018307	2.022860	2.028480	2.035376	.0006

o/o error	ln η^{-1}	Values of Ψ_1 for ρ: .0100	.01259	.01585	.01995	.02512	.03162	.03981	.05012	.06310	.07944	o/o error
.21	-2.0	0.4336	-0.2121	-2.3312	-9.086	-3.0442·10	-9.677·10	$-3.2397\cdot10^2$	$-1.0999\cdot10^3$	$-3.933\cdot10^3$	$-1.502\cdot10^4$	
.030	-1.6	0.7486	0.7195	0.6871	0.6253	0.4342	-0.2115	-2.3306	-9.087	-3.0449·10	-9.878·10	.048
	-1.4	0.8102	0.7792	0.7485	0.7196	0.6871	0.6250	0.4335	-0.2132	-2.3354	-9.0968	
	-1.2	0.8664	0.8396	0.8102	0.7794	0.7487	0.7197	0.6872	0.6250	0.4326	-0.2158	
.019	-1.1	0.8899	0.8664	0.8396	0.8103	0.7794	0.7489	0.7198	0.6874	0.6250	0.4321	.17
	-0.9	0.9269	0.9100	0.8899	0.8666	0.8400	0.8111	0.7800	0.7495	0.7205	0.6881	
.0018	-0.7	0.95244	0.94095	0.92698	0.91017	0.89016	0.86685	0.84034	0.81126	0.78085	0.75096	.0084
.0015	-0.5	0.96943	0.96185	0.95252	0.94109	0.92718	0.91047	0.89064	0.86758	0.84146	0.81300	.011
.0014	-0.3	0.98050	0.97559	0.96951	0.96198	0.95272	0.94139	0.92766	0.91122	0.89180	0.86940	.0023
	-0.1	0.98762	0.98448	0.98056	0.97571	0.96969	0.96228	0.95317	0.94213	0.92882	0.91305	
.0010	0.0	0.99015	0.98765	0.98453	0.98065	0.97583	0.96988	0.96257	0.95364	0.94285	0.92998	.0015
.0001	0.1	0.992170	0.990179	0.987694	0.984604	0.980761	0.976014	0.970169	0.963027	0.954371	0.944009	.0004
.0002	0.3	0.995063	0.993809	0.992245	0.990300	0.987884	0.984903	0.981237	0.976765	0.971360	0.964912	.0004
.0002	0.5	0.996898	0.996114	0.995139	0.993931	0.992436	0.990602	0.988362	0.985658	0.982434	0.978659	.0003

Table A--Continued

Subdivision A_5

o/o error	$\ln \eta^{-1}$	Values of $-\Psi_1^*$ for ρ .01000	.01259	.01585	.01995	.02512	.03162	.03981	.05012	.06310	.07944	o/o error
.032	-2.0	1.8703	4.6094	$1.3265\cdot10$	$4.0293\cdot10$	$1.2596\cdot10^2$	$4.0635\cdot10^2$	$1.3669\cdot10^3$	$4.850\cdot10^3$	$1.835\cdot10^4$	$7.48\cdot10^4$	1.5
.023	-1.6	0.8784	0.8455	0.8579	1.0557	1.8711	4.6104	$1.3268\cdot10$	$4.0304\cdot10$	$1.2599\cdot10^2$	$4.0636\cdot10$	.046
	-1.4	0.9423	0.9141	0.8785	0.8456	0.8583	1.0565	1.8726	4.6157	$1.3280\cdot10$	$4.0320\cdot10$	
	-1.2	0.9757	0.9623	0.9423	0.9142	0.8784	0.8456	0.8588	1.0579	1.8770	4.6278	
.014	-1.1	0.9844	0.9756	0.9623	0.9422	0.9141	0.8783	0.8457	0.8589	1.0595	1.8817	.067
.0017	-0.9	0.99369	0.99006	0.98436	0.97554	0.96208	0.94205	0.91376	0.87807	0.84584	0.86049	.016
	-0.7	0.99745	0.99596	0.99361	0.98994	0.98418	0.97527	0.96168	0.94160	0.91310	0.87749	
.0010	-0.5	0.99895	0.99834	0.99737	0.99584	0.99343	0.98966	0.98375	0.97460	0.96070	0.94012	.0066
	-0.3	0.99955	0.99929	0.99887	0.99822	0.99719	0.99558	0.99297	0.98894	0.98264	0.97291	
.0010	-0.1	0.99979	0.99967	0.99948	0.99917	0.99863	0.99792	0.99672	0.99480	0.99181	0.98712	.0018
	0.0	0.99985	0.99976	0.99962	0.99950	0.99905	0.99850	0.93763	0.99625	0.99408	0.99068	
.0001	0.1	0.999869	0.999821	0.999716	0.999550	0.999287	0.998871	0.998213	0.997172	0.995532	0.992950	.0004
.0001	0.3	0.999925	0.999881	0.999811	0.999701	0.999526	0.999250	0.998811	0.998119	0.997024	0.995298	.0003
.0001	0.5	0.999940	0.999905	0.999849	0.999761	0.999621	0.999401	0.999050	0.998497	0.997620	0.996237	.0003

o/o error	$\ln \eta^{-1}$	Values of Φ_1 for ρ .1000	.1259	.1585	.1995	.2512	.3162	.3981	.5012	.6310	.7944	o/o error
	-2.0	$4.0990\cdot10$	$8.6411\cdot10$	$2.0652\cdot10^2$	$5.677\cdot10^2$	$1.8169\cdot10^3$	$6.81\cdot10^3$	$2.97\cdot10^4$	$1.484\cdot10^5$			
.072	-1.8	$1.2684\cdot10$	$2.1710\cdot10$	$4.0953\cdot10$	$8.630\cdot10$	$2.0613\cdot10^2$	$5.66\cdot10^2$	$1.81\cdot10^3$ (2)	$6.77\cdot10^3$ (6)	$2.9\cdot10^4$ (17)	$1.4\cdot10^5$	41.
.023	-1.6	5.5176	8.0605	$1.26724\cdot10$	$2.1681\cdot10$	$4.0871\cdot10$	$8.6035\cdot10$	$2.052\cdot10^2$	$5.627\cdot10^2$	$1.79\cdot10^3$ (2)	$6.7\cdot10^3$	6.1
.030	-1.4	3.0966	4.0213	5.5123	8.0474	$1.26419\cdot10$	$2.16010\cdot10$	$4.0651\cdot10$	$8.5363\cdot10$	$2.0288\cdot10^2$	$5.535\cdot10^2$	.54
.0096	-1.2	2.0915	2.4949	3.0928	4.0135	5.4969	8.0134	$1.25626\cdot10$	$2.1403\cdot10$	$4.0099\cdot10$	$8.3673\cdot10$	.026
.012	-1.0	1.6104	1.8097	2.0888	2.4896	3.0834	3.9950	5.4583	7.9306	$1.2368\cdot10$	$2.0913\cdot10$	.022
.0067	-0.8	1.35694	1.46405	1.60834	1.80577	2.08196	2.4770	3.0596	3.9486	5.3625	7.7225	.014
.0075	-0.6	1.21441	1.27520	1.35495	1.46062	1.60267	1.79605	2.0647	2.4459	3.0009	3.8343	.024
.0082	-0.4	1.13099	1.16671	1.21273	1.27232	1.35024	1.45267	1.5890	1.7724	2.0226	2.3694	.031
.0009	-0.2	1.08050	1.10214	1.12932	1.16396	1.20829	1.26506	1.33814	1.43247	1.55463	1.71316	.0062
.0016	0.0	1.04997	1.06289	1.07914	1.09953	1.12515	1.15720	1.19728	1.24699	1.30827	1.38269	.041
.0005	0.1	1.039321	1.049365	1.061920	1.077561	1.097034	1.121088	1.15066	1.18661	1.22959	1.27966	.0031
	0.2	1.030921	1.038718	1.048401	1.060365	1.075102	1.093049	1.11470	1.14032	1.16981	1.20219	
.0005	0.3	1.024293	1.030329	1.037768	1.046873	1.057948	1.071202	1.08681	1.10466	1.12414	1.14372	.0035
	0.4	1.019048	1.023698	1.029378	1.036247	1.044467	1.054085	1.06504	1.07694	1.08884	1.09877	
.0005	0.5	1.014898	1.018455	1.022752	1.027868	1.033859	1.040643	1.04800	1.05532	1.06143	1.06408	.0038

o/o error	$\ln \eta^{-1}$	Values of Φ_1^* for ρ .1000	.1259	.1585	.1995	.2512	.3162	.3981	.5012	.6310	.7944	o/o error
	-2.0	$2.044\cdot10^2$	$4.75\cdot10^2$	$1.25\cdot10^3$	$3.81\cdot10^3$	$1.34\cdot10^4$						
	-1.9	$9.850\cdot10$	$2.0430\cdot10^2$	$4.749\cdot10^2$	$1.253\cdot10^3$	$3.81\cdot10^3$	$1.34\cdot10^4$	$5.5\cdot10^4$	$2.6\cdot10^5$			
.055	-1.7	$3.0614\cdot10$	$5.2480\cdot10$	$9.836\cdot10$	$2.0390\cdot10^2$	$4.736\cdot10^2$	$1.249\cdot10^3$	$3.79\cdot10^3$ (1)	$1.33\cdot10^4$	$5.4\cdot10^4$	$2.5\cdot10^5$	24.
.12	-1.4	9.372	$1.3042\cdot10$	$1.9276\cdot10$	$3.0518\cdot10$	$5.2252\cdot10$	$9.774\cdot10$	$2.0211\cdot10^2$	$4.677\cdot10^2$	$1.227\cdot10^3$	$3.69\cdot10^3$	1.
.026	-1.2	5.6262	7.094	9.355	$1.3010\cdot10$	$1.9202\cdot10$	$3.0339\cdot10$	$5.1801\cdot10$	$9.6523\cdot10$	$1.9848\cdot10^2$	$4.5574\cdot10^2$	.058
.0051	-1.0	3.9542	4.6348	5.6159	7.0724	9.3170	$1.29207\cdot10$	$1.90079\cdot10$	$2.9391\cdot10$	$5.0684\cdot10$	$9.3523\cdot10$	.025
.026	-0.8	3.1145	3.4640	3.9439	4.6163	5.5835	7.0118	9.139	$1.2684\cdot10$	$1.8501\cdot10$	$2.8741\cdot10$	.030
.010	-0.6	2.6592	2.8514	3.1067	3.4502	3.9203	4.5737	5.5063	6.8681	8.9191	$1.2116\cdot10$	.026
.045	-0.4	2.3976	2.5088	2.6509	2.8377	3.085	3.4134	3.8573	4.4650	5.3101	6.5061	.14
.0040	-0.2	2.24365	2.30865	2.39154	2.4975	2.6350	2.8088	3.0354	3.3293	3.7115	4.2082	.056
.0019	0.0	2.14986	2.18857	2.23719	2.29813	2.3746	2.4698	2.5881	2.7337	2.9102	3.1187	.073
.0007	0.1	2.117542	2.14741	2.18463	2.23081	2.28797	2.35798	2.44[illegible]95	2.54423	2.66156	2.79097	.0043
	0.2	2.092119	2.11511	2.14352	2.17836	2.22084	2.27172	2.3[illegible]93	2.40075	2.47549	2.54940	
.0007	0.3	2.072096	2.08973	2.11128	2.13736	2.16857	2.20504	2.[illegible]4645	2.29105	2.33469	2.36878	.005.
	0.4	2.056272	2.06970	2.08591	2.10517	2.12766	2.15300	2.18013	2.20650	2.22697	2.23195	
.0007	0.5	2.043772	2.05389	2.06591	2.07985	2.09555	2.11205	2.1[illegible]46	2.14095	2.14398	2.12727	.0057

Table A--Continued

Subdivision A_6

o/o error	ln η^{-1}	Values of Ψ_1 for ρ: .1000	.1259	.1585	.1995	.2512	.3162	.3981	.5012	.6310	.7944	o/o error
.052	-1.6	$-3.2395\cdot10^2$	$-1.0995\cdot10^3$	$-3.9301\cdot10^3$	$-1.500\cdot10^4$	$-6.19\cdot10^4$ (2)	$-2.78\cdot10^5$ (5)	$-1.83\cdot10^6$	$-7.3\cdot10^6$			24.
.042	-1.4	$-3.0471\cdot10$	$-9.883\cdot10$	$-3.2396\cdot10^2$	$-1.0988\cdot10^3$	$-3.924\cdot10^3$	$-1.436\cdot10^4$	$-6.16\cdot10^4$ (2)	$-2.76\cdot10^5$ (5)	$-1.35\cdot10^6$	$-7.2\cdot10^6$	23.
.067	-1.2	-2.3436	-9.1214	$-3.0532\cdot10$	$-9.895\cdot10$	$-3.2400\cdot10^2$	$-1.0974\cdot10^3$	$-3.912\cdot10^3$	$-1.488\cdot10^4$	$-6.11\cdot10^4$	$-2.73\cdot10^5$	3.6
.22	-1.0	0.4312	-0.2237	-2.3687	-9.188	$-3.0694\cdot10$	$-9.927\cdot10$	$-3.2422\cdot10^2$	$-1.0943\cdot10^3$	$-3.8614\cdot10^3$	$-1.466\cdot10^4$	.57
.013	-0.8	0.88882	0.6250	0.4265	-0.2419	-2.4249	-9.344	$-3.1077\cdot10$	$-1.0001\cdot10^2$	$-3.2450\cdot10^2$	$-1.0853\cdot10^3$	.044
.012	-0.6	0.75242	0.72432	0.69203	0.6257	0.4164	-0.2866	-2.5664	-9.734	$-3.2015\cdot10$	$-1.0174\cdot10^2$	
.0021	-0.4	0.81466	0.78594	0.75816	0.7319	0.69952	0.6265	0.38944	-0.3999	-2.9186	$-1.0667\cdot10$	.043
.0020	-0.2	0.87114	0.84699	0.82142	0.79601	0.77235	0.74987	0.71694	0.6257	0.3177	-0.6839	.11
.0015	0.0	0.91488	0.89757	0.87840	0.85822	0.83841	0.82102	0.80713	0.7928	0.7547	0.6121	.11
.0005	0.1	0.931807	0.917742	0.902055	0.88541	0.86892	0.85458	0.84444	0.83893	0.8301	0.7823	.027
	0.2	0.945844	0.934688	0.922270	0.90916	0.89634	0.88567	0.87951	0.88003	0.8847	0.8811	
.0001	0.3	0.957360	0.948726	0.939226	0.929409	0.920226	0.913442	0.911514	0.91741	0.93261	0.95119	.0078
	0.4	0.966751	0.960255	0.953290	0.946412	0.940590	0.937571	0.939880	0.95088	0.97260	1.00681	
.0001	0.5	0.974360	0.969651	0.964830	0.960495	0.957655	0.958077	0.964370	0.98015	1.00928	1.05318	.0082

o/o error	ln η^{-1}	Values of $-\Psi_1^*$ for ρ: .1000	.1259	.1585	.1995	.2512	.3162	.3981	.5012	.6310	.7944	o/o error
.082	-1.6	$1.3666\cdot10^3$	$4.847\cdot10^3$	$1.834\cdot10^4$	$7.47\cdot10^4$ (2)	$3.30\cdot10^5$	$1.60\cdot10^6$ (8)	$8.3\cdot10^6$ (18)	$4.6\cdot10^7$			28.
.012	-1.4	$1.26042\cdot10^2$	$4.0639\cdot10^2$	$1.3660\cdot10^3$	$4.841\cdot10^3$	$1.829\cdot10^4$	$7.44\cdot10^4$ (4)	$3.28\cdot10^5$	$1.58\cdot10^6$ (8)	$8.2\cdot10^6$ (17)	$4.6\cdot10^7$	35.
.045	-1.2	$1.3313\cdot10$	$4.0409\cdot10$	$1.2619\cdot10^2$	$4.0648\cdot10^2$	$1.3644\cdot10^3$	$4.827\cdot10^3$	$1.820\cdot10^4$	$7.38\cdot10^4$ (1)	$3.24\cdot10^5$	$1.55\cdot10^6$	5.8
.054	-1.0	1.8891	4.6620	$1.3402\cdot10$	$4.0624\cdot10$	$1.2662\cdot10^2$	$4.0684\cdot10^2$	$1.36106\cdot10^3$	$4.793\cdot10^3$	$1.796\cdot10^4$	$7.22\cdot10^4$	1.06
.014	-0.8	0.86222	1.0711	1.9174	4.7408	$1.3606\cdot10$	$4.1109\cdot10$	$1.2757\cdot10^2$	$4.0740\cdot10^2$	$1.3514\cdot10^3$	$4.7034\cdot10^3$	.069
.011	-0.6	0.87697	0.8466	0.8685	1.0948	1.9892	4.9405	$1.4124\cdot10$	$4.2331\cdot10$	$1.2987\cdot10^2$	$4.0825\cdot10^2$	.048
.0021	-0.4	0.93879	0.90980	0.87481	0.84848	0.88468	1.1543	2.1685	5.4289	$1.53503\cdot10$	$4.5041\cdot10$	.035
.0018	-0.2	0.97130	0.95586	0.93349	0.90349	0.87015	0.85458	0.92843	1.3086	2.6193	6.6152	.024
.0018	0.0	0.98537	0.97716	0.96471	0.94632	0.92607	0.88899	0.8617	0.8772	1.0518	1.7132	.074
.0005	0.1	0.988908	0.982621	0.971759	0.95846	0.93744	0.90939	0.87610	0.85846	0.9098	1.1920	.027
	0.2	0.991156	0.986113	0.978316	0.966451	0.948878	0.92415	0.89276	0.86254	0.8629	0.9811	
.0001	0.3	0.992584	0.988334	0.981735	0.971617	0.956420	0.934464	0.904938	0.87123	0.84985	0.89282	.0078
	0.4	0.993487	0.989745	0.983914	0.974929	0.961314	0.941322	0.913533	0.87914	0.84807	0.85450	
.0001	0.5	0.994059	0.990638	0.985297	0.977039	0.964455	0.945787	0.919327	0.88511	0.84955	0.83742	.0082

o/o error	ln η^{-1}	Values of Φ_1 for ρ: 1.000	1.259	1.585	1.995	2.512	3.162	3.981	5.012	6.310	7.944	o/o error
2.1	-1.4	$1.75\cdot10^3$	$6.5\cdot10^3$ (5.5)	$2.8\cdot10^4$ (15)	$1.4\cdot10^5$ (36)							
.11	-1.2	$1.971\cdot10^2$	$5.31\cdot10^2$	$1.657\cdot10^3$	$6.0\cdot10^3$ (4.3)	$2.5\cdot10^4$ (12)	$1.2\cdot10^5$ (29)					
	-1.0	$3.7754\cdot10$	$7.860\cdot10$	$1.8231\cdot10^2$	$4.78\cdot10^2$	$1.430\cdot10^3$	$4.93\cdot10^3$	$1.955\cdot10^4$	$8.87\cdot10^4$			
.017	-0.8	$1.1886\cdot10$	$1.9718\cdot10$	$3.5532\cdot10$	$7.003\cdot10$	$1.5193\cdot10^2$	$3.640\cdot10^2$	$9.86\cdot10^2$	$2.88\cdot10^3$ (1)	$8.8\cdot10^3$ (4.6)	$2.9\cdot10^4$	14.
.025	-0.6	5.129	7.222	$1.0752\cdot10$	$1.6972\cdot10$	$2.8412\cdot10$	$5.020\cdot10$	$9.25\cdot10$	$1.73\cdot10^2$	$3.1\cdot10^2$ (5.5)	$4.5\cdot10^2$	39.
.032	-0.4	2.858	3.577	4.579	6.076	8.281	$1.1424\cdot10$	$1.5592\cdot10$	$2.033\cdot10$	$2.584\cdot10$	$5.27\cdot10$	.95
.052	-0.2	1.919	2.184	2.520	2.931	3.395	3.822	4.004	3.589	2.332		.26
.068	0.0	1.471	1.571	1.676	1.767	1.806	1.725	1.434	0.90			5.5
.0041	0.1	1.33544	1.39272	1.44160	1.46239	1.42093	1.2651	0.938 (.27)	0.43			5.0
	0.2	1.23524	1.26306	1.27506	1.25230	1.16545	0.9759	0.656	0.25			
.0047	0.3	1.16012	1.16726	1.15427	1.10352	0.99003	0.7857	0.482 (.4)	0.15			11.
	0.4	1.10308	1.09537	1.06515	0.99565	0.86610	0.6561	0.370	0.10			
.0073	0.5	1.05939	1.04082	0.99807	0.91610	0.77662	0.565	0.295				2.6

Table A--Concluded

Subdivision A_7

o/o error	ln η^{-1}	Values of Φ_1^* for ρ: 1.000	1.259	1.585	1.995	2.512	3.162	3.981	5.012	6.310	7.944	o/o error
3.2	-1.4	$1.29\cdot10^4$	$5.2\cdot10^4$(8.6	$2.4\cdot10^5$ (22)								
.022	-1.2	$1.1832\cdot10^3$	$3.514\cdot10^3$	$1.20\cdot10^4$ (2)	$4.8\cdot10^4$(6.7	$2.2\cdot10^5$ (18)	$1.1\cdot10^6$ (40)					
.028	-1.0	$1.8965\cdot10^2$	$4.2703\cdot10^2$	$1.0790\cdot10^3$	$3.089\cdot10^3$	$1.010\cdot10^4$	$3.78\cdot10^4$ (3)	$1.62\cdot10^5$ (7)	$7.8\cdot10^5$ (16)	$4.2\cdot10^6$ (36)		
.035	-0.8	$4.7881\cdot10$	$8.616\cdot10$	$1.6857\cdot10^2$	$3.606\cdot10^2$	$8.470\cdot10^2$	$2.187\cdot10^3$	$6.19\cdot10^3$	$1.90\cdot10^4$	$6.2\cdot10^4$ (8)	$2.1\cdot10^5$	24.
.028	-0.6	$1.7291\cdot10$	$2.6029\cdot10$	$4.1431\cdot10$	$6.976\cdot10$	$1.2385\cdot10^2$	$2.296\cdot10^2$	$4.352\cdot10^2$	$8.05\cdot10^2$ (3)	$1.3\cdot10^3$ (15)		
.13	-0.4	8.228	$1.0749\cdot10$	$1.4477\cdot10$	$2.000\cdot10$	$2.7990\cdot10$	$3.874\cdot10$	$5.107\cdot10$	$6.131\cdot10$	$3.684\cdot10$	$4.014\cdot10^2$	.15
.020	-0.2	4.850	5.667	6.667	7.794	8.824	9.180	7.788	3.562	-2.37 (5)	$-1.53\cdot10$	4.9
.11	0.0	3.355	3.596	3.790	3.827	3.48	2.71 (1.4)	1.05 (8.2)				
.0052	0.1	2.92098	3.02586	3.0537	2.9115	2.4561	1.517	0.04	-1.6 (15)			
	0.2	2.60778	2:62275	2.5443	2.2922	1.7591	0.853	-0.35	-1.2			
.0064	0.3	2.37721	2.33177	2.1859	1.8711	1.3081	0.4582	-0.52 (4.2)	-0.8 (22)			
	0.4	2.20471	2.11755	1.9275	1.5762	1.0056	0.2236	-0.59	-0.55			
.0052	0.5	2.07373	1.95759	1.7379	1.3651	0.7968	0.057 (10)	-0.62 (9.7)				

o/o error	ln η^{-1}	Values of Ψ_1 for ρ: 1.000	1.259	1.585	1.995	2.512	3.162	3.981	5.012	6.310	7.944	o/o error
1.6	-1.0	$-5.97\cdot10^4$ (2)	$-2.55\cdot10^5$	$-1.27\cdot10^6$ (8)	$-6.6\cdot10^6$ (17)	$-3.8\cdot10^7$ (32)						
.088	-0.8	$-3.801\cdot10^3$	$-1.412\cdot10^4$	$-5.61\cdot10^4$	$-2.41\cdot10^5$	$-1.12\cdot10^6$ (3)	$-5.6\cdot10^6$ (6)	$-3.1\cdot10^7$ (6)	$-1.8\cdot10^8$ (17)	$-1.1\cdot10^9$ (25)		
.043	-0.6	$-3.2471\cdot10^2$	$-1.0558\cdot10^3$	$-3.595\cdot10^3$	$-1.274\cdot10^4$	$-4.74\cdot10^4$ (2)	$-1.86\cdot10^5$ (6)	$-7.7\cdot10^5$ (7)				
.038	-0.4	$-3.4161\cdot10$	$-1.0523\cdot10^2$	$-3.2175\cdot10^2$	$-9.88\cdot10^2$	$-3.047\cdot10^3$	$-9.32\cdot10^3$	$-2.72\cdot10^4$ (5)	$-6.6\cdot10^4$ (20)			
.15	-0.2	-3.757	$-1.2774\cdot10$	$-3.8394\cdot10$	$-1.0899\cdot10^2$	$-2.952\cdot10^2$	$-7.45\cdot10^2$ (3)	$-1.61\cdot10^3$ (6)				
.075	0.0	0.12141	-1.3861	-5.6383	$-1.6777\cdot10$	$-4.369\cdot10$	$-1.021\cdot10^2$	$-2.15\cdot10^2$ (4)	$-5.0\cdot10^2$ (28)			
.087	0.1	0.5814	-0.0915	-2.048	-7.157 (.3)	$-1.916\cdot10$	$-4.4\cdot10$ (5)	$-9.\cdot10$ (23)				
	0.2	0.8088	0.5020	-0.4638	-3.04	-9.03	$-2.1\cdot10$	$-4.6\cdot10$				
.0082	0.3	0.9423	0.8106	0.3079	-1.12 (1.3)	-4.46 (3.6)	$-1.1\cdot10$ (16)					
	0.4	1:03353	0.9936	0.7252	-0.133	-2.23	-6.4					
.0050	0.5	1.10213	1.1147	0.9732	0.410 (1.2)	-1.05 (4.6)	-4.1 (11)					

o/o error	ln η^{-1}	Values of $-\Psi_1^*$ for ρ: 1.000	1.259	1.585	1.995	2.512	3.162	3.981	5.012	6.310	7.944	o/o error
3.1	-1.0	$3.14\cdot10^5$	$1.49\cdot10^6$ (6)	$5.7\cdot10^6$ (18)	$4.2\cdot10^7$ (24)							
.15	-0.8	$1.7344\cdot10^4$	$6.47\cdot10^4$	$2.89\cdot10^5$ (1)	$1.32\cdot10^6$	$6.6\cdot10^6$(4.4)	$3.5\cdot10^7$(8.2)	$2.0\cdot10^8$ (14)	$1.2\cdot10^9$ (21)	$7.8\cdot10^9$ (25)		
.053	-0.6	$1.3249\cdot10^3$	$4.471\cdot10^3$	$1.579\cdot10^4$	$5.86\cdot10^4$	$2.29\cdot10^5$ (2)	$9.4\cdot10^5$(4.5)	$4.1\cdot10^6$ (12)	$1.8\cdot10^7$ (29)			
.034	-0.4	$1.3447\cdot10^2$	$4.0582\cdot10^2$	$1.2415\cdot10^3$	$3.838\cdot10^3$	$1.185\cdot10^4$	$3.54\cdot10^4$ (3)	$9.3\cdot10^4$ (12)				
.026	-0.2	$1.8173\cdot10$	$5.064\cdot10$	$1.3960\cdot10^2$	$3.744\cdot10^2$	$9.47\cdot10^2$	$2.08\cdot10^3$ (7)	$2.8\cdot10^3$ (49)				
.027	0.0	3.7280	9.263	$2.3468\cdot10$	$5.710\cdot10$	$1.277\cdot10^2$	$2.50\cdot10^2$ (3)	$4{:}6\cdot10^2$ (22)				
.026	0.1	2.1271	4.7539	$1.13864\cdot10$	$2.645\cdot10$	$5.59\cdot10$	$1.0\cdot10^2$ (19)					
	0.2	1.4476	2.8156	6.2718	$1.39\cdot10$	$2.80\cdot10$	$5.0\cdot10$					
.013	0.3	1.1401	1.9252	3.9434	8.34	$1.62\cdot10$	$2.8\cdot10$ (61)					
	0.4	0.9932	1.4904	2.8128	5.70	$1.08\cdot10$	$1.8\cdot10$					
.030	0.5	0.9196	1.2678	2.2363	3.83	8.2	$1.4\cdot10$ (29)					

Table B--Values of A_j, coefficients for Φ_0

$[A_1 \equiv 1.000]$

$\ln \eta^{-1}$	η	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
-2.0	$1.000\cdot10^{2}$	$1.000\cdot10^{2}$	$3.333\cdot10^{3}$	$5.554\cdot10^{4}$	$5.552\cdot10^{5}$	$3.700\cdot10^{6}$	$1.760\cdot10^{7}$	$6.279\cdot10^{7}$	$1.742\cdot10^{8}$	$3.864\cdot10^{8}$
-1.9	$7.935\cdot10$	$7.935\cdot10$	$2.103\cdot10^{3}$	$2.783\cdot10^{4}$	$2.212\cdot10^{5}$	$1.167\cdot10^{6}$	$4.417\cdot10^{6}$	$1.250\cdot10^{7}$	$2.751\cdot10^{7}$	$4.848\cdot10^{7}$
-1.8	$6.310\cdot10$	$6.310\cdot10$	$1.326\cdot10^{3}$	$1.394\cdot10^{4}$	$8.795\cdot10^{4}$	$3.694\cdot10^{5}$	$1.108\cdot10^{6}$	$2.489\cdot10^{6}$	$4.348\cdot10^{6}$	$6.070\cdot10^{6}$
-1.7	$5.010\cdot10$	$5.010\cdot10$	$8.374\cdot10^{2}$	$6.986\cdot10^{3}$	$3.496\cdot10^{4}$	$1.165\cdot10^{5}$	$2.774\cdot10^{5}$	$4.549\cdot10^{5}$	$6.848\cdot10^{5}$	$7.568\cdot10^{5}$
-1.6	$3.981\cdot10$	$3.981\cdot10$	$5.281\cdot10^{2}$	$3.501\cdot10^{3}$	$1.390\cdot10^{4}$	$3.680\cdot10^{4}$	$6.941\cdot10^{4}$	$9.802\cdot10^{4}$	$1.074\cdot10^{5}$	$9.439\cdot10^{4}$
-1.5	$3.162\cdot10$	$3.162\cdot10$	$3.331\cdot10^{2}$	$1.752\cdot10^{3}$	$5.527\cdot10^{3}$	$1.158\cdot10^{4}$	$1.732\cdot10^{4}$	$1.936\cdot10^{4}$	$1.676\cdot10^{4}$	$1.155\cdot10^{4}$
-1.4	$2.512\cdot10$	$2.512\cdot10$	$2.101\cdot10^{2}$	$8.777\cdot10^{2}$	$2.194\cdot10^{3}$	$3.644\cdot10^{3}$	$4.304\cdot10^{3}$	$3.799\cdot10^{3}$	$2.591\cdot10^{3}$	$1.403\cdot10^{3}$
-1.3	$1.995\cdot10$	$1.995\cdot10$	$1.324\cdot10^{2}$	$4.390\cdot10^{2}$	$8.694\cdot10^{2}$	$1.140\cdot10^{3}$	$1.063\cdot10^{3}$	$7.381\cdot10^{2}$	$3.942\cdot10^{2}$	$1.665\cdot10^{2}$
-1.2	$1.585\cdot10$	$1.585\cdot10$	$8.358\cdot10$	$2.193\cdot10^{2}$	$3.433\cdot10^{2}$	$3.557\cdot10^{2}$	$2.602\cdot10^{2}$	$1.409\cdot10^{2}$	$5.841\cdot10$	$1.901\cdot10$
-1.1	$1.259\cdot10$	$1.259\cdot10$	$5.266\cdot10$	$1.094\cdot10^{2}$	$1.351\cdot10^{2}$	$1.096\cdot10^{2}$	$6.258\cdot10$	$2.618\cdot10$	8.290	2.027
-1.0	$1.000\cdot10$	$1.000\cdot10$	$3.316\cdot10$	$5.444\cdot10$	$5.277\cdot10$	$3.337\cdot10$	$1.463\cdot10$	4.632	1.083	0.1892
-0.9	7.945	7.945	$2.086\cdot10$	$2.695\cdot10$	$2.036\cdot10$	9.880	3.254	0.7474	0.1194	$1.281/10^{2}$
-0.8	6.310	6.310	$1.309\cdot10$	$1.324\cdot10$	7.708	2.799	6.577/10	$9.817/10^{2}$	$3.078/10^{3}$	$4.556/10^{5}$
-0.7	5.010	5.010	8.208	6.436	2.812	7.247/10	1.063/10	$6.102/10^{3}$	$-6.356/10^{4}$	$-1.382/10^{4}$
-0.6	3.981	3.981	5.116	3.063	9.635/10	1.537/10	$6.172/10^{2}$	$-1.869/10^{3}$	$-2.911/10^{4}$	$-4.480/10^{6}$
-0.5	3.162	3.162	3.166	1.404	2.861/10	$1.342/10^{2}$	$-4.778/10^{3}$	$-7.725/10^{4}$	$-2.050/10^{6}$	$8.429/10^{6}$
-0.4	2.512	2.512	1.936	6.014/10	$5.420/10^{2}$	$-1.097/10^{2}$	$-2.604/10^{3}$	$-3.739/10^{5}$	$3.294/10^{5}$	$2.293/10^{6}$
-0.3	1.995	1.995	1.159	2.196/10	$-1.416/10^{2}$	$-9.224/10^{3}$	$-5.373/10^{4}$	$1.264/10^{4}$	$1.444/10^{5}$	$-7.822/10^{7}$
-0.2	1.585	1.585	6.708/10	$4.499/10^{2}$	$-2.635/10^{2}$	$-4.280/10^{3}$	$3.036/10^{4}$	$9.374/10^{5}$	$-8.885/10^{8}$	$-1.045/10^{6}$
-0.1	1.259	1.259	3.616/10	$-2.902/10^{2}$	$-2.173/10^{2}$	$-8.564/10^{4}$	$4.632/10^{4}$	$3.626/10^{5}$	$-3.214/10^{6}$	$-5.490/10^{7}$
0.0	1.000	1.000	1.666/10	$-5.555/10^{2}$	$-1.388/10^{2}$	$9.259/10^{4}$	$3.747/10^{4}$	$-3.100/10^{6}$	$-5.296/10^{6}$	$-8.277/10^{8}$
0.1	0.7943	0.7943	$4.364/10^{2}$	$-6.041/10^{2}$	$-6.980/10^{3}$	$1.644/10^{3}$	$2.284/10^{4}$	$-2.288/10^{5}$	$-3.677/10^{6}$	$1.8/10^{7}$
0.2	0.6309	0.6309	$-3.399/10^{2}$	$-5.615/10^{2}$	$-1.843/10^{3}$	$1.794/10^{3}$	$9.778/10^{5}$	$-2.983/10^{5}$	$-1.881/10^{6}$	$3.051/10^{7}$
0.3	0.5012	0.5012	$-8.293/10^{2}$	$-4.869/10^{2}$	$1.706/10^{3}$	$1.680/10^{3}$	$-4.48/10^{7}$	$-3.001/10^{5}$	$-4.105/10^{7}$	
0.4	0.3981	0.3981	-0.1138	$-4.073/10^{2}$	$4.069/10^{3}$	$1.466/10^{3}$	$-7.147/10^{5}$	$-2.719/10^{5}$	$7./10^{7}$	
0.5	0.3162	0.3162	-0.1333	$-3\ 337/10^{2}$	$5.610/10^{3}$	$1.231/10^{3}$	$-1.15/10^{4}$	$-2.328/10^{5}$	$1.791/10^{6}$	

Table C--Values of a_j, coefficients for Ψ_0

$[a_0 \equiv 1.000,\ a_1 \equiv 0.000]$

$\ln \eta^{-1}$	η	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9
-2.0	$1.000\cdot10^{2}$	$-3.000\cdot10^{4}$	$-1.555\cdot10^{6}$	$-3.240\cdot10^{7}$	$-3.739\cdot10^{8}$	$-2.762\cdot10^{9}$	$-1.424\cdot10^{10}$	$-5.416\cdot10^{10}$	
-1.9	$7.935\cdot10$	$-1.893\cdot10^{4}$	$-7.795\cdot10^{5}$	$-1.289\cdot10^{7}$	$-1.180\cdot10^{8}$	$-6.933\cdot10^{8}$	$-2.837\cdot10^{9}$	$-8.561\cdot10^{9}$	
-1.8	$6.310\cdot10$	$-1.194\cdot10^{4}$	$-3.906\cdot10^{5}$	$-5.134\cdot10^{6}$	$-3.736\cdot10^{7}$	$-1.740\cdot10^{8}$	$-5.654\cdot10^{8}$	$-1.354\cdot10^{9}$	
-1.7	$5.010\cdot10$	$-7.538\cdot10^{3}$	$-1.958\cdot10^{5}$	$-2.042\cdot10^{6}$	$-1.179\cdot10^{7}$	$-4.365\cdot10^{7}$	$-1.124\cdot10^{8}$	$-2.139\cdot10^{8}$	
-1.6	$3.981\cdot10$	$-4.756\cdot10^{3}$	$-9.815\cdot10^{4}$	$-8.134\cdot10^{5}$	$-3.731\cdot10^{6}$	$-1.094\cdot10^{7}$	$-2.237\cdot10^{7}$	$-3.371\cdot10^{7}$	
-1.5	$3.162\cdot10$	$-3.001\cdot10^{3}$	$-4.917\cdot10^{4}$	$-3.236\cdot10^{5}$	$-1.176\cdot10^{6}$	$-2.741\cdot10^{6}$	$-4.439\cdot10^{6}$	$-5.294\cdot10^{6}$	
-1.4	$2.512\cdot10$	$-1.894\cdot10^{3}$	$-2.465\cdot10^{4}$	$-1.286\cdot10^{5}$	$-3.717\cdot10^{5}$	$-6.850\cdot10^{5}$	$-8.779\cdot10^{5}$	$-8.255\cdot10^{5}$	
-1.3	$1.995\cdot10$	$-1.195\cdot10^{3}$	$-1.235\cdot10^{4}$	$-5.120\cdot10^{4}$	$-1.170\cdot10^{5}$	$-1.708\cdot10^{5}$	$-1.725\cdot10^{5}$	$-1.278\cdot10^{5}$	
-1.2	$1.585\cdot10$	$-7.542\cdot10^{2}$	$-6.188\cdot10^{3}$	$-2.033\cdot10^{4}$	$-3.683\cdot10^{4}$	$-4.235\cdot10^{4}$	$-3.365\cdot10^{4}$	$-1.949\cdot10^{4}$	
-1.1	$1.259\cdot10$	$-4.760\cdot10^{2}$	$-3.102\cdot10^{3}$	$-8.077\cdot10^{3}$	$-1.153\cdot10^{4}$	$-1.042\cdot10^{4}$	$-6.467\cdot10^{3}$	$-2.897\cdot10^{3}$	
-1.0	$1.000\cdot10$	$-3.005\cdot10^{2}$	$-1.554\cdot10^{3}$	$-3.200\cdot10^{3}$	$-3.597\cdot10^{3}$	$-2.536\cdot10^{3}$	$-1.213\cdot10^{3}$	$-4.127\cdot10^{2}$	
-0.9	7.945	$-1.898\cdot10^{2}$	$-7.787\cdot10^{2}$	$-1.264\cdot10^{3}$	$-1.110\cdot10^{3}$	$-6.037\cdot10^{2}$	$-2.180\cdot10^{2}$	$-5.415\cdot10$	
-0.8	6.310	$-1.199\cdot10^{2}$	$-3.899\cdot10^{2}$	$-4.977\cdot10^{2}$	$-3.383\cdot10^{2}$	$-1.386\cdot10^{2}$	$-3.618\cdot10$	-6.00	
-0.7	5.010	$-7.587\cdot10$	$-1.952\cdot10^{2}$	$-1.943\cdot10^{2}$	$-1.002\cdot10^{2}$	$-2.972\cdot10$	-5.03	-3.87/10	
-0.6	3.981	$-4.805\cdot10$	$-9.771\cdot10$	$-7.505\cdot10$	$-2.844\cdot10$	-5.49	-3.80/10	$4.80/10^{2}$	
-0.5	3.162	$-3.050\cdot10$	$-4.882\cdot10$	$-2.838\cdot10$	-7.33	-6.33/10	$8.82/10^{2}$	$2.27/10^{2}$	
-0.4	2.512	$-1.943\cdot10$	$-2.437\cdot10$	$-1.034\cdot10$	-1.502	1.137/10	$5.33/10^{2}$	$2.80/10^{3}$	
-0.3	1.995	$-1.244\cdot10$	$-1.213\cdot10$	-3.510	$-6.7/10^{2}$	1.215/10	$1.376/10^{2}$	$-1.31/10^{3}$	
-0.2	1.585	-8.030	-6.014	-1.002	1.794/10	$5.73/10^{2}$	$-2.4/10^{4}$	$-1.117/10^{3}$	
-0.1	1.259	-5.255	-2.963	-1.413/10	1.551/10	$1.853/10^{2}$	$-2.951/10^{3}$	$-4.87/10^{4}$	
0.0	1.000	-3.500	-1.444	1.158/10	$9.62/10^{2}$	$1.89/10^{3}$	$-2.436/10^{3}$	$-1.189/10^{4}$	
0.1	0.7943	-2.393	-0.6913	0.1639	$5.257/10^{2}$	$-3.636/10^{3}$	$-1.502/10^{3}$	$3.207/10^{5}$	$2.294/10^{5}$
0.2	0.6309	-1.694	-0.3205	0.1488	$2.646/10^{2}$	$-4.677/10^{3}$	$-8.087/10^{4}$	$7.538/10^{5}$	$1.311/10^{5}$
0.3	0.5012	-1.2536	-0.1402	0.1212	$1.232/10^{2}$	$-4.246/10^{3}$	$-3.945/10^{4}$	$7.144/10^{5}$	$6.571/10^{6}$
0.4	0.3981	-0.9755	$-5.391/10^{2}$	$9.663/10^{2}$	$5.084/10^{3}$	$-3.514/10^{3}$	$-1.706/10^{4}$	$6.611/10^{5}$	$2.977/10^{6}$
0.5	0.3162	-0.7999	$-1.406/10^{2}$	$7.823/10^{2}$	$1.580/10^{3}$	$-2.860/10^{3}$	$-5.820/10^{5}$	$5.436/10^{5}$	$1.018/10^{6}$

Table D--Values of A_j, coefficients for Φ_1

$[A_2 \equiv 1]$

$\ln \eta^{-1}$	η	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}
-2.0	$1.000\cdot10^{2}$	$5.000\cdot10$	$1.000\cdot10^{3}$	$1.110\cdot10^{4}$	$7.931\cdot10^{4}$	$3.962\cdot10^{5}$	[illegible]	$4.182\cdot10^{6}$	$9.491\cdot10^{6}$	[illegible]
-1.9	$7.934\cdot10$	$3.967\cdot10$	$6.310\cdot10^{2}$	$5.565\cdot10^{3}$	$3.155\cdot10^{4}$	$1.251\cdot10^{5}$	$3.676\cdot10^{5}$	$8.322\cdot10^{5}$	$1.497\cdot10^{6}$	$2.197\cdot10^{6}$
-1.8	$6.310\cdot10$	$3.155\cdot10$	$3.980\cdot10^{2}$	$2.787\cdot10^{3}$	$1.255\cdot10^{4}$	$3.953\cdot10^{4}$	$9.215\cdot10^{4}$	$1.655\cdot10^{5}$	$2.363\cdot10^{5}$	$2.747\cdot10^{5}$
-1.7	$5.010\cdot10$	$2.505\cdot10$	$2.512\cdot10^{2}$	$1.397\cdot10^{3}$	$4.990\cdot10^{3}$	$1.246\cdot10^{4}$	[illegible]	$3.280\cdot10^{4}$	$3.714\cdot10^{4}$	[illegible]
-1.6	$3.980\cdot10$	$1.990\cdot10$	$1.584\cdot10^{2}$	$6.995\cdot10^{2}$	$1.983\cdot10^{3}$	$3.931\cdot10^{3}$	$5.757\cdot10^{3}$	$6.492\cdot10^{3}$	$5.808\cdot10^{3}$	$4.245\cdot10^{3}$
-1.5	$3.162\cdot10$	$1.581\cdot10$	$9.990\cdot10$	$3.500\cdot10^{2}$	$7.871\cdot10^{2}$	$1.234\cdot10^{3}$	$1.433\cdot10^{3}$	$1.276\cdot10^{3}$	$9.018\cdot10^{2}$	$5.160\cdot10^{2}$
-1.4	$2.512\cdot10$	$1.256\cdot10$	$6.300\cdot10$	$1.751\cdot10^{2}$	$3.119\cdot10^{2}$	$3.874\cdot10^{2}$	$3.543\cdot10^{2}$	$2.489\cdot10^{2}$	$1.381\cdot10^{2}$	$6.192\cdot10$
-1.3	$1.995\cdot10$	9.975	$3.970\cdot10$	$8.748\cdot10$	$1.232\cdot10^{2}$	$1.206\cdot10^{2}$	$8.696\cdot10$	$4.750\cdot10$	$2.070\cdot10$	7.210
-1.2	$1.585\cdot10$	7.925	$2.502\cdot10$	$4.361\cdot10$	$4.845\cdot10$	$3.732\cdot10$	$2.101\cdot10$	8.982	2.996	7.963/10
-1.1	$1.259\cdot10$	6.295	$1.575\cdot10$	$2.168\cdot10$	$1.893\cdot10$	$1.136\cdot10$	4.951	1.619	4.067/10	$7.990/10^{2}$
-1.0	$1.000\cdot10$	5.000	9.900	$1.072\cdot10$	7.306	3.384	1.118	2.711/10	$4.894/10^{2}$	$6.520/10^{3}$
-0.9	7.934	3.967	6.210	5.259	2.761	9.641/10	2.328/10	$3.914/10^{2}$	$4.391/10^{3}$	$2.868/10^{4}$
-0.8	6.310	3.155	3.880	2.544	1.008	2.547/10	$4.080/10^{2}$	$3.716/10^{3}$	$6.971/10^{5}$	$-2.557/10^{5}$
-0.7	5.010	2.505	2.412	1.203	3.444/10	$5.615/10^{2}$	$4.064/10^{3}$	$-2.165/10^{4}$	$-7.157/10^{5}$	$-4.349/10^{6}$
-0.6	3.980	1.990	1.485	5.463/10	1.023/10	$6.708/10^{3}$	$-9.094/10^{4}$	$-1.991/10^{4}$	$-7.642/10^{6}$	$1.508/10^{6}$
-0.5	3.162	1.581	9.000/10	2.283/10	$1.944/10^{2}$	$-2.641/10^{3}$	$-6.682/10^{4}$	$-2.217/10^{5}$	$5.972/10^{6}$	$7.214/10^{7}$
-0.4	2.512	1.256	5.310/10	$7.843/10^{2}$	$-4.895/10^{3}$	$-2.575/10^{3}$	$-1.490/10^{4}$	$2.611/10^{5}$	$3.183/10^{6}$	$3.762/10^{8}$
-0.3	1.995	9.975/10	2.980/10	$1.068/10^{2}$	$-9.117/10^{3}$	$-1.179/10^{3}$	$8.175/10^{5}$	$2.158/10^{5}$	$4.436/10^{8}$	$-9.457/10^{8}$
-0.2	1.585	7.925/10	1.510/10	$-1.741/10^{2}$	$-7.356/10^{3}$	$-1.488/10^{4}$	$1.272/10^{4}$	$7.863/10^{6}$	$-1.163/10^{6}$	$-2.485/10^{8}$
-0.1	1.259	6.295/10	$5.850/10^{2}$	$-2.679/10^{2}$	$-4.494/10^{3}$	$3.861/10^{4}$	$1.010/10^{4}$	$-1.876/10^{6}$	$-1.205/10^{6}$	$5.495/10^{8}$
0.0	1.000	5.000/10	0.000	$-2.777/10^{2}$	$-1.984/10^{3}$	$5.952/10^{4}$	$5.879/10^{5}$	$-6.823/10^{6}$	$-8.227/10^{7}$	$1.001/10^{7}$
0.1	0.7943	3.9715/10	$-3.69088/10^{2}$	$-2.53213/10^{2}$	$-1.18451/10^{4}$	$6.28328/10^{4}$	$2.0678/10^{5}$	$-8.5068/10^{6}$	$-3.8855/10^{7}$	$7.3051/10^{8}$
0.2	0.6309	3.1545/10	$-6.01965/10^{2}$	$-2.17448/10^{2}$	$1.16996/10^{3}$	$5.80526/10^{4}$	$-8.10102/10^{6}$	$-8.43925/10^{6}$	$-2.89503/10^{8}$	$7.78030/10^{8}$
0.3	0.5012	2.5060/10	$-7.48799/10^{2}$	$-1.80922/10^{2}$	$2.02658/10^{3}$	$5.03091/10^{4}$	$-2.81904/10^{5}$	$-7.59070/10^{6}$	$2.33881/10^{7}$	$7.24550/10^{8}$
0.4	0.3981	1.9905/10	$-8.41516/10^{2}$	$-1.47806/10^{2}$	$2.58512/10^{3}$	$4.20973/10^{4}$	$-4.16655/10^{5}$	$-6.48781/10^{6}$	$4.14772/10^{7}$	$6.31301/10^{8}$
0.5	0.3162	1.581/10	$-9.00018/10^{2}$	$-1.19454/10^{2}$	$2.94455/10^{3}$	$3.45188/10^{4}$	$-5.0486/10^{5}$	$-5.38737/10^{6}$	$5.349/10^{7}$	$5.301/10^{8}$

Table E--Values of a_j, coefficients for Ψ_1

$[a_{-1} \equiv 1.000,\ a_2 \equiv 0.000]$

$\ln \eta^{-1}$	η	a_0	a_1	a_3	a_4	a_5	a_6	a_7	a_8
-2.0	$1.000\cdot10^{2}$	$-1.000\cdot10^{2}$	$1.000\cdot10^{4}$	$-4.167\cdot10^{7}$	$-1.300\cdot10^{9}$	$-1.814\cdot10^{10}$	$-1.503\cdot10^{11}$	$-8.371\cdot10^{11}$	$-3.369\cdot10^{12}$
-1.9	$7.944\cdot10$	$-7.944\cdot10$	$6.311\cdot10^{3}$	$-1.658\cdot10^{7}$	$-4.108\cdot10^{8}$	$-4.556\cdot10^{9}$	$-2.998\cdot10^{10}$	$-1.325\cdot10^{11}$	$-4.236\cdot10^{11}$
-1.8	$6.310\cdot10$	$-6.310\cdot10$	$3.981\cdot10^{3}$	$-6.606\cdot10^{6}$	$-1.300\cdot10^{8}$	$-1.145\cdot10^{9}$	$-5.980\cdot10^{9}$	$-2.098\cdot10^{10}$	$-5.323\cdot10^{10}$
-1.7	$5.012\cdot10$	$-5.012\cdot10$	$2.513\cdot10^{3}$	$-2.629\cdot10^{6}$	$-4.109\cdot10^{7}$	$-2.874\cdot10^{8}$	$-1.191\cdot10^{9}$	$-3.320\cdot10^{9}$	$-6.677\cdot10^{9}$
-1.6	$3.981\cdot10$	$-3.981\cdot10$	$1.586\cdot10^{3}$	$-1.047\cdot10^{6}$	$-1.301\cdot10^{7}$	$-7.218\cdot10^{7}$	$-2.376\cdot10^{8}$	$-5.249\cdot10^{8}$	$-8.409\cdot10^{8}$
-1.5	$3.162\cdot10$	$-3.162\cdot10$	$1.001\cdot10^{3}$	$-4.173\cdot10^{5}$	$-4.111\cdot10^{6}$	$-1.812\cdot10^{7}$	$-4.732\cdot10^{7}$	$-8.286\cdot10^{7}$	$-1.044\cdot10^{8}$
-1.4	$2.512\cdot10$	$-2.512\cdot10$	$6.315\cdot10^{2}$	$-1.662\cdot10^{5}$	$-1.302\cdot10^{6}$	$-4.549\cdot10^{6}$	$-9.416\cdot10^{6}$	$-1.304\cdot10^{7}$	$-1.299\cdot10^{7}$
-1.3	$1.995\cdot10$	$-1.995\cdot10$	$3.985\cdot10^{2}$	$-6.630\cdot10^{4}$	$-4.118\cdot10^{5}$	$-1.142\cdot10^{6}$	$-1.870\cdot10^{6}$	$-2.045\cdot10^{6}$	$-1.606\cdot10^{6}$
-1.2	$1.585\cdot10$	$-1.585\cdot10$	$2.517\cdot10^{2}$	$-2.644\cdot10^{4}$	$-1.305\cdot10^{5}$	$-2.864\cdot10^{5}$	$-3.703\cdot10^{5}$	$-3.186\cdot10^{5}$	$-1.957\cdot10^{5}$
-1.1	$1.259\cdot10$	$-1.259\cdot10$	$1.590\cdot10^{2}$	$-1.057\cdot10^{4}$	$-4.134\cdot10^{4}$	$-7.177\cdot10^{4}$	$-7.304\cdot10^{4}$	$-4.911\cdot10^{4}$	$-2.339\cdot10^{4}$
-1.0	$1.000\cdot10$	$-1.000\cdot10$	$1.005\cdot10^{2}$	$-4.233\cdot10^{3}$	$-1.313\cdot10^{4}$	$-1.796\cdot10^{4}$	$-1.430\cdot10^{4}$	$-7.438\cdot10^{3}$	$-2.699\cdot10^{3}$
-0.9	7.944	-7.944	$6.360\cdot10$	$-1.700\cdot10^{3}$	$-4.174\cdot10^{3}$	$-4.483\cdot10^{3}$	$-2.763\cdot10^{3}$	$-1.090\cdot10^{3}$	$-2.921\cdot10^{2}$
-0.8	6.310	-6.310	$4.030\cdot10$	$-6.868\cdot10^{2}$	$-1.333\cdot10^{3}$	$-1.115\cdot10^{3}$	$-5.228\cdot10^{2}$	$-1.512\cdot10^{2}$	$-2.763\cdot10$
-0.7	5.012	-5.012	$2.562\cdot10$	$-2.795\cdot10^{2}$	$-4.274\cdot10^{2}$	$-2.751\cdot10^{2}$	$-9.49\cdot10$	$-1.853\cdot10$	-1.779
-0.6	3.981	-3.981	$1.635\cdot10$	$-1.153\cdot10^{2}$	$-1.384\cdot10^{2}$	$-6.696\cdot10$	$-1.589\cdot10$	-1.589	$6.4/10^{2}$
-0.5	3.162	-3.162	$1.050\cdot10$	$-4.846\cdot10$	$-4.521\cdot10$	$-1.585\cdot10$	-2.144	$7.75/10^{2}$	$5.32/10^{2}$
-0.4	2.512	-2.512	6.810	$-2.091\cdot10$	$-1.506\cdot10$	-3.518	$-7.03/10^{2}$	$8.93/10^{2}$	$1.013/10^{2}$
-0.3	1.995	-1.995	4.480	-9.381	-5.123	-6.51/10	1.139/10	$3.019/10^{2}$	$-3.1/10^{5}$
-0.2	1.585	-1.585	3.010	-4.425	-1.797	$-3.82/10^{2}$	$7.059/10^{2}$	$8.72/10^{3}$	$-1.045/10^{3}$
-0.1	1.259	-1.259	2.085	-2.228	-6.493/10	$6.196/10^{2}$	$3.257/10^{2}$	$2.30/10^{4}$	$-6.536/10^{4}$
0.0	1.000	-1.000	1.500	-1.2083	-2.416/10	$5.879/10^{2}$	$1.3869/10^{2}$	$-1.034/10^{3}$	$-3.170/10^{4}$
0.1	0.7943	-7.943/10	1.13091	-7.11463/10	$-9.07103/10^{2}$	$4.24540/10^{2}$	$5.68850/10^{3}$	$-1.01179/10^{3}$	$-1.40069/10^{4}$
0.2	0.6309	-6.309/10	8.98035/10	-4.56370/10	$-3.28073/10^{2}$	[illegible]	[illegible]	$-7.7682/10^{4}$	$-5.80959/10^{5}$
0.3	0.5012	-5.012/10	7.51201/10	-3.18760/10	$-1.00391/10^{2}$	[illegible]	[illegible]	[illegible]	[illegible]
0.4	0.3981	-3.981/10	6.58484/10	-2.41121/10	$-1.08669/10^{3}$	$1.56198/10^{4}$	[illegible]	$-4.2916/10^{4}$	[illegible]
0.5	0.3162	-3.162/10	5.99982/10	-1.95820/10	$2.22482/10^{3}$	[illegible]	[illegible]	[illegible]	[illegible]

Table F--Values of p_L and q_L

$\ln \eta^{-1}$	η	p_0	q_0	p_1	q_1
-2.0	$1.00\cdot10^{2}$	$2.00\cdot10^{2}$	$9.519220\cdot10^{2}$	$6.66733333\cdot10^{5}$	$2.6177964\cdot10^{6}$
-1.9	$7.935\cdot10$	$1.587\cdot10^{2}$	$7.1864\cdot10^{2}$	$3.33133650\cdot10^{5}$	$1.23094\cdot10^{6}$
-1.9	$7.934\cdot10$	$1.5868\cdot10^{2}$	$7.18534\cdot10^{2}$	$3.33007731\cdot10^{5}$	$1.230431\cdot10^{6}$
-1.9	$7.944\cdot10$	$1.5888\cdot10^{2}$	$7.19639\cdot10^{2}$	$3.34268352\cdot10^{5}$	$1.235508\cdot10^{6}$
-1.8	$6.310\cdot10$	$1.2620\cdot10^{2}$	$5.425556\cdot10^{2}$	$1.67535127\cdot10^{5}$	$5.806604\cdot10^{5}$
-1.7	$5.010\cdot10$	$1.0020\cdot10^{2}$	$4.07663\cdot10^{2}$	$8.3867734\cdot10^{4}$	$2.71334\cdot10^{5}$
-1.7	$5.012\cdot10$	$1.0024\cdot10^{2}$	$4.078653\cdot10^{2}$	$8.39681877\cdot10^{4}$	$2.716920\cdot10^{5}$
-1.6	$3.981\cdot10$	$7.962\cdot10$	$3.0562949\cdot10^{2}$	$4.20880901\cdot10^{4}$	$1.2649269\cdot10^{5}$
-1.6	$3.980\cdot10$	$7.960\cdot10$	$3.0553266\cdot10^{2}$	$4.20563947\cdot10^{4}$	$1.2638682\cdot10^{5}$
-1.5	$3.162\cdot10$	$6.324\cdot10$	$2.2818914\cdot10^{2}$	$2.10973784\cdot10^{4}$	$5.854988\cdot10^{4}$
-1.4	$2.512\cdot10$	$5.024\cdot10$	$1.697223\cdot10^{2}$	$1.05841345\cdot10^{4}$	$2.693972\cdot10^{4}$
-1.3	$1.995\cdot10$	$3.990\cdot10$	$1.2560001\cdot10^{2}$	$5.30673325\cdot10^{3}$	$1.2285954\cdot10^{4}$
-1.2	$1.585\cdot10$	$3.170\cdot10$	$9.249844\cdot10$	$2.66515108\cdot10^{3}$	$5.558414\cdot10^{3}$
-1.1	$1.259\cdot10$	$2.518\cdot10$	$6.768031\cdot10$	$1.33880465\cdot10^{3}$	$2.484947\cdot10^{3}$
-1.0	$1.000\cdot10$	$2.000\cdot10$	$4.915700\cdot10$	$6.73333333\cdot10^{2}$	$1.0955081\cdot10^{3}$
-0.9	7.945	$1.589\cdot10$	$3.540762\cdot10$	$3.39638289\cdot10^{2}$	$4.751070\cdot10^{2}$
-0.9	7.934	$1.5868\cdot10$	$3.533667\cdot10$	$3.38244171\cdot10^{2}$	$4.726931\cdot10^{2}$
-0.9	7.944	$1.5888\cdot10$	$3.540118\cdot10$	$3.39511392\cdot10^{2}$	$4.748874\cdot10^{2}$
-0.8	6.310	$1.2620\cdot10$	$2.52232\cdot10$	$1.71699727\cdot10^{2}$	$2.01140\cdot10^{2}$
-0.7	5.010	$1.0020\cdot10$	$1.772739\cdot10$	$8.7174334\cdot10$	$8.24187\cdot10$
-0.7	5.012	$1.0024\cdot10$	$1.773845\cdot10$	$8.72761078\cdot10$	$8.25489\cdot10$
-0.6	3.981	7.962	$1.227152\cdot10$	$4.47155501\cdot10$	$3.23189\cdot10$
-0.6	3.980	7.960	$1.226646\cdot10$	$4.46831947\cdot10$	$3.22847\cdot10$
-0.5	3.162	6.324	8.31010	$2.31842984\cdot10$	$1.167228\cdot10$
-0.4	2.512	5.024	5.4706	$1.22420545\cdot10$	3.5474
-0.3	1.995	3.990	3.457850	6.62343325	0.55303
-0.2	1.585	3.170	2.06021	3.71125111	-0.41657
-0.1	1.259	2.518	1.11298	2.16974465	-0.63922
0.0	1.000	2.000	0.49816	1.33333333	-0.61234
0.1	0.7943	1.5886	0.12497	8.63622527/10	-0.519363
0.2	0.6309	1.2618	-0.079511	5.88013441/10	-0.421915
0.3	0.5012	1.0024	-0.173879	4.18068108/10	-0.337377
0.4	0.3981	0.7962	-0.202915	3.07461550/10	-0.268226
0.5	0.3162	0.6324	-0.197339	2.31876298/10	-0.212887

Table G--Values of $\ln_e 2\rho$

ρ	$\ln_e 2\rho$	ρ	$\ln_e 2\rho$	ρ	$\ln_e 2\rho$
0.1000	-1.60943791	0.2512	-0.688358662	0.6310	0.232697764
0.1259	-1.3791206	0.3162	-0.458233174	0.7944	0.4629[illegible]3013
0.1585	-1.14885350	0.3981	-0.227904869	5.0000	2.30258509
0.1995	-0.918793350	0.5012	0.002397125		

Table H--Values of C_L and D_L

$\ln \eta^{-1}$	η	C_0	D_0	C_1	D_1
-2.0	$1.000\cdot10^{2}$	$9.153/10^{136}$	$1.0925\cdot10^{135}$	$3.051/10^{134}$	$1.0925\cdot10^{133}$
-1.9	$7.944\cdot10$	$9.396/10^{108}$	$1.0640\cdot10^{107}$	$2.488/10^{106}$	$1.3497\cdot10^{[illegible]}$
-1.8	$6.310\cdot10$	$1.631/10^{85}$	$6.1312\cdot10^{84}$	$3.430/10^{84}$	$9.718[illegible]\cdot10^{82}$
-1.7	$5.012\cdot10$	$7.381/10^{68}$	$1.3548\cdot10^{67}$	$1.233/10^{66}$	$2.7034\cdot10^{65}$
-1.6	$3.981\cdot10$	$7.622/10^{54}$	$1.3120\cdot10^{53}$	$1.011/10^{52}$	$3.2970\cdot10^{51}$
-1.5	$3.162\cdot10$	$1.008/10^{42}$	$9.9206\cdot10^{41}$	$1.062/10^{41}$	$3.1387\cdot10^{40}$
-1.4	$2.512\cdot10$	$6.734/10^{34}$	$1.4850\cdot10^{33}$	$5.642/10^{33}$	$5.9080\cdot10^{31}$
-1.3	$1.995\cdot10$	$6.699/10^{27}$	$1.4927\cdot10^{26}$	$4.460/10^{26}$	$7.4737\cdot10^{24}$
-1.2	$1.585\cdot10$	$2.372/10^{21}$	$4.2158\cdot10^{20}$	$1.255/10^{20}$	$2.6560\cdot10^{19}$
-1.1	$1.259\cdot10$	$5.924/10^{17}$	$1.6880\cdot10^{16}$	$2.493/10^{16}$	$1.3370\cdot10^{15}$
-1.0	1.000 10	$1.8015/10^{13}$	$5.5509\cdot10^{12}$	$6.034/10^{13}$	$5.5241\cdot10^{11}$
-0.9	7.944	$1.027/10^{10}$	$9.7371\cdot10^{9}$	$2.740/10^{10}$	$1.2165\cdot10^{9}$
-0.8	6.310	$1.5502/10^{8}$	$6.4507\cdot10^{7}$	$3.300/10^{8}$	$1.0100\cdot10^{7}$
-0.7	5.012	$8.149/10^{7}$	$1.2271\cdot10^{6}$	$1.387/10^{6}$	$2.4032\cdot10^{5}$
-0.6	3.981	$1.8515/10^{5}$	$5.4010\cdot10^{4}$	$2.533/10^{5}$	$1.3159\cdot10^{4}$
-0.5	3.162	$2.1725/10^{4}$	$4.6029\cdot10^{3}$	$2.401/10^{4}$	$1.3882\cdot10^{3}$
-0.4	2.512	$1.4875/10^{3}$	$6.7226\cdot10^{2}$	$1.340/10^{3}$	$2.4875\cdot10^{2}$
-0.3	1.995	$6.737/10^{3}$	$1.4843\cdot10^{2}$	$5.011/10^{3}$	$6.6519\cdot10$
-0.2	1.585	$2.173/10^{2}$	$4.6019\cdot10$	$1.357/10^{2}$	$2.4563\cdot10$
-0.1	1.259	$5.392/10^{2}$	$1.8546\cdot10$	$2.889/10^{2}$	$1.1537\cdot10$
0.0	1.000	1.085/10	9.2165	$5.113/10^{2}$	6.5192
0.1	0.7944	1,848/10	5.4113	$7.868/10^{2}$	4.2365
0.2	0.6310	2.7692/10	3.6112	1.0915/10	3.0539
0.3	0.5012	3.757/10	2.6617	1.4008/10	2.3796
0.4	0.3981	4.726/10	2.1160	1.6956/10	1.9659
0.5	0.3162	5.6195/10	1.7795	1.9644/10	1.6969

Table I_0

ρ	F_0	F'_0	$\rho F'_0/F_0$
0.1	0.0998	0.9950	0.997
0.2	0.1987	0.9800	0.986
0.3	0.2956	0.9554	0.970
0.4	0.3894	0.9210	0.946
0.5	0.4794	0.8775	0.915
0.6	0.5647	0.8254	0.877
0.7	0.6442	0.7648	0.831
0.8	0.7174	0.6966	0.777
0.9	0.7833	0.6216	0.714
1.0	0.8414	0.5404	0.642
1.1	0.8912	0.4536	0.560
1.2	0.9320	0.3613	0.465
1.3	0.9635	0.2675	0.361
1.4	0.9855	0.1700	0.242
1.5	0.9975	0.0708	0.106
1.6	0.9996	-0.0292	-0.047
1.7	0.9917	-0.1288	-0.221
1.8	0.9739	-0.2272	-0.420
1.9	0.9463	-0.3233	-0.649
2.0	0.9094	-0.4160	-0.915
2.1	0.8632	-0.5048	-1.228
2.2	0.8085	-0.5885	-1.601
2.3	0.7457	-0.6664	-2.055
2.4	0.6755	-0.7374	-2.620
2.5	0.5984	-0.8010	-3.346
2.6	0.5154	-0.8570	-4.323
2.7	0.4274	-0.9040	-5.711
2.8	0.3350	-0.9422	-7.875
2.9	0.2393	-0.9710	-11.77
3.0	0.1411	-0.9900	-21.05
3.1	0.0416	-0.9992	-74.46
3.2	-0.0583	-0.9983	54.80
3.3	-0.1577	-0.9875	20.66
3.4	-0.2555	-0.9668	12.87
3.5	-0.3508	-0.9364	9.343
3.6	-0.4426	-0.8967	7.294
3.7	-0.5298	-0.8481	5.923
3.8	-0.6119	-0.7910	4.912
3.9	-0.6878	-0.7260	4.117
4.0	-0.7569	-0.6537	3.455
4.1	-0.8183	-0.5749	2.880
4.2	-0.8716	-0.4902	2.362
4.3	-0.9162	-0.4007	1.881
4.4	-0.9517	-0.3072	1.420
4.5	-0.9775	-0.2108	0.970
4.6	-0.9936	-0.1122	0.519
4.7	-0.9999	-0.0124	0.058
4.8	-0.9962	0.0875	-0.422
4.9	-0.9824	0.1866	-0.931
5.0	-0.9589	0.2836	-1.479
5.1	-0.9258	0.3780	-2.082
5.2	-0.8834	0.4685	-2.758
5.3	-0.8323	0.5544	-3.530
5.4	-0.7728	0.6347	-4.435
5.5	-0.7055	0.7087	-5.525
5.6	-0.6312	0.7756	-6.881

Table I_1

ρ	F_1	F'_1	$\rho F'_1/F_1$
0.1	0.0033	0.0665	1.998
0.2	0.0133	0.1323	1.992
0.3	0.0297	0.1964	1.982
0.4	0.0525	0.2582	1.968
0.5	0.0813	0.3169	1.950
0.6	0.1157	0.3718	1.927
0.7	0.1555	0.4221	1.901
0.8	0.1999	0.4674	1.870
0.9	0.2488	0.5069	1.834
1.0	0.3012	0.5403	1.794
1.1	0.3566	0.5670	1.751
1.2	0.4143	0.5867	1.699
1.3	0.4737	0.5992	1.644
1.4	0.5339	0.6040	1.584
1.5	0.5942	0.6013	1.518
1.6	0.6540	0.5909	1.446
1.7	0.7116	0.5710	1.364
1.8	0.7682	0.5470	1.282
1.9	0.8213	0.5139	1.189
2.0	0.8708	0.4741	1.089
2.1	0.9158	0.4270	0.979
2.2	0.9560	0.3740	0.861
2.3	0.9906	0.3151	0.732
2.4	1.0189	0.2509	0.591
2.5	1.0404	0.1823	0.438
2.6	1.0552	0.1096	0.270
2.7	1.0623	0.0340	0.086
2.8	1.0618	-0.0442	-0.117
2.9	1.0535	-0.1240	-0.341
3.0	1.0370	-0.2046	-0.592
3.1	1.0126	-0.2850	-0.873
3.2	0.9801	-0.3646	-1.190
3.3	0.9397	-0.4424	-1.554
3.4	0.8917	-0.5178	-1.974
3.5	0.8362	-0.5897	-2.468
3.6	0.7738	-0.6575	-3.059
3.7	0.7049	-0.7203	-3.781
3.8	0.6300	-0.7777	-4.691
3.9	0.5496	-0.8288	-5.881
4.0	0.4645	-0.8730	-7.518
4.1	0.3753	-0.9098	-9.939
4.2	0.2827	-0.9389	-13.95
4.3	0.1876	-0.9598	-22.00
4.4	0.1065	-0.9723	-40.17
4.5	-0.0064	-0.9760	-686.
4.6	-0.1038	-0.9711	43.04
4.7	-0.2004	-0.9572	22.45
4.8	-0.2950	-0.9348	15.21
4.9	-0.3871	-0.9033	11.43
5.0	-0.4754	-0.8638	9.085
5.1	-0.5595	-0.8161	7.439
5.2	-0.6364	-0.7606	6.195
5.3	-0.7114	-0.6981	5.201
5.4	-0.7778	-0.6288	4.377
5.5	-0.8370	-0.5532	3.635
5.6	-0.8883	-0.4726	2.979

Table I_2

ρ	F_2	F'_2	$\rho F'_2/F_2$
0.1	0.0001	0.0020	2.999
0.2	0.0005	0.0080	2.994
0.3	0.0018	0.0161	2.987
0.4	0.0042	0.0314	2.977
0.5	0.0082	0.0485	2.964
0.6	0.0140	0.0689	2.948
0.7	0.0221	0.0924	2.929
0.8	0.0326	0.1185	2.908
0.9	0.0459	0.1469	2.883
1.0	0.0620	0.1771	2.855
1.1	0.0813	0.2087	2.824
1.2	0.1038	0.2413	2.789
1.3	0.1296	0.2743	2.752
1.4	0.1587	0.3072	2.711
1.5	0.1909	0.3396	2.666
1.6	0.2265	0.3707	2.619
1.7	0.2651	0.4003	2.567
1.8	0.3065	0.4276	2.511
1.9	0.3505	0.4523	2.452
2.0	0.3969	0.4741	2.388
2.1	0.4452	0.4920	2.321
2.2	0.4952	0.5059	2.248
2.3	0.5464	0.5155	2.170
2.4	0.5981	0.5205	2.089
2.5	0.6500	0.5203	2.001
2.6	0.7022	0.5151	1.907
2.7	0.7529	0.5046	1.810
2.8	0.8027	0.4885	1.704
2.9	0.8506	0.4669	1.592
3.0	0.8959	0.4397	1.472
3.1	0.9383	0.4072	1.345
3.2	0.9771	0.3694	1.210
3.3	1.0120	0.3264	1.064
3.4	1.0423	0.2786	0.909
3.5	1.0675	0.2262	0.742
3.6	1.0873	0.1696	0.562
3.7	1.1013	0.1096	0.368
3.8	1.1092	0.0461	0.158
3.9	1.1106	-0.0199	-0.070
4.0	1.1052	-0.0881	-0.319
4.1	1.0929	-0.1578	-0.592
4.2	1.0734	-0.2286	-0.894
4.3	1.0472	-0.2994	-1.229
4.4	1.0137	-0.3700	-1.606
4.5	0.9732	-0.4391	-2.030
4.6	0.9263	-0.5059	-2.512
4.7	0.8720	-0.5716	-3.081
4.8	0.8119	-0.6332	-3.744
4.9	0.7454	-0.6913	-4.544
5.0	0.6736	-0.7448	-5.529
5.1	0.5968	-0.7935	-6.781
5.2	0.5151	-0.8364	-8.444
5.3	0.4236	-0.8735	-10.77
5.4	0.3407	-0.9040	-14.33
5.5	0.2469	-0.9275	-20.50
5.6	0.1553	-0.9437	-34.[illegible]

Table I_3

ρ	F_3	F'_3	$\rho F'_3/F_3$
0.1	0.0000	0.0000	3.999
0.2	0.0000	0.0003	3.995
0.3	0.0001	0.0010	3.990
0.4	0.0002	0.0024	3.982
0.5	0.0006	0.0047	3.972
0.6	0.0012	0.0080	3.960
0.7	0.0022	0.0125	3.945
0.8	0.0038	0.0185	3.928
0.9	0.0060	0.0259	3.909
1.0	0.0090	0.0350	3.888
1.1	0.0130	0.0458	3.864
1.2	0.0182	0.0583	3.838
1.3	0.0247	0.0725	3.809
1.4	0.0328	0.0884	3.778
1.5	0.0425	0.1060	3.744
1.6	0.0540	0.1252	3.708
1.7	0.0676	0.1459	3.669
1.8	0.0834	0.1678	3.623
1.9	0.1012	0.1906	3.583
2.0	0.1214	0.2147	3.536
2.1	0.1441	0.2394	3.489
2.2	0.1694	0.2642	3.431
2.3	0.1972	0.2891	3.372
2.4	0.2271	0.3142	3.320
2.5	0.2597	0.3384	3.258
2.6	0.2952	0.3615	3.184
2.7	0.3321	0.3840	3.122
2.8	0.3716	0.4046	3.049
2.9	0.4131	0.4233	2.972
3.0	0.4562	0.4397	2.891
3.1	0.5009	0.4537	2.808
3.2	0.5466	0.4646	2.720
3.3	0.5936	0.4723	2.626
3.4	0.6411	0.4766	2.528
3.5	0.6889	0.4772	2.424
3.6	0.7365	0.4737	2.315
3.7	0.7834	0.4661	2.201
3.8	0.8296	0.4544	2.081
3.9	0.8742	0.4381	1.954
4.0	0.9170	0.4174	1.821
4.1	0.9575	0.3924	1.680
4.2	0.9954	0.3626	1.530
4.3	1.0300	0.3285	1.371
4.4	1.0612	0.2900	1.202
4.5	1.0877	0.2482	1.027
4.6	1.1102	0.2018	0.836
4.7	1.1282	0.1519	0.633
4.8	1.1407	0.0990	0.417
4.9	1.1477	0.0427	0.182
5.0	1.1490	-0.0157	-0.068
5.1	1.1445	-0.0766	-0.341
5.2	1.1337	-0.1389	-0.637
5.3	1.1168	-0.2025	-0.961
5.4	1.0932	-0.2667	-1.317
5.5	1.0633	-0.3309	-1.712
5.6	1.0270	-0.3948	-2.153

Table I_4

ρ	F_4	F'_4	$\rho F'_4/F_4$
0.1	0.0000	0.0000	[illegible]
0.2	0.0000	[illegible]	4.996
0.3	[illegible]	[illegible]	4.992
0.4	0.0000	0.0001	4.985
0.5	[illegible]	0.0003	[illegible]
0.6	0.0001	[illegible]	4.967
0.7	0.0002	[illegible]	[illegible]
0.8	0.0003	[illegible]	4.942
0.9	0.0006	0.0033	4.926
1.0	0.0010	0.0050	4.906
1.1	0.0016	0.0072	4.889
1.2	0.0025	0.0100	4.868
1.3	0.0036	0.0136	4.844
1.4	0.0052	0.0179	4.819
1.5	0.0073	0.0232	4.792
1.6	0.0099	0.0294	4.763
1.7	0.0131	0.0366	4.733
1.8	0.0172	0.0450	4.699
1.9	0.0222	0.0545	4.663
2.0	0.0282	0.0641	4.552
2.1	0.035	0.0758	4.54
2.2	0.0438	0.0900	4.52
2.3	0.0536	0.1036	4.45
2.4	0.0644	0.1196	4.46
2.5	0.0771	0.1362	4.41
2.6	0.0924	0.1528	4.30
2.7	0.1078	0.1724	4.31
2.8	0.1261	0.1914	4.25
2.9	0.1463	0.2115	4.19
3.0	0.1685	0.2314	4.12
3.1	0.1926	0.2525	4.06
3.2	0.2188	0.2732	3.996
3.3	0.2471	0.2940	3.926
3.4	0.2777	0.3145	3.851
3.5	0.3102	0.3344	3.773
3.6	0.3448	0.3535	3.691
3.7	0.3808	0.3717	3.612
3.8	0.4190	0.3884	3.522
3.9	0.4585	0.4040	3.436
4.0	0.4996	0.4174	3.342
4.1	0.5417	0.4289	3.246
4.2	0.5854	0.4379	3.142
4.3	0.6296	0.4443	3.034
4.4	0.6749	0.4473	2.916
4.5	0.7189	0.4487	2.809
4.6	0.7635	0.4464	2.690
4.7	0.8083	0.4400	2.558
4.8	0.8513	0.4317	2.434
4.9	0.8942	0.4179	2.290
5.0	0.9350	0.4011	2.145
5.1	0.9742	0.3804	1.991
5.2	1.0110	0.3561	1.831
5.3	1.0453	0.3277	1.662
5.4	1.0765	0.2958	1.484
5.5	1.1043	0.2602	1.296
5.6	1.1284	0.221[illegible]	1.097

and to the right along a diagonal line. If the energy is kept fixed and the distance between particles is varied one moves along a horizontal row of each table.

In Tables B, C, D, and E are listed the coeffiients A_j, a_j that were used in computing the Tables A. These coefficients are useful for computing the series for values of ρ spaced at closer intervals than in Tables A. These coefficients correspond to the values of η which are listed in the second columns of Tables B and C and approximately to the values of $\ln_{10}(1/\eta)$ given in the first columns of these tables. The difference between the exact antilogarithms of these numbers and the approximate values used is often of no significance. There are cases, however, where considerable accuracy is required in the values of the series for the computation of Θ and Θ^*. In such cases it is desirable to use consistently the same value of η both in the coefficients and in the quantities p, q. For this reason the extra columns for η were included in the Tables.

In Table F the values of p and q are given for the values of η which were used in the preceding Tables. In some cases there are therefore several values of η for each approximate value of $\ln_{10}(1/\eta)$. The accuracy of these numbers is usually greater than necessary. Tables G and H are self-explanatory. In tables I_0, I_1, I_2, I_3, I_4 the values of the internal functions computed by means of Bessel functions of half-integral order are given for $L=0$, 1, 2, 3, 4, respectively.

It is our pleasant duty to acknowledge the assistance of Messrs. I. S. Lowen, M. Ostrofsky, and D. P. Johnson in some of the numerical calculations.

University of Wisconsin,
Madison, Wisconsin

New York University,
New York, New York

NOTES

(See also pages 412 and 432)

35. *Bulletin of Society of Petroleum Geophysicists*—In accordance with the action of the American Association of Petroleum Geologists at the twentieth annual meeting at Wichita, Kansas, in March 1935, the Division of Geophysics is undertaking independent publication of geophysical papers instead of having them published as previously in the Association *Bulletin*. Therefore, papers such as those heretofore appearing in the January 1935 *Bulletin* of the Association geophysics symposium will hereafter be printed in a new journal to be known as the *Bulletin of the Society of Petroleum Geophysicists*. B. B. Weatherby of the Geophysical Research Corporation, Tulsa, Oklahoma, President of the Society, announces that the new *Bulletin* will be published twice a year. Members and associates of the Society will receive the new *Bulletin* without payment in addition to their dues. Members and associates of the Association in good standing have the privilege of subscribing to the new *Bulletin* at the special rate of $4.00 per year. Non-members may subscribe at a somewhat higher rate to be decided later. Subscriptions and inquiries should be sent to Gerald H. Westby, Secretary Treasurer of the Division, Seismograph Service Corporation, Kennedy Building, Tulsa, Oklahoma. Manuscript contributions should be sent to F. M. Kannenstine, Editor of the Division, 2011 Esperson Building, Houston, Texas.

36. *Exhibition of the Carnegie Institution of Washington*—The annual exhibition representing the results of research activities of the Carnegie Institution of Washington, was held at the Administrative Building of the Institution, December 14, 15, and 16, 1935. Among the 14 exhibits by the various departments of the Institution of especial interest to the geophysicist were the following: The natural electric currents in the Earth, by the Department of Terrestrial Magnetism, illustrating particularly the extensive electrical eddies in the Earth which have been recently disclosed (especially by the results obtained during the Second International Polar Year 1932-33); the astronomer's instruments, by the Mount Wilson Observatory; an apparatus for securing core-samples from the ocean-bottom, by the Geophysical Laboratory. In the course of the Exhibition seven public lectures were given among which was one by O. H. Gish on "The natural electrical currents in the Earth."

37. *Personalia*—We have learned from a notice in *L'Astronomie* of the death on March 11, 1933, of *Georges Le Cadet*, at the age of 69 years. Although a large part of his scientific activity lay in the fields of astronomy and meteorology, he will be remembered by readers of this JOURNAL for his investigations of the variation of the potential gradient of the atmosphere with altitude, the results of which were embodied in a thesis entitled "Etude du champ éléctrique de l'atmosphère" (1898). In 1906 he was placed in charge of the Observatory of Indo-China at Phu-Lien where he remained for over twenty years engaged in the study of the meteorological and climatological aspects of the country.

Prof. Dr. *C. Dorno*, of Davos, Switzerland, meteorologist and geophysicist, widely known for his outstanding investigations of solar and sky radiation as well as for his studies of electricity and radioactivity of high-mountain atmosphere, observed his seventieth birthday on August 3, 1935.

Major General *Adolphus W. Greely*, meteorologist and arctic explorer, died in Washington, D. C., October 20, 1935, at the age of 91 years.

W. J. Green left Washington, D. C., December 11, 1935, for Watheroo, Western Australia, where he will relieve *W. C. Parkinson* as observer-in-charge of the Watheroo Magnetic Observatory, February 1, 1936; Mr. Parkinson will then have been in charge of the Observatory for five years. After taking leave of absence he will reoccupy, during 1936-37, many magnetic stations on Pacific islands to determine secular changes.

PROVISIONAL SUNSPOT-NUMBERS FOR SEPTEMBER TO NOVEMBER, 1935

(Dependent alone on observation at Zürich Observatory and its station at Arosa)

Day	Sep.	Oct.	Nov.	Day	Sep.	Oct.	Nov.
1	30	*E*..[c]	*E*22[c]	17	34[d]	*M*95[cc]	*E*..[ac]
2	36	*M*73[ac]	17	18	33[a]	84	94[a]
3	37	68[a]	*E*42[cd]	19	31	79[a]	91
4	47[a]	52	46[aa]	20	20	59[a]	70
5	48	61	46	21	18	..	71
6	47	57[a]	41	22	*M*21[c]	..	52[b]
7	48	53	59[d]	23	..[a]	..	65
8	*W*47[c]	40	59	24	*M*56[c]	..	45
9	53[d]	0	67[b]	25	*E*59[c]	..	44
10	32	7	61	26	69[d]	55[b]	*M*56[cd]
11	23	25[d]	*E*68[cd]	27	80[d]	..	58[a]
12	*E*23[c]	*M*34[c]	68	28	71	..	58
13	33	*M*60[dc]	97[abd]	29	78	..	64
14	*E*29[c]	53[a]	*E*98[c]	30	61	13	56
15	26	69[d]	117	31		13	
16	33	68	110				
				Means..	42.2	50.8	63.5
				No. days	29	22	29

Mean for the quarter July to September, 1935, 35.5 (87 days)

[a]Passage of an average-sized group through the central meridian.
[b]Passage of a large group through the central meridian.
[c]New formation of a new center of activity: *E*, on the eastern part of the Sun's disc; *W*, on the western part; *M*, in the central-circle zone.
[d]Entrance of a large or average-sized center of activity on the east limb.

W. BRUNNER

EIDGEN. STERNWARTE,
Zürich, Switzerland

AMERICAN *URSI* BROADCASTS OF COSMIC DATA[1] JULY TO SEPTEMBER, 1935

The data for terrestrial magnetism, sunspots, and solar constant are the same as given in previous tables.

The first three columns of the Table give (1) the magnetic character according to the scale 0-2 of the International Commission of Terrestrial Magnetism and Atmospheric Electricty, (2) the type featuring the day other than normal by the letters *b*, *p*, *o*, and *i* for days marked by bay, rapid pulsations, long-period oscillations, and irregular oscillations, respectively, and (3) the hour and minute of Greenwich mean time marking the beginning of a storm, the end of the storm being indicated in the footnote to the Table. The next two columns give the data relating to sunspots: (1) the number of groups of spots and (2) the total number of spots. It is to be noted that sunspot-numbers such as those from Zürich can be obtained from the number of groups and spots given in the Table by the formula $N=k\ (10g+s)$, where k for Mount Wilson is about 0.7.

[1]For previous announcements see Terr. Mag., 35, 184-185 and 252-253 (1930); 36, 74, 141, 258-259, and 358-360 (1931); 37, 85-89, 189-192, 408-411, and 484-487 (1932); 38, 60-63, 148-151, 262-265, 335-339 (1933); 39, 73-77, 159-163, 244-247, 353-356 (1934); 40, 111-115, 220-222 and 334-336 (1935).

Mount Wilson Observatory is now supplying corrections and additions to the sunspot-data which are broadcast in the *URSI*gram. So far as possible, these additional and corrected values will be used in this tabular summary and will be designated as such in footnotes to the Table.

Beginning January 1, 1934, the magnetic information of the *URSI*-gram is for Cheltenham, Maryland, instead of Tucson, Arizona. In addition to this change in observatory, the data cover the 24 hours ending 8 A.M., 75° west meridian mean time, instead of the 24 hours ending at 7 A. M., 105° west meridian mean time.

The columns headed solar constant show (1) the value in calories of the solar constant, and (2) by letters *s*, *f*, and *u* whether the determination was satisfactory, fair, or unsatisfactory, respectively.

In accordance with information received from Dr. C. G. Abbot, Secretary of the Smithsonian Institution, transfer from Table Mountain to Montezuma solar-constant values was made as of October 23, 1934. Table Mountain for a considerable time has been 0.012 calorie above

Summary American URSI daily broadcasts of cosmic data, July to September, 1935

Date	July							August							September							Date
	Magnetism			Sun-spot		Solar constant		Magnetism			Sun-spot		Solar constant		Magnetism			Sun-spot		Solar constant		
	Character	Type	G. M. T. begin. distur.	Groups	Number	Value	Character	Character	Type	G. M. T. begin. distur.	Groups	Number	Value	Character	Character	Type	G. M. T. begin. distur.	Groups	Number	Value	Character	
			h *m*			*cal.*				*h* *m*			*cal.*				*h* *m*			*cal.*		
1	0	...	.. ..	2*	18*	1.939	*s*	0	..	.. ..	1	3	......	...	1	...	.. ..	3	7	1.940	*f*	1
2	0	.	. .			1.935	*s*	0	.		2	5	1.923	*f*	0			4	6	1.912	*u*	2
3	0	...	.. ..	2	11	......	...	0	...	.. ..	2	3	1.936	*f*	0	...	.. ..	5*	9*	1.916	*u*	3
4	0	...	.. ..	3*	12*	1.950	*f*	0	...	.. ..	3	5	1.928	*f*	0	...	.. ..	5	14	1.931	*f*	4
5	0	...	.. ..	3	5	1.947	*f*	0	...	.. ..	3	6	1.933	*f*	1	*i*	18 00	5*	20*	......	...	5
6	0	...	.. ..	2	9	1.940	*f*	0	...	.. ..	3	13	1.939	*s*	0	...	.. ..	5	20	1.933	*f*	6
7	0	...	.. ..	4	11	1.935	*f*	1	...	.. ..	4	12	......	...	0	...	.. ..	5	17	1.929	*f*	7
8	1	*i*	21 10	4	14	1.937	*s*	0	...	.. ..	5*	10*	......	...	0	...	.. ..	4*	14*	......	...	8
9	1	...	.. ..	4	21	1.933	*f*	0	...	.. ..	4	8	1.942	*f*	0	...	.. ..	5	15	......	...	9
10	0	...	.. ..	4	19	1.923	*f*	0	...	.. ..	4*	9*	1.936	*f*	1	...	.. ..	3	14	......	...	10
11	0	...	.. ..	5	10	1.930	*s*	0	...	.. ..	5*	10*	1.935	*f*	1	*i*	18 27	3	9	......	...	11
12	0	...	.. ..	6*	19	1.924	*f*	0	...	.. ..	5	13	1.931	*s*	2	*i*	.. ..	4*	9*	1.951	*f*	12
13	0	...	.. ..	6	22	1.932	*f*	0	...	.. ..	3*	5*	1.942	*f*	0	...	.. ..	3	8	1.960	*f*	13
14	0	...	.. ..	6	19	1.931	*f*	0	...	.. ..	5	7	1.933	*f*	0	...	.. ..	4*	13*	1.950	*s*	14
15	0	...	.. ..	4	15	......	...	...	...	.. ..	2	3	1.943	*s*	0	...	.. ..	3*	16*	1.941	*s*	15
16	0	...	.. ..	4	22	......	...	0	...	.. ..	4*	6*	1.949	*s*	1	...	.. ..	3*	20	1.932	*f*	16
17	0	...	.. ..	4	20	......	...	0	...	.. ..	3	6	1.931	*f*	1	...	.. ..	3	21	1.945	*f*	17
18	0	...	.. ..	5	17	1.931	*f*	0	...	.. ..	2	7	1.934	*f*	1	...	.. ..	3	17	1.949	*f*	18
19	1	...	.. ..	4	24	......	...	0	...	.. ..	4	10	1.934	*f*	1	...	.. ..	3	12	1.958	*f*	19
20	0	...	.. ..	3	23	......	...	1	...	.. ..	3	31	1.931	*f*	0	...	.. ..	2	4	......	...	20
21	0	...	.. ..	2	20	......	...	1	...	.. ..	3	25	......	...	0	...	.. ..	2*	4	......	...	21
22	1	...	.. ..	2	15	1.930	*f*	1	...	.. ..	2	18	1.931	*f*	0	...	.. ..	3	13	......	...	22
23	1	...	.. ..	2*	18	......	...	1	...	.. ..	2	12	1.929	*f*	1	*o*	2 54	4	28	1.928	*u*	23
24	0	...	.. ..	2	12	......	...	0	...	.. ..	3	4	1.933	*s*	1	*i*	.. ..	5	35	......	...	24
25	2	*i*	20 34	2	9	......	...	1	...	.. ..	...	...	1.933	*f*	2	*o*	1 48	8*	27	......	...	25
26	1	...	.. ..	2	5	......	...	0	...	.. ..	3	5	1.925	*s*	1	...	.. ..	9*	34*	......	...	26
27	0	...	.. ..	1	1	......	...	0	...	.. ..	3	6	1.933	*f*	0	...	.. ..	7	45	1.930	*u*	27
28	0	...	.. ..	1	1	......	...	1	*i*	17 30	4	12	1.934	*f*	0	...	.. ..	9*	29	1.938	*s*	28
29	1	...	.. ..	0	0	......	...	0	...	.. ..	3	12	1.927	*f*	0	...	.. ..	8*	40*	......	...	29
30	0	...	.. ..	1	2	......	...	0	...	.. ..	3	6	1.930	*f*	0	...	.. ..	9	36*	1.926	*f*	30
31	0	...	.. ..	1	2	1.956	*f*	0	...	.. ..	4	10	1.929	*f*								31
Mean	0.3	...	.. ..	3.0	13.2	1.936	...	0.2	...	.. ..	3.2	9.3	1.933	...	0.5	...	.. ..	4.6	18.5	1.937	...	Mean

*A revision of value originally broadcast.

Greenwich mean time for ending of storms: 6^h, July 9; 12^h, July 26; 7^h, August 29; 4^h, September 6; 7^h 54^m, September 12; 5^h 21^m, September 24; 12^h, September 25.

Kennelly-Heaviside Layer heights, Washington, D. C., July to September, 1935
(Nearest hour, Greenwich mean time, of all observations is 17)

Date	Freq.	Ht.	Date	Freq.	Ht.	Date	Freq.	Ht.	Date	Freq.	Ht.
1935	*kc/sec*	*km*	*1935*	*kc/sec*	*km*	*1935*	*kc/sec*	*km*	*1935*	*kc/sec*	*km*
Jul. 4	3,000	110	Jul. 24	5,900	660	Aug. 14	5,600	430	Sep. 4	7,100	470
" "	3,800	110	" "	6,100	*	" "	6,100	430	" "	7,200	*
" "	3,860	110	Jul. 31	3,000	110	" "	6,200	490	Sep. 11	2,500	120
" "	3,860	140	" "	3,500	110	" "	6,300	930	" "	3,000	130
" "	4,000	110	" "	3,800	120	" "	6,400	*	" "	3,430	140
" "	4,000	140	" "	3,800	250	Aug. 21	3,000	120	" "	3,430	290
" "	4,400	150	" "	4,000	230	" "	3,300	140	" "	3,700	260
Jul. 10	2,500	130	" "	4,380	370	" "	3,400	300	" "	4,300	360
" "	3,500	130	" "	4,500	470	" "	3,550	210	" "	4,500	330
" "	3,800	140	" "	4,600	420	" "	3,900	280	" "	4,700	330
" "	4,400	*	" "	4,700	380	" "	4,100	*	" "	4,800	310
" "	4,500	110	" "	5,100	360	" "	4,400	590	" "	5,100	290
" "	4,500	310	" "	5,400	400	" "	4,500	600	" "	5,400	320
" "	4,700	450	" "	5,500	470	" "	4,600	570	" "	5,900	330
" "	4,900	370	" "	5,600	580	" "	4,700	*	" "	6,500	340
" "	5,100	120	" "	5,700	440	" "	5,000	290	" "	6,600	770
" "	5,100	350	" "	5,900	380	" "	5,100	*	" "	6,800	800
" "	5,500	340	" "	6,100	420	" "	5,200	560	" "	6,900	380
" "	5,700	370	" "	6,200	490	" "	5,300	620	" "	7,000	410
" "	5,900	490	" "	6,300	*	" "	5,400	*	" "	7,100	470
" "	6,100	390	Aug. 7	2,800	110	Aug. 28	2,500	110	" "	7,200	*
" "	6,100	510	" "	3,300	130	" "	3,400	110	Sep. 18	2,500	120
" "	6,500	400	" "	3,400	*	" "	3,500	260	" "	3,030	120
" "	6,700	450	" "	3,480	170	" "	3,650	230	" "	3,030	180
" "	6,900	*	" "	3,700	130	" "	4,000	270	" "	3,400	130
Jul. 17	3,000	110	" "	3,820	250	" "	4,400	350	" "	3,400	240
" "	3,320	110	" "	3,900	220	" "	4,600	590	" "	3,600	200
" "	3,330	150	" "	4,250	490	" "	4,700	910	" "	4,400	500
" "	3,600	150	" "	4,400	430	" "	4,800	420	" "	4,500	440
" "	3,700	*	" "	4,500	510	" "	5,000	450	" "	4,700	420
" "	4,300	240	" "	4,900	380	" "	5,100	540	" "	4,900	450
" "	4,500	330	" "	5,100	520	" "	5,200	460	" "	5,000	530
" "	4,700	470	" "	5,300	630	" "	5,500	460	" "	5,100	460
" "	4,900	410	" "	5,500	*	" "	5,600	470	" "	5,500	460
" "	5,100	390	" "	5,700	500	" "	5,700	490	" "	5,600	490
" "	5,300	400	" "	5,900	520	" "	5,800	550	" "	5,700	590
" "	5,800	410	" "	6,100	600	" "	5,900	*	" "	5,800	*
" "	5,800	540	" "	6,300	*	Sep. 4	2,500	120	Sep. 25	3,000	120
" "	7,100	420	Aug. 14	3,000	120	" "	3,400	120	" "	3,340	130
" "	6,300	380	" "	3,400	130	" "	3,500	160	" "	3,340	320
" "	6,600	410	" "	3,480	130	" "	3,550	*	" "	3,700	130
" "	6,700	*	" "	3,480	230	" "	3,600	260	" "	3,700	200
Jul. 24	3,000	110	" "	3,650	130	" "	3,700	220	" "	4,100	380
" "	3,500	130	" "	3,650	160	" "	4,500	330	" "	4,300	*
" "	3,900	160	" "	3,690	260	" "	4,700	330	" "	4,350	740
" "	4,000	*	" "	3,850	230	" "	4,900	300	" "	4,400	670
" "	4,400	320	" "	4,300	340	" "	5,300	310	" "	4,500	570
" "	4,500	620	" "	4,500	300	" "	5,600	320	" "	4,600	580
" "	4,700	450	" "	4,800	440	" "	6,000	330	" "	4,700	680
" "	4,900	460	" "	4,900	440	" "	6,400	330	" "	4,800	320
" "	5,100	840	" "	5,300	330	" "	6,500	340	" "	4,900	360
" "	5,300	*	" "	5,500	440	" "	6,900	380	" "	5,200	590
" "	5,700	600	" "	5,500	540	" "	7,000	410	" "	5,400	670
									" "	5,500	*

* = No value obtained.

Montezuma, and above the scale of 1913 to 1930. Hence the value of October 23 and succeeding values are on a scale 0.012 calorie lower than previous ones.

The table of Kennelly-Heaviside Layer heights is self-explanatory.

C. C. Ennis

Department of Terrestrial Magnetism,
Carnegie Institution of Washington,
Washington, D. C.

A NEW EXPERIMENTAL METHOD FOR STUDY OF THE UPPER ATMOSPHERE

The idea of utilizing the light scattered from a powerful searchlight-beam directed to high altitudes has frequently been suggested for exploring the upper atmosphere. Examination of this possibility was made here some years ago, and no doubt elsewhere at earlier dates, but, in addition to technical difficulties connected with instrumental sensitivity, the background level of the light of the night sky constitutes a fundamental limitation. The light scattered from a powerful searchlight-beam in the absence of clouds or dust (molecular scattering only) would be submerged in this background luminosity even for altitudes which are reached by instrument-carrying balloons. However, an audacious suggestion was made in 1930 by E. H. Synge,[1] who proposed to concentrate the beams from an assemblage of several hundred searchlights (for example, with the help of the military authorities) on a common volume of the upper air. Utilizing a 36-inch mirror in conjunction with the photo-cell at the receiver, Synge calculated that with one hundred large searchlights it should be possible to measure the light scattered by the atmosphere (cloud-free) at a height of 30 km with an accuracy of possibly one per cent. The stimulus of this large-scale proposal led one of us (M. A. T.) shortly thereafter to discuss the problem at a meeting of the Washington Atomic-Physics Colloquium and to point out that by using a *modulated searchlight-beam*, thereby placing an "identification-tag" on the scattered light it is desired to observe, a method of wholly new possibilities emerged from the older idea. Not only does this make available the convenience of resonance-instruments for high-sensitivity observations (alternating-current amplification), but the background limit is no longer the total light of the night sky entering the receiver but only that Fourier component of the (variable) sky-light which has the same frequency as the searchlight-modulation. On this basis it was suggested that the use of a single modulated searchlight, with a 36-inch receiving mirror, gave promise of making accessible to examination the unexplored levels of the atmosphere at heights above 30 km.

During the past several months we have undertaken a further study of the possibilities of this method. It appears that rather accurate measurements of the light returned by molecular scattering should be feasible for altitudes up to about 70 km, and approximate values should be obtainable up to 80 km. It is evident that this affords a direct means of attacking the important problem of the atmospheric density at great altitudes, as well as studying high clouds and possible fluorescence even to the very top of the atmosphere. These altitude figures are of great interest since very little information of any kind is available for

[1]E. H. Synge, London Phil. Mag., 9, 1014-1020 (1930).

the region of the atmosphere which lies above the limits of instrument-carrying balloons (about 30 km) and below the lowest auroral levels (about 80 km), and, in addition to the intrinsic meteorological significance of the observations themselves, data on the lower zones are highly important to recent investigations relating to the auroral and ionosphere-regions of the upper air.

The sensitivity-limits may be estimated in the following way. A standard 10-kw searchlight with a 36-inch 110° mirror gives about 50,000 lumens (approximately 75 watts luminous power) in a beam which under special conditions may have as low as 1° divergence. Aimed 60° above the horizontal, the diameter of the beam at 30-km height is 600 meters. At the receiver, some kilometers distant from the source, if a similar mirror elevated 60° is used with a photo-cell of the same area as the arc-crater, the divergence is again 1° and the volume of the atmosphere at 30-km height which scatters light to the receiver is approximately a 600-meter length of the searchlight beam. [It may be advantageous to use a slightly larger aperture at the receiver.] The scattering-coefficient for a gas is $k = 32\pi^3 (\mu - 1)^2/3N\lambda^4$, where μ for air is 1.0002926 at $\lambda = 5.9 \times 10^{-5}$ cm. Thus in air under standard conditions $8.6 \times 10^{-8} \sim 10^{-7}$ of a light-beam is scattered per cm of path-length (independent of area). Hence if the density at 30-km height is one-fiftieth the sea level density 1.2×10^{-4} of the beam is scattered in all directions (somewhat non-uniformly) by the 600-meter section seen by the receiver. This corresponds to a light-source emitting $1.2 \times 10^{-4} \times 75 = 0.009$ watt, viewed by a 36-inch mirror at 35-km distance. By solid-angle ratios, 4×10^{-13} watt or about 3×10^{-10} lumen then enters the photo-cell. With a sensitivity of 35 microamperes per lumen the latter responds with a current of 1×10^{-14} ampere. Estimates of the background light of the night sky vary somewhat, but an intensity one hundred times the light scattered from the beam at 30 km appears to be a fair estimate. With this background (assumed steady) an added increment (due to the beam) of 10^{-17} ampere can be detected, and 5×10^{-17} ampere can be measured to about two per cent, using a modulation-frequency of one cycle per second. This is one two-hundredth of the current estimated above for the scattering from a height of about 30 km. Assuming a ten-fold decrease in density for each 20-km additional height (and recalling the inverse-square decrease, partially compensated by length of beam-section viewed), this would correspond to measurements up to a height of 70 km with an accuracy of about two per cent, provided the amplitude of the Fourier component of the night-sky light at 1.0 cycle per second does not exceed 10^{-4} of the total. With the narrow band-width of 0.01 cycle in the amplifier this assumption seems plausible. These limits can obviously be extended by using more than one searchlight, but the limits of current-measurement have been extended as far as is practical. The use of a synchronous rectifier is assumed at the receiver; without this refinement measurements at 50 km to about two per cent should be possible.

With regard to the significance which observations by this method may be expected to have, it is perhaps too early to say much. However, using very rough spectral resolution (filters) to determine the magnitudes of the Rayleigh scattering and the white-light scattering separately, data on the variation of atmospheric density with height—a basic unknown in present-day upper-air studies—should be obtained directly. Numerous

problems connected with water-vapor, turbulence, winds, dust, and the variation of the night-sky light itself, as well as certain very interesting possibilities with regard to fluorescence and absorption relate themselves immediately to observations of this type. In order to avoid haze and polar auroræ as far as possible, regions of dry atmosphere and low latitude are evidently most suitable for these experiments. We are grateful to Professor S. Chapman and Dr. J. Bartels for discussion and encouragement.

Laboratory tests of a photo-cell and amplifier used with an artificial light-source have substantially checked the sensitivity limits given above, and preparations for field tests are under way.

M. A. TUVE, E. A. JOHNSON, O. R. WULF

DEPARTMENT OF TERRESTRIAL MAGNETISM,
CARNEGIE INSTITUTION OF WASHINGTON (M. A. T., E. A. J.),

BUREAU OF CHEMISTRY AND SOILS,
U. S. DEPARTMENT OF AGRICULTURE (O. R. W.),

Washington, D. C., October 15, 1935

PROVISIONAL SOLAR AND MAGNETIC CHARACTER-FIGURES, MOUNT WILSON OBSERVATORY, JULY, AUGUST, AND SEPTEMBER, 1935

Greenwich mean time							Range
Beginning				Ending			Hor. int.
1935		*h*	*m*	*d*	*h*	*m*	γ
July	7	21	12*	8	13	..	112
Sept.	10	18	..	12	8	..	147

*Regular clock not recording; time may be in error by two or three minutes.

The magnetic storm of July 7 began with a sudden decrease of 4 gammas in the horizontal intensity followed immediately by an increase of 27 gammas. The most active spot-group on July 7 was in latitude 34° south and longitude 45° east. A smaller group in latitude 22° north first appeared on July 7, 65° east of the central meridian, and developed its maximum area on July 9.

The most active group on September 10 was in latitude 18° south and longitude 72° west. This group first appeared on September 7 in longitude 27° west, and was moderately active from September 7 until it disappeared at the west limb on September 11.

On July 24 at $20^h\ 37^m$, G. M. T., the horizontal intensity increased suddenly by 22 gammas. This was followed by irregular oscillations with a maximum amplitude of 88 gammas which ended July 25 at about 12^h. On Sepetember 25 a similar disturbance occurred with a total amplitude of 120 gammas. Although the range in *H* was larger than 100 gammas on September 25 the variations were not like those of the usual magnetic storm.

July 1935

Day	K_2 A	K_2 B	HαB A	HαB B	HαD A	HαD B	No. groups	Mag'c char.
1	1	0.5	1	0	2	1	2	0
2								0
3	1	0.5	1	0	1	0	2	0
4	1	0	1	0	1	0	3	0
5	1	0	1	0	1	0	3	0
6	1	0	1	0	1	0	2	0
7	1	0	1	0	1	0	4	0.5
8	1	0.5	1	0	1	0	4	1
9	2	1	2	1	1	1	4	0.5
10	2	1	2	1	1	1	4	0
11	2	1	2	1	2	0	5^{l}	0.5
12	2	1	2	1	2	0	6^{h}	0
13	2	2	2	2	1	0	6	0
14	2	1	2	1	2	0	6^{i}	0.5
15	2	0.5					4	0
16	1	0.5	2	0	1	0	4	0
17	1	1	2	1	1	0	4^{l}	0
18	1	0.5	1	0	1	0	5^{b}	0
19	1	0.5	$2^{d,d}$	0	1	0	4	0.5
20	1	1	2^{d}	1^{c}	1	0	3	0
21	2	2	2	2	1	0	2^{i}	0
22	1	2	2	2	1	0	2	0.5
23	1	0.5	1	0.5	2	1	2	0
24	1	0	2	0	2	0	2	1
25	1	0	2	0	2	0.5	2	1
26	1	0.5	2	1	2	1	2	0
27	1	0.5	1	0.5	2	2	1	0
28	1	0	1	0	1	1	1	0
29	1	0	1	0	1	1	0	0.5
30	1	0	1	0	1	1	1	0
31	1	0	1	0	1	0	1	0
Mean	1.3	0.6	1.5	0.5	1.3	0.4	3.0	0.2

August 1935

Day	K_2 A	K_2 B	HαB A	HαB B	HαD A	HαD B	No. groups	Mag'c char.
1	1	0	1	0	1	0.5	1	0
2	1	0	1	0	1	0	2	0
3	1	1	1	1	1	0	2	0
4	1	1	1	1	1	0	3^{i}	0
5	1	1	1	1	1	0	3	0
6	1	1	1	0.5	1	0	3	0
7	2	1	2	0	1	0	4	0
8	2	1	2	1	1	0	5	0
9	2	2	2	1.5	1	0.5	4	0
10	2	2	2	1	1	0	4^{i}	0
11	2	2	2	1	2	1	5	0
12	2	1	2	1	1?	0.5?	5	0
13							3	0
14	2	0.5	2	0.5	1	0	5	0
15							2	0
16	2	1	2	1	1	0	4^{a}	0
17	2	1	2	1	1	0	3	0
18	2	2	2	2	1	0	2	0
19	2	2	1	1	1	0	4	0.5
20	2	2	2	3^{c}	1	0	$3^{a,g}$	0.5
21	2	1	2^{d}	1	1	0	3	0.5
22	1	1	2	1	2	0	2	0.5
23	1	0	2	0	1	0	2	0.5
24	1	0	1	0	1	0	3	0
25								0.5
26							3	0
27	1	0	2	0	1	1	3	0.5
28							4	0.5
29	2	1	2	1	2	0	3	0
30	2	2	2	2^{c}	2	0	3	0
31	2	2	2	2	1	1	4^{i}	0.5
Mean	1.6	1.1	1.7	0.9	1.2	0.2	3.2	0.1

September 1935

Day	K_2 A	K_2 B	HαB A	HαB B	HαD A	HαD B	No. groups	Mag'c char.
1	2	1	2	1^{c}	1	1	3	0.5
2	2	1	2	1	2	2	4	0
3	2	2	2	2	2	1	5	0
4	2	2	2.5	2	2	2	5^{a}	0.5
5	2.5	3	3	3	2	1	5	0
6	2.5	2.5	3	3	2	1	5	0
7	2	1.5	2.5	2	2	0.5	5^{b}	0
8	2	0.5	2	0	2	0	4	0
9	2	0	2	0	1	1	5	0
10	2	0	1	0	1	0	3	0.5
11	2	0	1	0	1	0	3	1.5
12	2	0	1	0	1	0	4	1
13	2	1	2	1	2	0	3	0
14	2	1	2	1	2	0	4	0
15	2	2	2	2	1	0	3	0.5
16	2	3	2	3	1	0	3	0.5
17	2	2	3	2	1	0	3	0.5
18	2	2	2.5	2	1	0	3	1
19							3	0.5
20	2	0	2.5	0	1	0	2	0
21	2	0	2	0	1	0	2	0
22	2	2	2	2	2	0	3^{a}	0
23	2	3	2.5	3	2	0	4^{h}	1
24	2	3	2.5	2.5	1	0	5^{b}	0.5
25	2	2	2.5	2	1	0.5	8	1
26	2	1	3	0	1	1	9	0.5
27	3	1	3	0	1	1	7	0.5
28	2	1	3	1	1	0	9	0.5
29	3	1	2	1	1	0	8	0.5
30	3	2	2	2	2	2	9	0.5
31								
Mean	2.1	1.4	2.2	1.3	1.4	0.5	4.6	0.4

NOTE—For an explanation of these tables see this JOURNAL, 35, 47-49 (1930).
?Indicates an uncertain value which should be given low weight.
a, b Formation of a new group which later developed to average size or larger; (a) less than 30° from the center of the disc, (b) more than 30° from the center of the disc.
c, d Very bright hydrogen-flocculi; (c) less than 30° from the center of the disc, (d) more than 30° from the center of the disc.
g, h, i, j, l Passage of a large or average group across the central meridian within 15°, 20°, 25°, 30°, 40° of the center of the disc, respectively.

SETH B. NICHOLSON
ELIZABETH E. STERNBERG
*Carnegie Institution of Washington, Mount Wilson Observatory,
Pasadena, California*

CORRELATION OF AURORAL AND MAGNETIC ACTIVITIES FOR DIFFERENT PERIODS OF THE NIGHT AT CHESTERFIELD, CANADA, 1932-33

In the September number of this JOURNAL (p. 267), correlation of auroral and magnetic activities was discussed using the twelve-hour night period as a basis. At the suggestion of Professor S. Chapman, correlation-coefficients (r) have been worked out for three-hour periods, to determine whether the high degree of correlation found for twelve-hour periods obtained during different parts of the night. In the following summary of results I_A and I_M are mean auroral intensity and magnetic character-number, respectively.

Period, local mean time		Correlation-coefficient r	Ratio r/p. e.	I_A	I_M
From	To				
h	h				
18	21	+0.68	14	0.8	0.7
21	24	+0.70	15.5	1.1	1.2
24	3	+0.53	8	1.3	1.1
3	6	+0.31	3.5	0.8	0.4
18	6	+0.63	11.5	1.0	0.8

The correlation is good for all except the morning hours 3^h to 6^h. During these hours magnetic activity decreases more rapidly than auroral activity. The value of I_A for the morning hours is as great as for the evening hours 18^h to 21^h, when conditions for observation of aurora are the same. It appears then that at this station, although auroral and magnetic activities are strongly associated during the greater part of the night, auroral activity continues into the morning hours when much quieter magnetic conditions prevail.

A difference observed in auroral displays of the morning hours as compared with those seen at other times was in the direction of auroral arcs and bands. Usually these ran from northwest toward southeast but during the period just before sunrise a sudden change to a direction at right angles, namely, northeast toward southwest, was frequently observed. This change was always preceded by a lull in auroral activity.

F. T. DAVIES

DEPARTMENT OF TERRESTRIAL MAGNETISM,
CARNEGIE INSTITUTION OF WASHINGTON,
Washington, D. C.

PRINCIPAL MAGNETIC STORMS

Sitka Magnetic Observatory

July to September, 1935[1]

(Latitude 57° 03'.0 N., longitude 135° 20'.1 or 9h 01m.3 W. of Gr.)

No storms of importance were recorded at the Sitka Magnetic Observatory during the third quarter of 1935.

John Hershberger, *Observer-in-Charge*

Cheltenham Magnetic Observatory

July to September, 1935[1]

(Latitude 38° 44'.0 N., longitude 76° 50'.5 or 5h 07m.4 W. of Gr.)

July 24-26—A mild disturbance beginning July 21 reached storm-proportions July 24, when, at 20h 33m, there occurred an abrupt decrease in horizontal intensity of about 22 gammas followed almost immediately by an increase of about 106 gammas. At the same time the declination also changed suddenly but through a small amplitude. The disturbance continued until 4h on July 26. There were three bays in all three elements on July 25 between 5h and 9h. The ranges during the storm were: Declination 31′; horizontal intensity 154 gammas; and vertical intensity 128 gammas.

September 10-12—A mild storm began September 10 at 18h 27m. It was characterized by short-period oscillations in all three elements until about September 12 at 0h 30m when the perturbations changed to long-period fluctuations. The storm ended September 12 at 9h. The ranges were: Declination 33′; horizontal intensity 106 gammas; and vertical intensity 110 gammas.

September 24-26 A storm began very gradually September 24 about 11h, and ended as gradually about 22h on September 26. The fluctuations were for the most part irregular, the distinguishing feature of the storm being a double bay in declination from 3h 20m to 5h 00m, September 25, with a range of 33′. In the same interval, the horizontal intensity had a single bay with a range of 96 gammas and the vertical intensity had a single bay with a range of 72 gammas. From about 8h 10m to 9h 30m a bay occurred in all three elements with ranges 36′ for declination, 74 gammas for horizontal intensity, and 104 gammas for vertical intensity. The ranges during the storm were: Declination 39′; horizontal intensity 136 gammas; and vertical intensity 144 gammas.

Albert K. Ludy, *Observer-in-Charge*

Huancayo Magnetic Observatory

July to September, 1935

(Latitude 12° 02'.7 S., longitude 75° 20'.4 or 5h 01m.4 W. of Gr.)

Greenwich mean time						Range		
Beginning			Ending			Decl'n	Hor. int.	Ver. int.
1935	*h*	*m*	*d*	*h*	*m*	′	γ	γ
July 7	21	09	9	24	..	7	154	38
July 24	20	35	25	24	..	8	150	33
September 10	18	30	12	08	..	6	310	49
September 23	01	33	25	20	..	7	258	34

[1]Communicated by R. S. Patton, Director, United States Coast and Geodetic Survey.

July 7-9—A minor disturbance, chiefly important for a sudden commencement in all three elements at 21^h 09^m G. M. T., with an increase in H of 35 gammas in 5 minutes, an increase in Z, in the same interval, of 5 gammas, and a decrease in D of $0'.5$. Thereafter, minor perturbations were superposed on the normal trend in H until 6^h July 8, at which time a deep bay began which reached a minimum at 12^h 28^m. Following this bay, the diurnal maximum was greatly depressed. The diurnal maximum on the following day, July 9, was almost completely suppressed, but the remainder of the day showed minor fluctuations superposed on the normal diurnal trend. D and Z, throughout the disturbance, showed only minor perturbations not greatly affecting the regular diurnal course.

July 24-25—A minor disturbance, with conspicuous sudden commencement in all three elements at 20^h 35^m G. M. T. H first decreased 3 gammas in one minute, then increased 62 gammas in 3 minutes; Z increased 7 gammas in 4 minutes; and D decreased $0'.7$. Following the commencement, H showed several long-period bays of range about 30 or 40 gammas and of duration about two hours from crest to crest, until 12^h on July 25. Thereafter, small rapid fluctuations were superposed on a somewhat depressed diurnal maximum. At 15^h, following the maximum, H resumed a normal trend, although the maximum of July 26 was much lower than usual. D and Z showed considerable perturbation, though diurnal trends were well maintained.

September 10-12—This disturbance (character 1) appears to have begun at 18^h 30^m G. M. T. with a small peak in H and slight gradual increases in D and Z, over a period of 10 minutes. Thereafter, H was very irregular, with small, rapid fluctuations superposed on larger peaks and bays. Throughout September 11, all three elements were quite disturbed, although D and Z maintained their general diurnal trends. H was particularly active during the daylight hours of that day between 13^h and 22^h, G. M. T. From 6^h to 8^h on September 12, H was disturbed by several bays and peaks of long period while D and Z were comparatively quiet. All elements became markedly quiet and smooth at 8^h on September 12.

September 23-25—This disturbance (character 1) began with a sudden commencement in H and Z but none in D. H increased 10 gammas in 8 minutes and Z increased 2 gammas in the same period. H thereafter exhibited minor long-period fluctuations until 12^h 42^m, after which, until 15^h 50^m, it showed several bays and peaks of 20-minute period with ranges of 50 to 75 gammas. D during this period was unusually disturbed, there being several bays and peaks of ranges of $4'$ to $6'$. After 15^h 50^m, and until the end of the disturbance, D and Z were little disturbed; H however, was unusually low between 20^h on September 23 and 3^h on September 24 and again between 2^h and 9^h on September 25. The end of the disturbance is not very definite.

O. W. Torreson, *Observer-in-Charge*

Apia Observatory

July to September, 1935

(Latitude 13° 48'4. S., longitude 171° 46'.5 or 11^h $27^m.1$ W. of Gr.)

July 7-10—A magnetic disturbance commenced with a sudden rise in H of 16 gammas at 21^h 07^m July 7, to reach a value of 35051 gammas.

After oscillating about this value the trace began to descend from about $5^h\ 00^m$ July 8 and reached a minimum of 34910 gammas at about $10^h\ 10^m$ July 8. The trace then rose slowly and continued more or less disturbed until July 10.

July 19-20—The H-trace was slightly disturbed from early July 19 until July 20.

August 19—A sudden small increase in H, and a corresponding increase in the numerical value of Z, occurred at $5^h\ 22^m$ August 19 and both traces then continued moderately disturbed for several days.

August 27-28—A slight disturbance commenced with sudden rises in H and in the numerical value of Z at $17^h\ 31^m$ August 27 but lasted only until August 28.

September 9-12—A magnetic storm, which began indefinitely early September 9, developed throughout September 10 and, September 11, caused a very disturbed trace in H. H decreased to reach a minimum of 34913 gammas at $5^h\ 37^m$ September 12 and then gradually returned to normal. The range of the disturbance in H was 148 gammas.

September 23-25—A disturbance, beginning indefinitely September 23, caused moderate fluctuations in H on that and the following day. The value of H began to decrease sharply at $2^h\ 00^m$ September 25 and continued, reaching a minimum at about $11^h\ 20^m$ on that day. The trace was subnormal during the remainder of September, the range in H throughout the disturbance amounting to 126 gammas.

H. F. BAIRD, *Acting Director*

WATHEROO MAGNETIC OBSERVATORY

JULY TO SEPTEMBER, 1935

(Latitude 30° 19′.1 S., longitude 115° 52′.6 or $7^h\ 43^m.5$ E. of Gr.)

September 10-12—This was a disturbance of moderate intensity presenting some unusual features. The exact time of beginning is doubtful. At $21^h\ 50^m$ G. M. T., September 10, there was a sharp but small movement shown in all three elements, and this possibly marked the commencement of the disturbance. The horizontal intensity increased 5 gammas, the vertical intensity decreased 7 gammas, and the westerly declination decreased 1′.2; all these changes took place in about two minutes of time. However, for the ensuing three hours the traces fluctuated very mildly until $1^h\ 20^m$, September 11, when there was a sudden impulse in all three elements, the amplitude being practically equal to that described above but the movement occupying about 30 seconds of time. Thence onward the traces showed irregular fluctuations, rapid but generally of only moderate amplitude. Between $13^h\ 10^m$ and $14^h\ 20^m$, September 11, there was a remarkable westerly swing in the declination; the highest value, 19′ above the normal, occurred at $13^h\ 30^m$. A similar swing occurred in vertical intensity, but the horizontal-intensity trace showed nothing outstanding. Irregular flucuations ensued until 8^h, September 12, after which normal conditions prevailed. The ranges recorded were: Declination, 20′.2; horizontal intensity, 95 gammas; vertical intensity, 160 gammas.

W. C. PARKINSON, *Observer-in-Charge*

LIST OF RECENT PUBLICATIONS

BY H. D. HARRADON

A—Terrestrial and Cosmical Magnetism

AMERICAN GEOPHYSICAL UNION. Transactions of the American Geophysical Union. Sixteenth annual meeting, April 25 and 26, 1935, Washington, D. C. Washington, D. C., Nation. Res. Council, August, 1935 (530 with illus.). 25 cm. [These transactions are published in two parts. Part I contains the introduction, and reports and papers presented at the General Assembly and sections of Geodesy, Seismology, Meteorology, Terrestrial Magnetism and Electricity, Oceanography, and Volcanology. Part II contains the reports and papers presented at the meetings of the Section of Hydrology.]

APPLETON, E. V., AND L. J. INGRAM. Magnetic storms and upper-atmospheric ionization. Nature, London, v. 136, Oct. 5, 1935 (548-549).

BRUNNER, W. Abhängigkeit der Korrelation zwischen Sonnenfleckenzahlen und erdmagnetischen Aktivitätszahlen innerhalb der 11-jährigen Periode. Astr. Mitt., Zürich, Nr. 133, 1935 (131-136).

CHAPMAN, S. Terrestrial magnetism and the Earth's interior. Observatory, London, v. 58, No. 735, 1935 (249-250). [Brief letter calling attention to the importance of terrestrial magnetism in furnishing information regarding the physical state of the Earth's interior.]

DE BILT, INSTITUT MÉTÉOROLOGIQUE DES PAYS-BAS. Caractère magnétique numérique des jours. Tome XIII. Octobre-décembre 1934, suppléments 1933, 1934, errata 1931-1934. De Bilt, 1935, 40 pp. 24 cm. [Published under the auspices of the Association of Terrestrial Magnetism and Electricity of the International Union of Geodesy and Geophysics.]

Caractère magnétique numérique des jours. Tome XIV. Janvier-mars 1935. De Bilt, 1935, 20 pp. 24 cm. [Published under the auspices of the Association of Terrestrial Magnetism and Electricity of the International Union of Geodesy and Geophysics.]

Der magnetische Charakter des Jahres 1934. Der numerische magnetische Charakter der Tage 1934. Met. Zs., Braunschweig, Bd. 52, Heft 9, 1935 (339).

EBLÉ, L. Observations magnétiques faites au Val-Joyeux pendant l'année 1933. Paris, Ann. Inst. Phys. Globe, T. 13, 1935 (1-28).

EGYPT, PHYSICAL DEPARTMENT. Meteorological report for the year 1932. Cairo, Ministry Pub. Works, Physical Dept., 1935 (xiv+221). [Contains values of the magnetic elements at Helwan Observatory for 1932.]

EICKELBERG, E. W. Need for more detailed magnetic observations shown by increased application of results. Trans. Amer. Geophys. Union, 16th Annual Meeting, Pt. 1, Washington, D. C., 1935 (168-171).

FLEMING, J. A. Magnetic and electric investigations of the Department of Terrestrial Magnetism of the Carnegie Institution of Washington, April 1934 to March 1935. Trans. Amer. Geophys. Union, 16th Annual Meeting, Pt. 1, Washington, D. C., 1935 (195-197)

GALLO, J. Report of the magnetic work at the Astronomical Observatory of Tacubaya during the year 1934. Trans. Amer. Geophys. Union, 16th Annual Meeting, Pt. 1, Washington, D. C., 1935 (189-191).

GREENLEE, H. R. Researches related to terrestrial magnetism and electricity of the United States Naval Research Laboratory. Trans. Amer. Geophys. Union, 16th Annual Meeting, Pt. 1, Washington, D. C., 1935 (193).

IMMLER, W. Der Magnetkompass beim Kurvenflug. Ann. Hydrogr., Berlin, Jahrg. 63, Heft 6, 1935 (241-244).

INDIA, SURVEY OF. Geodetic report 1934. Published by order of Brigadier H. J. Couchman, Surveyor General of India. Dehra Dun, Geod. Branch Office, 1935 (vii+143 with 21 charts and 10 pls.). 25 cm. [Contains a brief report on magnetic observations at Dehra Dun for 1933, with tables giving the mean monthly values of the magnetic elements for 1932 and 1933 with annual changes for the period; the mean hourly deviations from the monthly means; classification of the magnetic character of all days of 1933.]

JOHNSON, W. R., Jr., AND H. W. STRALEY, III. An attempt to locate the boundaries of the Durham Triassic Basin with a magnetometer. Trans. Amer. Geophys. Union, 16th Annual Meeting, Pt. 1, Washington, D. C., 1935 (176-181).

KALINOWSKI, ST. Wyniki spostrzeżeń magnetycznych w Swidrze. (Résultats des observations magnétiques à Swider) 1930-1933. Warszawa, Trav. Obs. Mag. Swider, No. 6, 1935 (xv+156). 28 cm.

KOENIGSBERGER, J. G. Residual magnetism and the measurement of geologic time. Preprint, Rep. XVI Internat. Geol. Cong., Washington, 1933, 7 pp. 27 cm. [Issued May 1935.]

Die Abhängigkeit der natürlichen remanenten Magnetisierung bei Eruptivgesteinen von deren Alter und Zusammensetzung. Beitr. angew. Geophysik, Leipzig, Bd. 5, Heft 2, 1935 (193-246).

LONDON, METEOROLOGICAL OFFICE. Annual report of the Director of the Meteorological Office presented by the Meteorological Committee to the Air Council for the year ended March 31, 1935. London, H. M. Stationery Office, 1935 (47). 24 cm. [Contains brief reports on Kew, Eskdalemuir, Aberdeen, Lerwick, and Valentia observatories.]

NICHOLSON, S. B. Researches at Mount Wilson Observatory of the Carnegie Institution of Washington relating to terrestrial magnetism. Trans. Amer. Geophys. Union, 16th Annual Meeting, Pt. 1, Washington, D. C., 1935 (191-192).

PARIS, BUREAU DES LONGITUDES. Annuaire pour l'an 1935 avec des notices scientifiques. Paris, Gauthier-Villars (vii+558+A.24+B.22+C.12+D.4+E.4+F.48). 20 cm. [Contains isogonic charts of France for epoch January 1, 1931, and tables of magnetic declination at various stations in France reduced to the same epoch. A table of mean annual values for observatories in all parts of the world is also given. "L'année polaire 1932-1933. Historique et organisation matérielle" by J. Charcot, and "Organisation générale et travaux scientifiques" by Ch. Maurain, appear among the Scientific Notices.]

PARIS, INSTITUT DE PHYSIQUE DU GLOBE. Annales de l'Institut de Physique du Globe de l'Université de Paris et du Bureau Central de Magnétisme Terrestre. Publiées par les soins de Ch. Maurain. Tome XIII. Paris, Les Presses Universitaires de France, 1935 (iii+142 avec 2 pages de graphiques). 31 cm.

PARIS, SERVICE GÉOGRAPHIQUE DE L'ARMÉE. Détermination d'éléments magnétiques au Maroc par le Service Géographique de l'Armée. Paris, Ann. Inst. Phys. Globe, T. 13, 1935 (60-62).

PATTON, R. S. Magnetic work of the United States Coast and Geodetic Survey, April 1934 to March 1935. Trans. Amer. Geophys. Union, 16th Annual Meeting, Pt. 1, Washington, D. C., 1935 (192-193).

POISSON, CH., J. COZE, ET H. GRUYELLE. Observations magnétiques faites à l'Observatoire de Tananarive (Madagascar) pendant l'année 1933. Paris, Ann. Inst. Phys. Globe, T. 13, 1935 (63-65).

PUSCHKOW, N. W. The new theories of terrestrial magnetism. Repr. Met. Vest., Leningrad, No. 3/4, 1935 (23-29). [Text in Russian.]

ROMAN, I., AND T. C. SERMON. Some magnetic gradiometer developments during 1934-35. Trans. Amer. Geophys. Union, 16th Annual Meeting, Pt. 1, Washington, D. C., 1935 (181-188).

ROSE, N. Observations variométriques de la station à Hornsund. Leningrad, Librairie Glavsevmorpout, 1935, 54 pp. 35 cm. [Institut Arctique de l'U. R. S. S. Matériaux pour l'exploration de l'arctique, No. 7. Missions scientifiques pour la mesure d'un arc de méridien au Spitzberg entreprises en 1899-1900. Texte russe et français.]

ROTHÉ, J. P. Observations magnétiques au Scoresby Sund pendant l'Année Polaire 1932-1933. Paris, Ann. Inst. Phys. Globe, T. 13, 1935 (99-140).

SAVORNIN, A. Nouvelles mesures de déclinaison dans le sud de Madagascar. Paris, Ann. Inst. Phys. Globe, T. 13, 1935 (66-70)

STEWART, R. M. Magnetic work of the Dominion Observatory, Canada. Trans. Amer. Geophys. Union, 16th Annual Meeting, Pt. 1, Washington, D. C., 1935 (189).

Tabesse, E. Observations magnétiques faites à l'Observatoire de Nantes pendant l'année 1933. Paris, Ann. Inst. Phys. Globe, T. 13, 1935 (29-38).

Toperczer, M. Zur Methodik magnetischer Landesaufnahmen. Beitr. Geophysik, Leipzig, Bd. 45, Heft 1/2, 1935 (184-188)

Tromsö, Auroral Observatory. Results of magnetic observations for the year 1934. By L. Harang and E. Tönsberg. Bergen, Pub. Inst. Kosmisk Fysikk, No. 7, 1935 (31). 31 cm.

United States Coast and Geodetic Survey. Magnetic declination in Florida 1935. Washington, D. C., Dept. Comm., U. S. Coast Geod. Surv., Serial 564, 1935 (37 with 1 map). 23 cm.

Magnetic declination in South Carolina in 1935. Washington, D. C., Dept. Comm., U. S. Coast Geod. Surv., 1935 (47 with 1 isogonic map). 27 cm. [Mimeographed copy.]

Magnetic declination in Georgia in 1935. Washington, D. C., Dept. Comm., U. S. Coast Geod. Surv., 1935 (64 with 1 isogonic map). 27 cm. [Mimeographed copy.]

Magnetic declination in Alabama in 1935. Washington, D. C., Dept. Comm., U. S. Coast Geod. Surv., 1935 (46 with 1 isogonic map). 27 cm. [Mimeographed copy.]

Magnetic declination in Mississippi in 1935. Washington, D. C., Dept. Comm., U. S. Coast Geod. Surv., 1935 (33 with 1 isogonic map). 27 cm. [Mimeographed copy.]

Zi-ka-wei, Observatoire de. Observations magnétiques faites aux deux stations: Lu-kia-pang, Zô-sè. Tome XVIII. Années 1932-1933. Zi-ka-wei—Chang-hai, Imprimerie de la Mission Catholique, 1935 (42+92 avec illus.). 31 cm. [Contains description of the new magnetic station at Zô-sè.]

Observations magnétiques faites à la station de Zô-sè. Tome XIX, Année 1934. Zi-ka-wei—Chang-hai, Imprimerie de la Mission Catholique, 1935 (36 avec 2 graphiques). 31 cm.

B—Terrestrial and Cosmical Electricity

Alexander, W. H. The distribution of thunderstorms in the United States, 1904-1933. Mon. Weath. Rev., Washington, D. C., v. 63, No. 5, 1935 (157-158 with 13 charts).

Ambronn, R. Elektrische Hilfsmittel in der geophysikalischen Bodenforschung. Elektrot. Zs., Berlin, Bd. 56, Nr. 21, 1935 (581-585).

Belluigi, A. Misure di resistività e porosità elettrica di terreni lungo i fori di sonda non tubati. Boll. Com. Geod. Geof., Consiglio Naz. Ricerche, Milano, Ser. 2, Anno 5, No. 3, 1935 (180-189). [L'A. propone un nuovo tipo di apparecchio, che descrive accuratamente, per permettere misure esatte di resistività e porosità lungo i fori di sonda non tubati, riconosciuta l'insufficienza degli altri prima descritti.]

Berkner, L. V. The ionization of the Earth's upper atmosphere. Trans. Amer. Geophys. Union, 16th Annual Meeting, Pt. 1, Washington, D. C., 1935 (20-26).

Brown, G. S. Cosmic ray burst phenomena. Sci. Mon., New York, N. Y., v. 41, No. 4, 1935 (347-349).

Brown, J. G. Research in atmospheric electricity at Stanford University. Trans. Amer. Geophys. Union, 16th Annual Meeting, Pt. 1, Washington, D. C., 1935 (191).

Bruins, E. M. Zur komischen Korpuskularstrahlung im erdmagnetischen Felde. Physica. The Hague, v. 2, No. 8, 1935 (879-891).

Clay, J. Results of the Dutch Cosmic Ray Expedition 1933. VII. Positive and negative primaries. North-south asymmetry. Difference of decrease in lead at different latitudes. Physica, The Hague, v. 2, No. 8, 1935 (861-869).

The nature of cosmic rays. London, Proc. R. Soc., A. v. 151, No. 872, 1935 (202-210).

Compton, A. H. Cosmic rays. Sci. Amer., New York, N. Y., v. 153, No. 3, 1935 (133).

DANFORTH, W. E. Analysis of cosmic ray deflection experiments. II. Philadelphia, Pa., J. Frank. Inst., v. 220, No. 3, 1935 (377-386).

DAUVILLIER, A. Etude du champ électrique terrestre, de l'ionisation atmosphérique et du courant vertical au Scoresby Sund, pendant l'Année Polaire. Résumé; J. Physique et Le Radium, Paris, T. 6, No. 7, 1935 (103 S.—104 S.).

DÉJARDIN, G. A propos de la présence du nouveau système de bandes de l'azote, dites de Vegard-Kaplan, dans les spectres du ciel nocturne et de l'auréole boréale. Résumé, J. Physique et Le Radium, Paris, T. 6, No. 7, 1935 (110 S.-111 S.).

DOAN, R. L. Increased frequency of cosmic-ray bursts with reduced shielding. Phys. Rev., Lancaster, Pa., v. 48, No. 5, 1935 (470-471).

ENNIS, C. C. Correlations of auroral and magnetic activity at Little America, First Byrd Antarctic Expedition. Trans. Amer. Geophys. Union, 16th Annual Meeting, Pt. 1, Washington, D. C., 1935 (165-168).

EVANS, R. D., AND R. W. RAITT. The radioactivity of the Earth's crust and its influence on cosmic-ray electroscope observations made near the ground. Phys. Rev., Lancaster, Pa., v. 48, No. 3, 1935 (171-176).

FARMER, F. T., AND J. A. RATCLIFFE. Measurements of the absorption of wireless waves in the ionosphere. London, Proc. R. Soc., A, v. 151, No. 873, 1935 (370-383).

FULLER, V. R. Auroral and ionospheric station at the Alaska Agricultural College and School of Mines. Trans. Amer. Geophys. Union, 16th Annual Meeting, Pt. 1, Washington, D. C., 1935 (193-194).

GEIGER, H. Die Sekundäreffekte der kosmischen Ultrastrahlung. Ergeb. exakt. Naturwiss., Berlin, Bd. 14, 1935 (42-78).

GOTT, J. P. On the electric charge collected by water-drops falling through a cloud of electrically charged particles in a vertical electric field. London, Proc. R. Soc., A, v. 151, No. 874, 1935 (665-684).

GUNN, R. The electricity of rain and thunderstorms. Abstract: Trans. Amer. Geophys. Union, 16th Annual Meeting, Pt. 1, Washington, D. C., 1935 (175-176). [This paper is published in full in Terr. Mag., v. 40, 1935 (79-106).]

HARRINGTON, J. The cosmic ray puzzle. Sci. Amer., New York, N. Y., v. 153, No. 3, 1935 (131-132).

ISRAËL-KÖHLER, H. Luftelektrische Messungen in Leiden (Holland). Bioklim. Beibl., Braunschweig, Jahrg. 2, Heft 3, 1935 (129-133).

JOHNSON, T. H. Contribution of cosmic radiation to the ionization of the upper atmosphere. Trans. Amer. Geophys. Union, 16th Annual Meeting, Pt. 1, Washington, D. C., 1935 (35-37).
Earth-magnetic analysis of the primary cosmic radiation. Trans. Amer. Geophys. Union, 16th Annual Meeting, Pt. 1, Washington, D. C., 1935 (171-175).
Progress of the directional survey of cosmic-ray intensities and its application to the analysis of the primary cosmic radiation. Phys. Rev., Lancaster, Pa., v. 48, No. 4, 1935 (287-299).

JOYCE, J. W. Induction prospecting for shallow ore-deposits and small metallic objects. Washington, D. C., Dept. Int., U. S. Bur. Mines, Inf. Cir., 6854 (18 with 22 figs.) 27 cm.

MCGREGOR, C. J. Auroral observations at Point Barrow, Alaska, during the International Polar Year. Trans. Amer. Geophys. Union, 16th Annual Meeting, Pt. 1, Washington, D. C., 1935 (147-151).

MATHIAS, E. La théorie de Dauzère sur la conductibilité de l'air dans les régions exposées à la foudre. Paris, C.-R. Acad. sci., T. 201, No. 5, 1935 (314-317).

NARKIEWICZ-JODKO, K., AND S. ZIEMECKI. Cosmic-ray observations on the Atlantic Ocean. Acta Phys. Polon., Warszawa, v. 3, 1934 (297-300). [English text with Polish summary.]

NEWMAN, F. H., AND H. J. WALKE. Note on cosmic-ray ions and the shower-producing radiation. Phil. Mag., London, v. 20, No. 132, 1935 (263-266).

Petrucci, G. Sulla prospezione elettrica di più strati conduttori. Boll. Com. Geod. Geof., Consiglio Naz. Ricerche, Milano, Ser. 2, Anno 5, No. 3, 1935 (190-203). [Si riportano i diagrammi di resistenza eseguiti, su alcuni *modelli* sperimentali col metodo di misura diretto della resistenza con corrente alternata. Si mostra la possibilità della prospezione di diversi casi geologici ed in particolare della prospezione di più strati conduttori.]

Priebsch, J. A. Versuche über die Sekundäreffekte der kosmischen Ultrastrahlung auf dem Hafelekar (2300 m ü. d. M.). Zs. Physik, Berlin, Bd. 95, Heft 1/2, 1935 (102-114).

Reich, H. Beobachtung über die Wirkung eines Blitzeinschlages auf ein Magnetometer. Beitr. angew. Geophysik, Leipzig, Bd. 5, Heft 2, 1935 (247-250).

Rooney, W. J. Seasonal variation in earth-currents at Tucson, Arizona. Abstract: Trans. Amer. Geophys. Union, 16th Annual Meeting, Pt. 1, Washington, D. C., 1935 (188). [This paper is published in full in Terr. Mag., v. 40, 1935 (183-192).]

Salles, Ed. Observations du champ électrique au Val-Joyeux pendant l'année 1933. Paris, Ann. Inst. Phys. Globe, T. 13, 1935 (39-46).

Scherhag, R. Untersuchungen über die Nachtgewitter im nordwestdeutschen Küstengebiet. Ann. Hydrogr., Berlin, Jahrg. 63, Heft 8, 1935 (318-319).

Schwegler, A. Sekundäreffekte der kosmischen Primärstrahlung. Zs. Physik, Berlin, Bd. 96, Heft 1/2, 1935 (62-75).

Scrase, F. J. The sampling errors of the Aitken nucleus-counter. London, Q. J. R. Met. Soc., v. 61, No. 261, 1935 (367-379).
Some measurements of the variation of potential gradient with height near the ground at Kew Observatory. London, Met. Office, Geophys. Mem., No. 67, 1935 (11 with 2 pls.).

Shepard, E. R. Subsurface exploration by earth-resistivity and seismic methods. Trans. Amer. Geophys. Union, 16th Annual Meeting, Pt. 1, Washington, D. C., 1935 (78-91).

Smith-Rose, R. L. The electrical properties of soil at frequencies up to 100 megacycles per second; with a note on the resistivity of ground in the United Kingdom. London, Proc. Phys. Soc., v. 47, No. 262, 1935 (923-931).

Soru, E., und N. Wolokewitsch. Ueber Unipolarisierung der künstlichen Luftbeladung in Räumen. Bioklim. Beibl., Braunschweig, Jahrg. 2, Heft 3, 1935 (124-129).

Southworth, G. C. Earth-potential measurements made during the International Polar Year. Abstract: Trans. Amer. Geophys. Union, 16th Annual Meeting, Pt. 1, Washington, D. C., 1935 (188). [Terr. Mag., **40**, 237-254 (1935).]

Stefenescu, S. S. Sur la mesure des résistivités apparentes par la méthode de la spire circulaire. Beitr. angew. Geophysik, Leipzig, Bd. 5, Heft 2, 1935 (182-192). [In accordance with the general formulas for variable fields, the exact expression of the vertical component of the magnetic field in the center of a circular current situated on the plane surface of homogeneous soil is determined. A formula is deduced for the apparent earth resistivity of a region with horizontal strata. Two practical examples are studied with the aid of that formula.]

Steinmaurer, R. Messungen der kosmischen Ultrastrahlung in Innsbruck (590 m) und auf dem Hafelekar (2300 m) im Jahre 1934. Beitr. Geophysik, Leipzig, Bd. 45, Heft 1/2, 1935 (148-183).

Stevenson, E. C., and J. C. Street. Cosmic-ray showers produced by electrons. Phys. Rev., Lancaster, Pa., v. 48, No. 5, 1935 (464-465).

Swann, W. F. G. Conservation of direction in the impact of high-energy particles. Philadelphia, Pa., J. Frank. Inst., v. 220, No. 3, 1935 (373-376).
The corpuscular theory of the primary cosmic radiation. Phys. Rev., Lancaster, Pa., v. 48, No. 8, 1935 (641-648).

Swann, W. F. G., and D. B. Cowie. The effect of primary cosmic-ray energy upon burst production. Phys. Rev., Lancaster, Pa., v. 48, No. 8, 1935 (649-652).

Thellier, O. Noyaux de condensation et particules en suspension dans l'atmosphère. Paris, C.-R. Acad. sci., T. 201, No. 5, 1935 (348-350).

VEGARD, L. The aurora and night-sky spectra and the upper atmosphere—report on recent work. Trans. Amer. Geophys. Union, 16th Annual Meeting, Pt. 1, Washington, D. C., 1935 (38-40).

WAIT, G. R. The intermediate ion of the atmosphere. Phys. Rev., Lancaster, Pa., v. 48, No. 4, 1935 (383).

ZEILLER, O. Das Durchdringungsvermögen der kosmischen Sekundärstrahlen. Zs. Physik, Berlin, Bd. 96, Heft 1/2, 1935 (121-136).

C—*Miscellaneous*

AÏTOFF, MLLE M. Sur les variations de la répartition spectrale de l'énergie solaire. Paris, Ann. Inst. Phys. Globe, T. 13, 1935 (92-98).

ANGENHEISTER, G. Geophysical explorations in the uppermost layer of the Earth's crust and their practical importance. Research and Progress, Berlin, v. 1, No. 3, 1935 (126-131).

BELLUIGI, A. The need of revising the apparatus employed in the measurement of an electrochemical effect of the ground. Beitr. angew. Geophysik, Leipzig, Bd. 5, Heft 2, 1935 (169-177).

BRAZIER, C. E. Résumé des observations et travaux actinométriques effectués à l'Observatoire du Parc Saint-Maur en 1933. Paris, Ann. Inst. Phys. Globe, T. 13, 1935 (51-59).

BRAZIER, C. E., ET L. GÉNAUX. Résumé des observations séismologiques faites à l'Observatoire du Parc Saint-Maur en 1933. Paris. Ann. Inst. Phys. Globe, T. 13, 1935 (46-47).

BRUNNER, W. Beiträge zur Photometrie des Nachthimmels unter besonderer Berücksichtigung des Zodiakallichtes und der Dämmerungserscheinungen. Zürich, Pub. Eidgen. Sternwarte Bd. 4, 1935 (v+120). 31 cm.

CHAPMAN, S. The lunar tide in the Earth's atmosphere. London, Proc. R. Soc., A, v. 151, No. 872, 1935 (105-117).

The lunar atmospheric tide over Canada, 1897 to 1932. London, Q. J. R. Met. Soc., v. 61, No. 261, 1935 (359-366).

DAVIS, W. Cosmic-data ursigrams. Trans. Amer. Geophys. Union, 16th Annual Meeting, Pt. 1, Washington, D. C., 1935 (41-49).

DELLINGER, J. H. Ionophere-researches of the National Bureau of Standards. Trans. Amer. Geophys. Union, 16th Annual Meeting, Pt. 1, Washington, D. C., 1935 (192).

A new radio transmission phenomenon. Phys. Rev., Lancaster, Pa., v. 48, No. 8, 1935 (705).

FLEMING, J. A. The sixteenth annual meeting of the American Geophysical Union. Science, New York, N. Y., v. 82, Oct. 18, 1935 (373-374).

FOWLE, F. E. Further ozone-measures and their connection with the sunspot-cycle. Trans. Amer. Geophys. Union, 16th Annual Meeting, Pt. 1, Washington, D. C., 1935 (164-165).

FUCHS, J. Der Aufbau der Stratosphäre nach elektrophysikalischen Untersuchungen. Met. Zs., Braunschweig, Bd. 52, Heft 8, 1935 (278-284).

GILLILAND, T. R. Multifrequency ionosphere recording and its significance. New York, N. Y., Proc. Inst. Radio Eng., v. 23, No. 9, 1935 (1076-1101).

HABERT. Rapport préliminaire sur les travaux de la Mission Française de l'Année Polaire 1932-1933 au Scoresby-Sund. Ann. hydrog., Paris, Ser. 3, T. 13, 1934 (153-179). [Contains brief summaries of the work in terrestrial magnetism and electricity, aurora, cosmic rays, etc.]

HOLLINGSWORTH, J. The structure of the ionosphere. London, Proc. Phys. Soc., v. 47, No. 262, 1935 (843-851).

HULBURT, E. O. Theory of the ionosphere. Abstract: Trans. Amer. Geophys. Union, 16th Annual Meeting, Pt. 1, Washington, D. C., 1935 (31). [Complete paper is published in Terr. Mag., v. 40, 1935 (193-200).]

Kosmath, W., und O. Gerke. Das radioaktive Klima und das radioaktive Milieu von Badgastein, seine bioklimatische und balneologische Bedeutung. Wien, Sitzber. Ak. Wiss, Mat,-naturw. Kl., Abt. 2a, Bd. 144, Heft 5 u. 6, 1935 (339-355).

Ladenburg, R. W. Light absorption and distribution of atmospheric ozone. J. Optical Soc., Lancaster, Pa., v. 25, No. 9, 1935 (259-269).

London, Department of Scientific and Industrial Research. The investigation of atmospheric pollution. Report on observations in the year ended 31st March 1934. Twentieth report. London, H. M. Stationery Office, 1935 (vii+108). 28 cm.

Lovera, G. L'ozono atmosferico. Boll. Com. Geod. Geof., Consiglio Naz. Ricerche, Milano, Ser. 2, Anno 5, No. 3, 1935 (153-169). [Esposti brevemente i vari metodi di misura dell' ozono atmosferico totale, della sua distribuzione nell'atmosfera, e del contenuto in ozono degli strati prossimi al suolo, si riferiscono i principali risultati sinora ottenuti.]

Löwy, H. Applied geophysics in dry regions. Beitr. angew. Geophysik, Leipzig, v. 5, No. 2, 1935 (257-259).

Maurain, Ch. Sur les caractéristiques numériques relatives aux facteurs climatiques chaleur et lumière. Paris, Ann. Inst. Phys. Globe, T. 13, 1935 (71-91).

Mildner, P., und M. Rötschke. Messungen des Staubgehaltes in der freien Atmosphäre. Met. Zs., Braunschweig, Bd. 52, Heft 9, 1935 (326-329).

Mount Wilson Observatory. Summary of Mount Wilson magnetic observations of sunspots for May and June, and July and August 1935. Pub. Astr. Soc. Pacific, San Francisco, Cal., v. 47, 1935 (233-234; 276-278).

Pettit, E. Ultra-violet solar radiation and atmospheric ozone. Trans. Amer. Geophys. Union, 16th Annual Meeting, Pt. 1, Washington, D. C., 1935 (37-38).

Rayleigh, Lord, and H. S. Jones. The light of the night sky: Analysis of the intensity variations at three stations. London, Proc. R. Soc., A, v. 151, No. 872, 1935 (22-55).

Regener, E. Erfahrungen und Ergebnisse mit Registrierballonen und Registrierapparaten in der Stratosphäre. Beitr. Physik frei. Atmos., Leipzig, Bd. 22, Heft 4, 1935 (249-260).

Reich, H. Ueber die deutsche geophysikalische Reichaufnahme. Beitr. angew. Geophysik, Leipzig, v. 5, No. 2, 1935 (143-151).

Schmid, F. Das Zodiakallicht vom 47. Breitegrad Nord bis 39. Breitegrad Süd. Beitr. Geophysik, Leipzig, Bd. 45, Heft 1/2, 1935 (5-34).

Stoyko, N., et R. Jouaust. Sur la propagation des ondes radioélectriques dans la région des aurores polaires. Paris, C.-R. Acad. sci., T. 201, No. 2, 1935 (133-134).

Wulf, O. R. The absorption of light in the atmosphere and its relation to atmospheric ozone. Abstract: Trans. Amer. Geophvs. Union, 16th Annual Meeting, Pt. 1, Washington, D. C., 1935 (20).